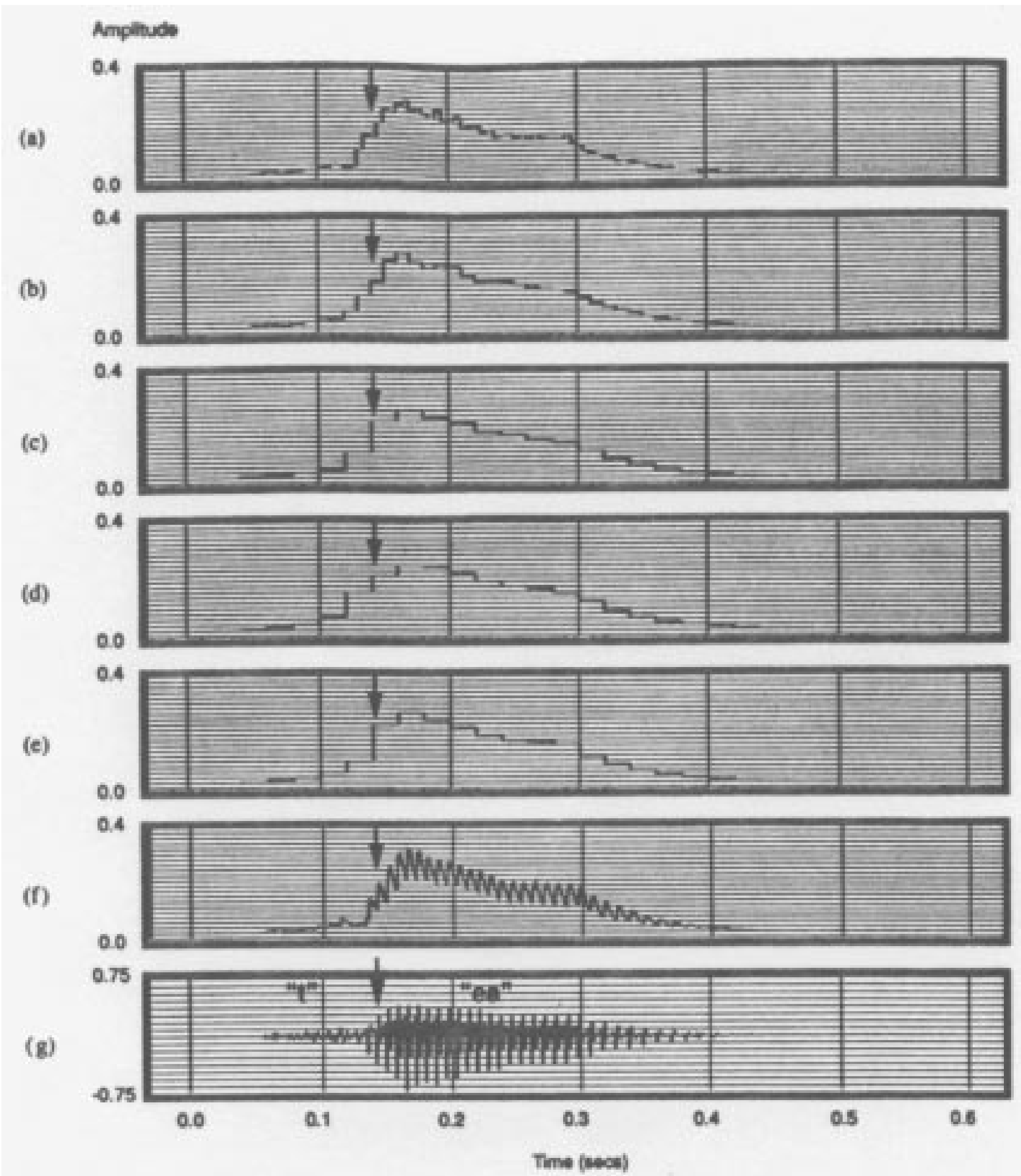
Short-Term Measurements

What is the point of this lecture?



$T_f = 5\ ms$
$T_w = 10\ ms$

$T_f = 10\ ms$
$T_w = 20\ ms$

$T_f = 20\ ms$
$T_w = 30\ ms$

$T_f = 20\ ms$
$T_w = 30\ ms$
$Hamm.\ Win.$

$T_f = 20\ ms$
$T_w = 60\ ms$
$Hamm.\ Win.$

Recursive
50 Hz LPF

Speech Signal

Time-Domain Windowing

Let $\{x(n)\}$ denote a sequence to be analyzed. Let's limit the duration of $\{x(n)\}$ to L samples:

$$\hat{x}(n) = x(n)w(n)$$

where $w(n)$ is a rectangular window and is defined as

$$w(n) = \begin{cases} 1, & 0 \le n \le L-1 \\ 0, & otherwise \end{cases}.$$

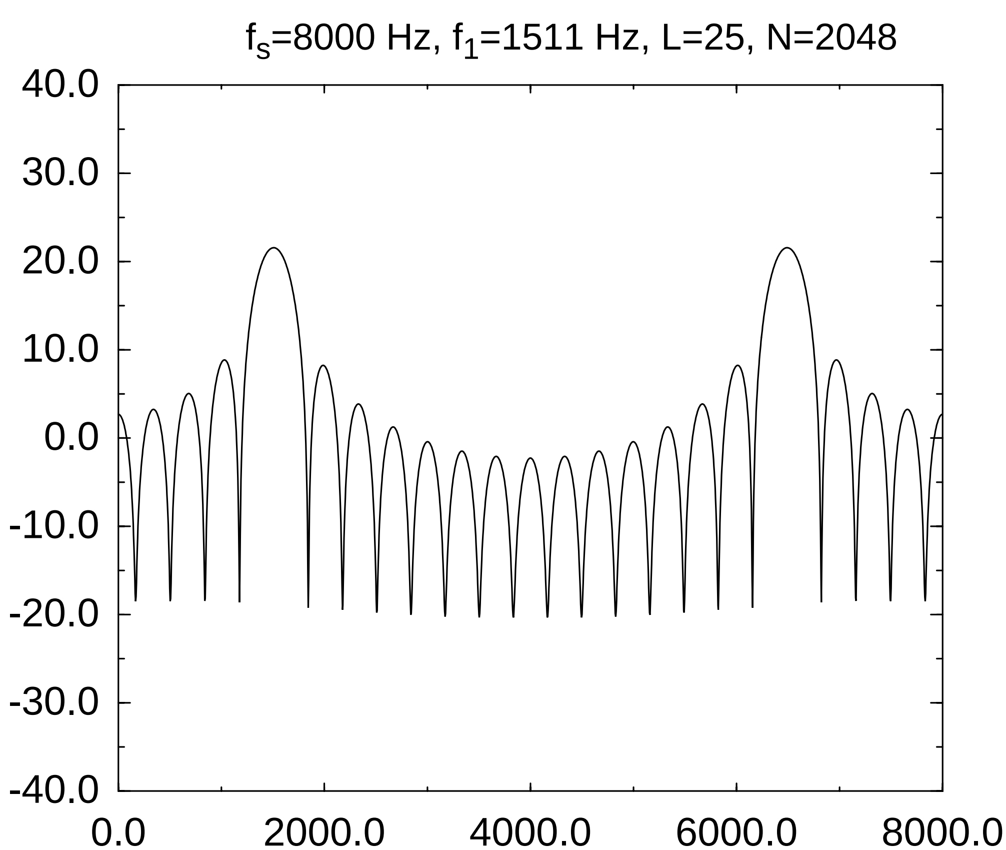
The Fourier transform of $w(n)$ is given by:

$$W(\omega) = \frac{\sin(\omega(L/2))}{\sin(\omega/2)} e^{-j\omega((L-1)/2)}$$

The transform of $\hat{x}(n)$ is given by:

$$\hat{X}(\omega) = \frac{1}{2}[W(\omega - \omega_o) + W(\omega + \omega_o)].$$

This introduces frequency domain aliasing (the so-called picket fence effect):



Time/Frequency Properties of Windows

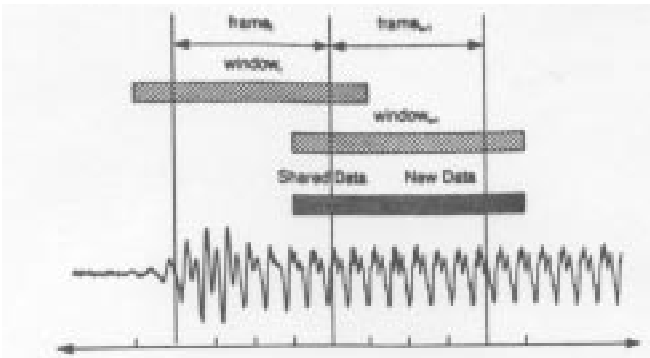
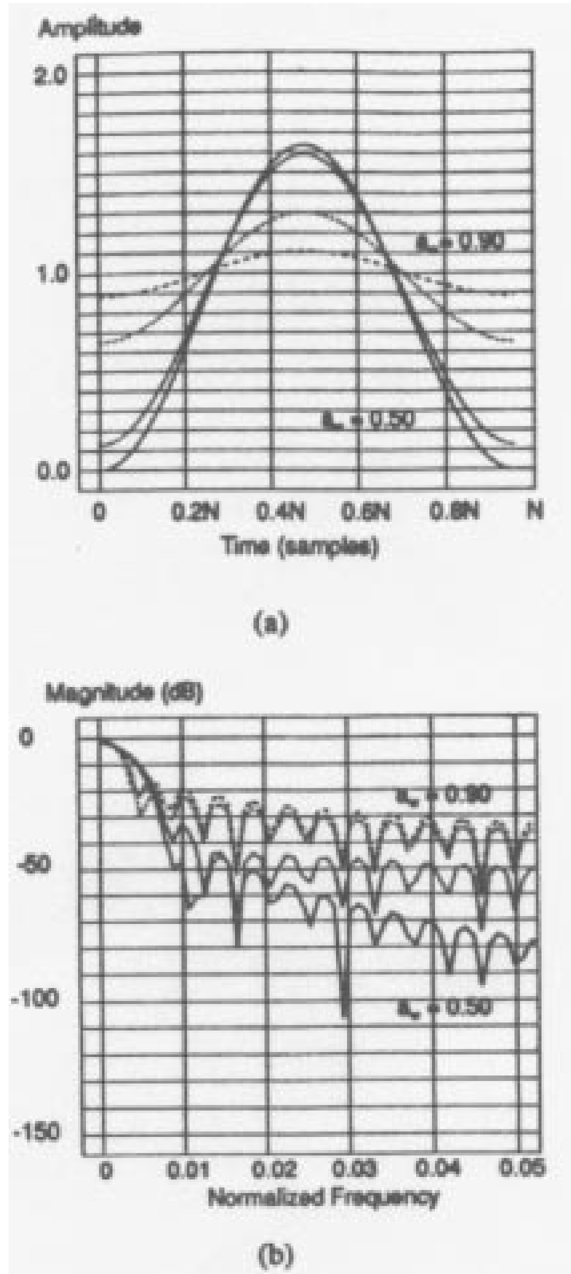


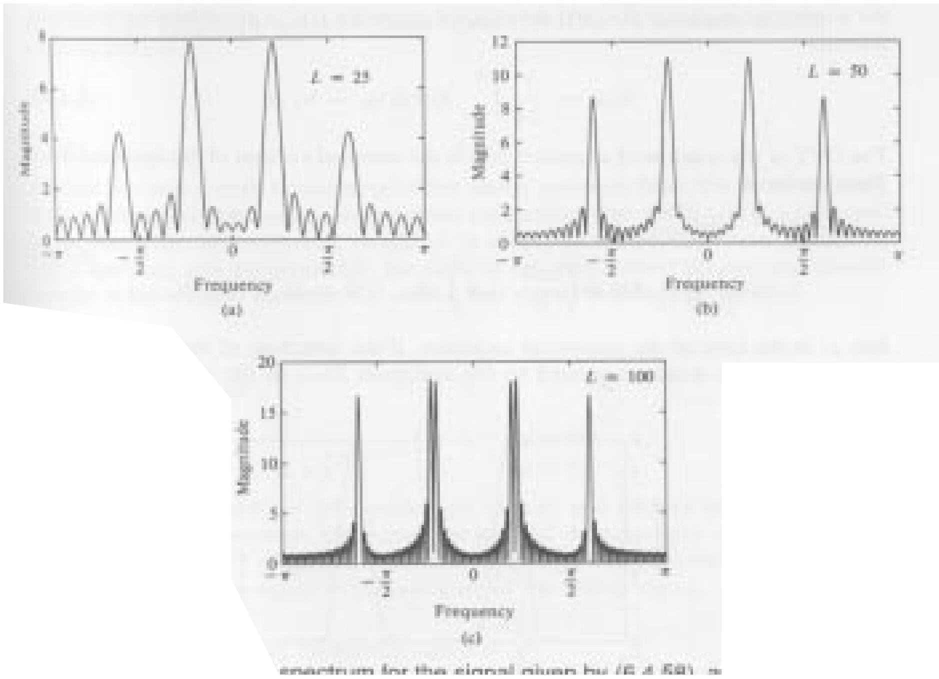
Fig. 6. A frame-based overlapping analysis is depicted. In this case, a 33% overlap is shown. One-third of the data used in each analysis frame is shared with the previous frame. Note that only one-third of the data are unique to the current frame—the remaining two-thirds are shared between adjacent frames.

$$\%Overlap = \frac{(T_w - T_f)}{T_w} \times 100\%$$

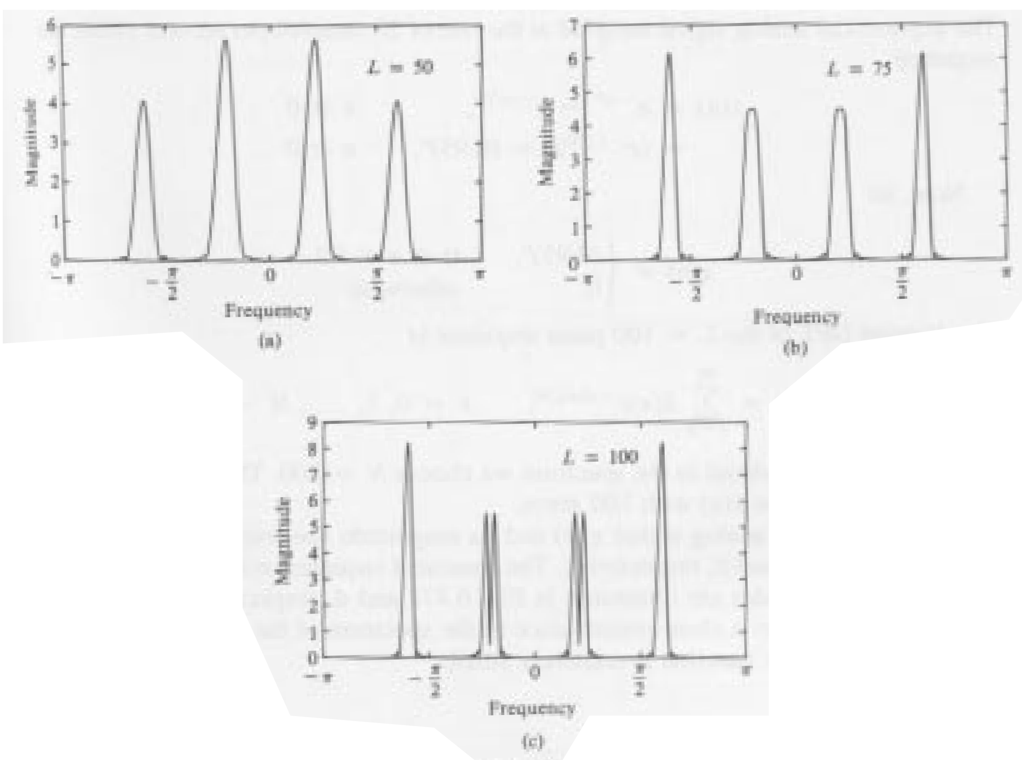


Improvements Via Better Windows

Rectangular Window:



Hanning Window:



Popular Windows

1. Rectangular:

$$w(k) = \begin{cases} 1, & |k| \le N \\ 0, & otherwise \end{cases}$$

2. Generalized Hanning:

$$w_H(k) = w(k)\left[\alpha + (1-\alpha)\cos\left(\frac{2\pi}{N}k\right)\right] \qquad 0 < \alpha < 1$$

$$\alpha = 0.54, \qquad Hamming\ window$$

$$\alpha = 0.50, \qquad Hanning\ window$$

3. Bartlett

$$w_B(k) = w(k)\left[1 - \frac{|k|}{N+1}\right]$$

4. Kaiser

$$w_K(k) = w(k) I_0\left(\alpha\sqrt{1 - \frac{K^2}{N}}\right) / I_0(\alpha)$$

5. Chebyshev:

$$w_N(k) = 2(x_0^2 - 1) w_{N-1}(k) + x_0^2 [w_{N-1}(k-1) + w_{N-1}(k+1)] - w_{N-2}(k)$$

6. Gaussian

$$w_G(k) = \begin{cases} \exp\left[-\frac{1}{2}k^2 \tan^2\left(\frac{\theta_0}{2}\right)\right] & |k| < N \\ w_G(N-1) / \left[2N\sin^2\left(\frac{\theta_0}{2}\right)\right] & |k| < N \\ 0 & |k| > N \end{cases}$$

There are many others. The most important characteristics are the width of the main lobe and the attenuation in the stop-band (height of highest sidelobe). The Hamming window is used quite extensively.

Recursive-in-Time Approaches

Define the short-term estimate of the power as:

$$P(n) = \frac{1}{N_s}\sum_{m=0}^{N_s-1}\left(w(m)s(n-\frac{N_s}{2}+m)\right)^2$$

We can view the above operation as a moving-average filter applied to the sequence $s^2(n)$.

This can be computed recursively using a linear constant-coefficient difference equation:

$$P(n) = -\sum_{i=1}^{N_a} a_{pw}(i)P(n-i) + \sum_{j=1}^{N_b} b_{pw}(j)s^2(n-j)$$

Common forms of this general equation are:

$$P(n) = \alpha P(n-1) + s^2(n)$$ (Leaky Integrator)

$$P(n) = \alpha P(n-1) + (1-\alpha)s^2(n)$$ (First-order weighted average)

$$P(n) = \alpha P(n-1) + \beta P(n-2) + s^2(n) + \gamma s^2(n-1)$$ (2^{nd}-order Integrator)

Of course, these are nothing more than various types of low-pass filters, or adaptive controllers. How do we compute the constants for these equations?

In what other applications have we seen such filters?

Relationship to Control Systems

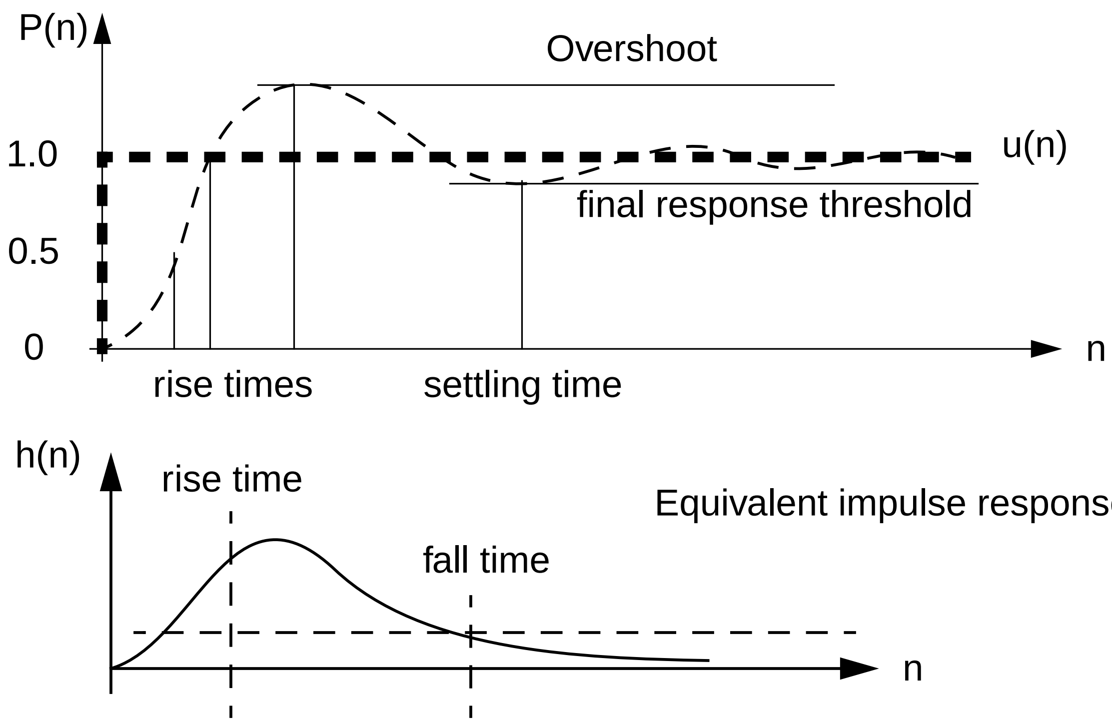
The first-order systems can be related to physical quantities by observing that the system consists of one real pole:

$$H(z) = \frac{1}{1 - \alpha z^{-1}}$$

α can be defined in terms of the bandwidth of the pole.

For second-order systems, we have a number of alternatives. Recall that a second-order system can consist of at most one zero and one pole and their complex conjugates. Classical filter design algorithms can be used to design the filter in terms of a bandwidth and an attenuation.

An alternate approach is to design the system in terms of its unit-step response:



There are many forms of such controllers (often known as servo-controllers). One very interesting family of such systems are those that correct to the velocity and acceleration of the input. All such systems can be implemented as a digital filter.