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Objectives

Overview:
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Typical Front End

Sampling:
Theorem
Derivation
Graphical
Reconstruction
Bandlimited
Aliasing
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Conditioning

On-Line Resources:
Signal Modeling
Applet
Theorem

LECTURE 08: SAMPLING

- Objectives:
  - Introduce a typical front end
  - Understand sampling issues
  - Understand the impact of aliasing
  - Appreciate the need for signal preprocessing
  - Understand frame-based processing

A good reference textbook on these topics is:

J.G. Proakis and D.G. Manolakis, *Digital Signal Processing: Principles, Algorithms, and Applications*, Prentice Hall, Upper Saddle River, New Jersey, USA, ISBN: 0-13-373762-4, 1996 (third edition).

ECE 8463: FUNDAMENTALS OF SPEECH RECOGNITION

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Modern speech understanding systems merge interdisciplinary technologies from Signal Processing, Pattern Recognition, Natural Language, and Linguistics into a unified statistical framework. These systems, which have applications in a wide range of signal processing problems, represent a revolution in Digital Signal Processing (DSP). Once a field dominated by vector-oriented processors and linear algebra-based mathematics, the current generation of DSP-based systems rely on sophisticated statistical models implemented using a complex software paradigm. Such systems are now capable of understanding continuous speech input for vocabularies of hundreds of thousands of words in operational environments.

In this course, we will explore the core components of modern statistically-based speech recognition systems. We will view speech recognition problem in terms of three tasks: signal modeling, network searching, and language understanding. We will conclude our discussion with an overview of state-of-the-art systems, and a review of available resources to support further research and technology development.

Tar files containing a compilation of all the notes are available. However, these files are large and will require a substantial amount of time to download. A tar file of the html version of the notes is available here. These were generated using wget:

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wget -np -k -m http://www.isip.msstate.edu/publications/courses/ece_8463/lectures/current
```

A pdf file containing the entire set of lecture notes is available here. These were generated using Adobe Acrobat.

Questions or comments about the material presented here can be directed to help@isip.msstate.edu.

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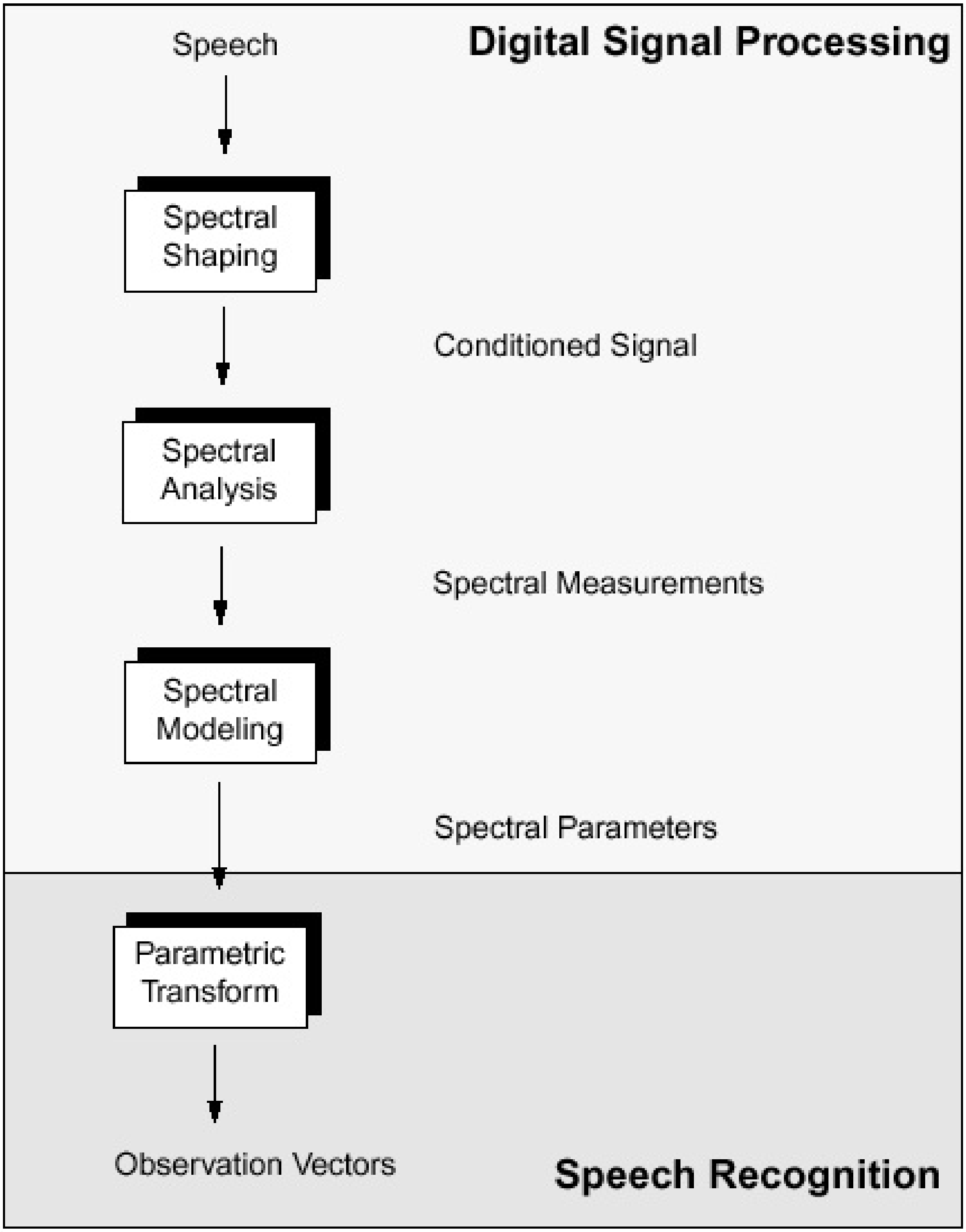
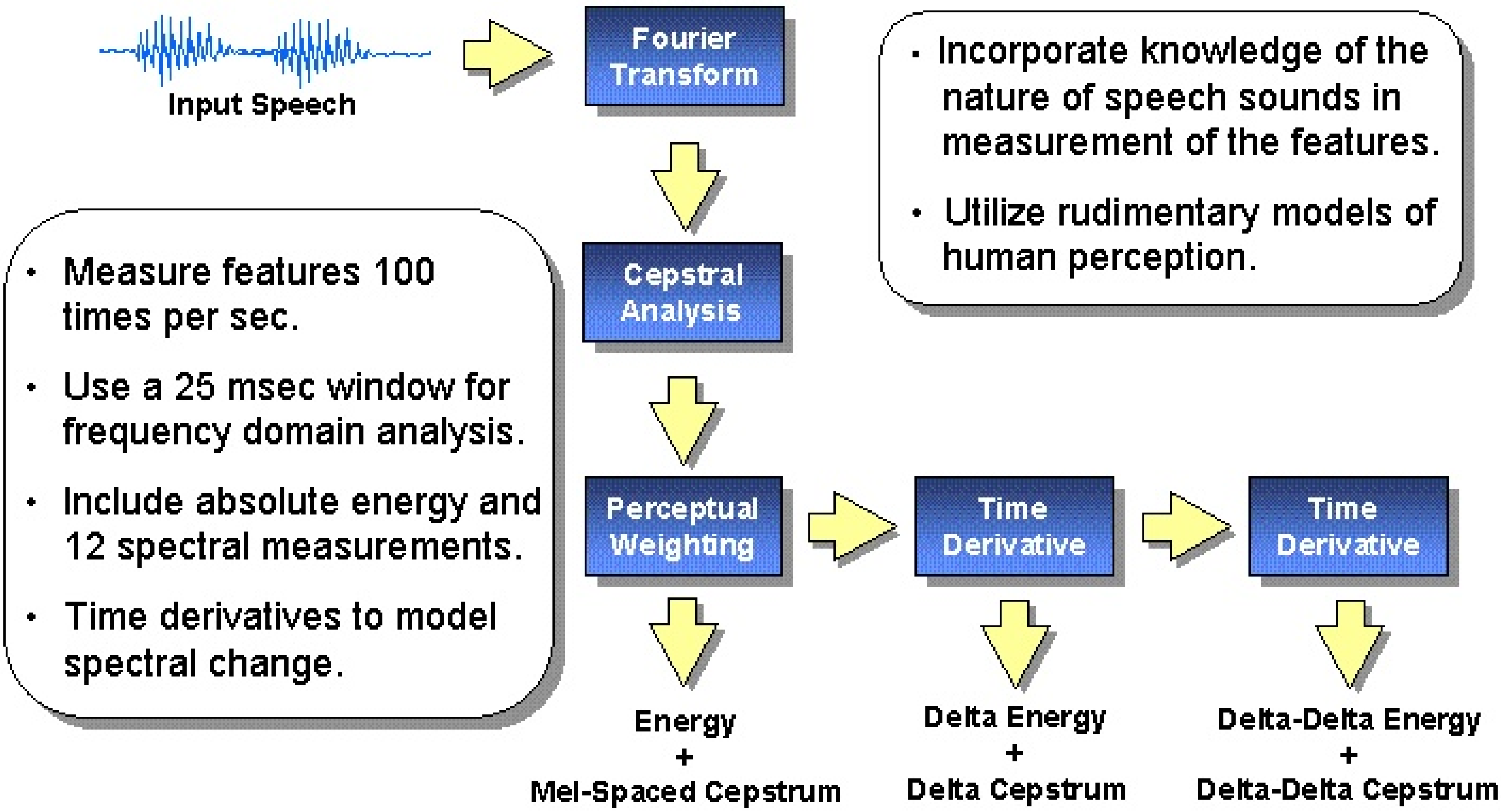
SIGNAL PROCESSING COMPONENTS
IN SPEECH RECOGNITION



A TYPICAL SPEECH RECOGNITION FRONT END



THE SAMPLING THEOREM

Theorem: If the highest frequency contained in an analog signal $x_a(t)$ is $F_{max} = B$, and the signal is sampled at a frequency $F_s > 2B$, then the analog signal can be *exactly* recovered from its samples using the following reconstruction formula:

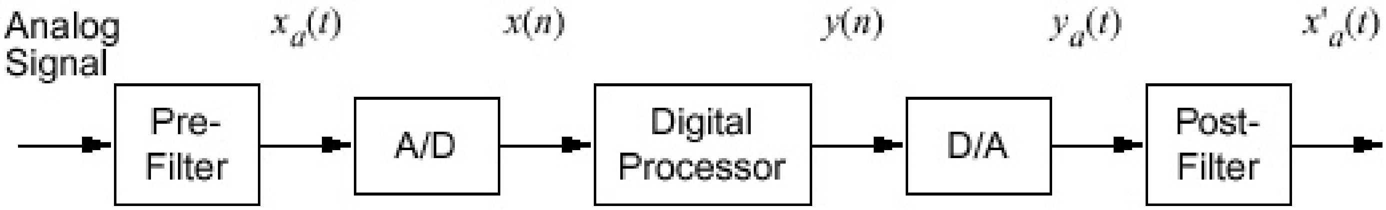
$$x_a(t) = \sum_{n=-\infty}^{\infty} x_a(nT) \frac{\sin((\pi/T)(t-nT))}{(\pi/T)(t-nT)}$$

Note that at the original sample instances (t = nT), the reconstructed analog signal is equal to the value of the original analog signal because the sinc functions take on values of zero at multiples of the sample period. At times between the sample instances, the signal is the weighted sum of shifted sinc functions.

DERIVATION OF THE SAMPLING THEOREM

Recall a discrete-time signal is given by:

$$x(n) = x_a(nT), \qquad -\infty < n < \infty$$



If $x_a(t)$ is an aperiodic signal with finite energy, its spectrum is given by:

$$X_a(F) = \int_{-\infty}^{\infty} x_a(t) e^{-j2\pi F t} dt$$

The signal can be recovered from the inverse Fourier transform:

$$x_a(t) = \int_{-\infty}^{\infty} X_a(F) e^{j2\pi F t} dF$$

The spectrum of the discrete-time signal is given by:

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

or, equivalently,

$$X(f) = \sum_{n=-\infty}^{\infty} x(n) e^{-j2\pi f n}$$

The signal can be recovered from its spectrum:

$$x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega) e^{j\omega n} d\omega$$

$$= \int_{-1/2}^{1/2} X(f) e^{j2\pi f n} df$$

Recall that $t = nT = \frac{n}{F_s}$. This allows us to write the inverse transform as:

$$x(n) \equiv x_a(nT) = \int_{-\infty}^{\infty} X_a(F) e^{j2\pi n(F/F_s)} dF$$

From this, we can conclude that

$$\int_{-1/2}^{1/2} X(f) e^{j2\pi fn} df = \int_{-\infty}^{\infty} X_a(F) e^{j2\pi n(F/F_s)} dF$$

We know that $f = \frac{F}{F_s}$. We can make a change of variables and write:

$$\frac{1}{F_s} \int_{-F_s/2}^{F_s/2} X\left(\frac{F}{F_s}\right) e^{j2\pi n(F/F_s)} df = \int_{-\infty}^{\infty} X_a(F) e^{j2\pi n(F/F_s)} dF$$

We can express the integral on the right as a sum of integrals:

$$\int_{-\infty}^{\infty} X_a(F) e^{j2\pi F/F_s} dF = \sum_{k=-\infty}^{\infty} \int_{(k-1/2)F_s}^{(k+1/2)F_s} X_a(F) e^{j2\pi n(F/F_s)} dF$$

By interchanging the order of integration and summation, and invoking the periodicity of the complex exponential, we can write:

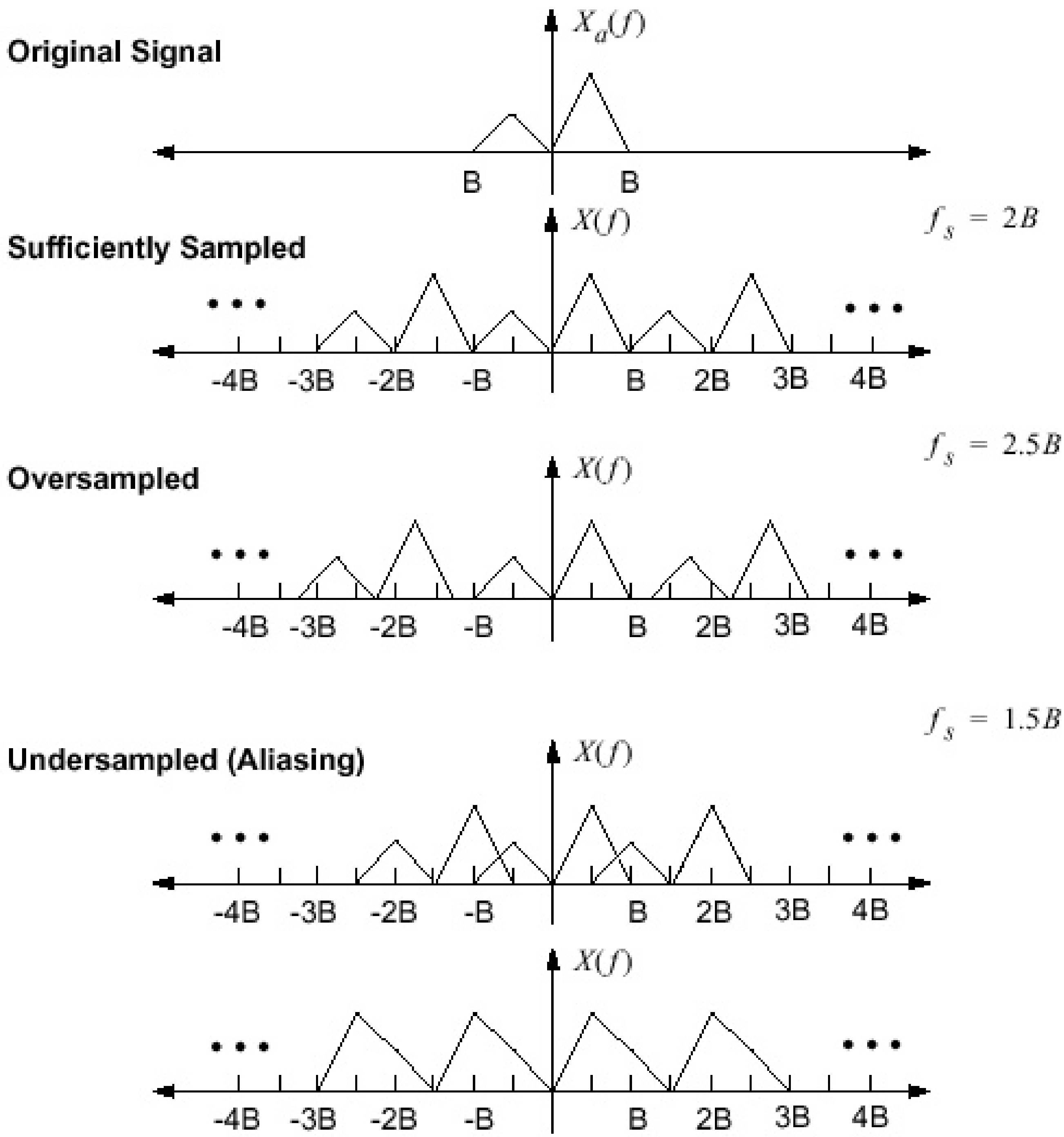
$$\frac{1}{F_s} \int_{-F_s/2}^{F_s/2} X\left(\frac{F}{F_s}\right) e^{j2\pi n(F/F_s)} df = \int_{-F_s/2}^{F_s/2} \left[\sum_{k=-\infty}^{\infty} X_a(F - kF_s) \right] e^{j2\pi n(F/F_s)} dF$$

By equating terms inside the integral, we have:

$$X\left(\frac{F}{F_s}\right) = F_s \sum_{k=-\infty}^{\infty} X_a(F - kF_s)$$

What does this imply about the spectrum of the sampled signal?

GRAPHICAL INTERPRETATION OF THE SAMPLING THEOREM

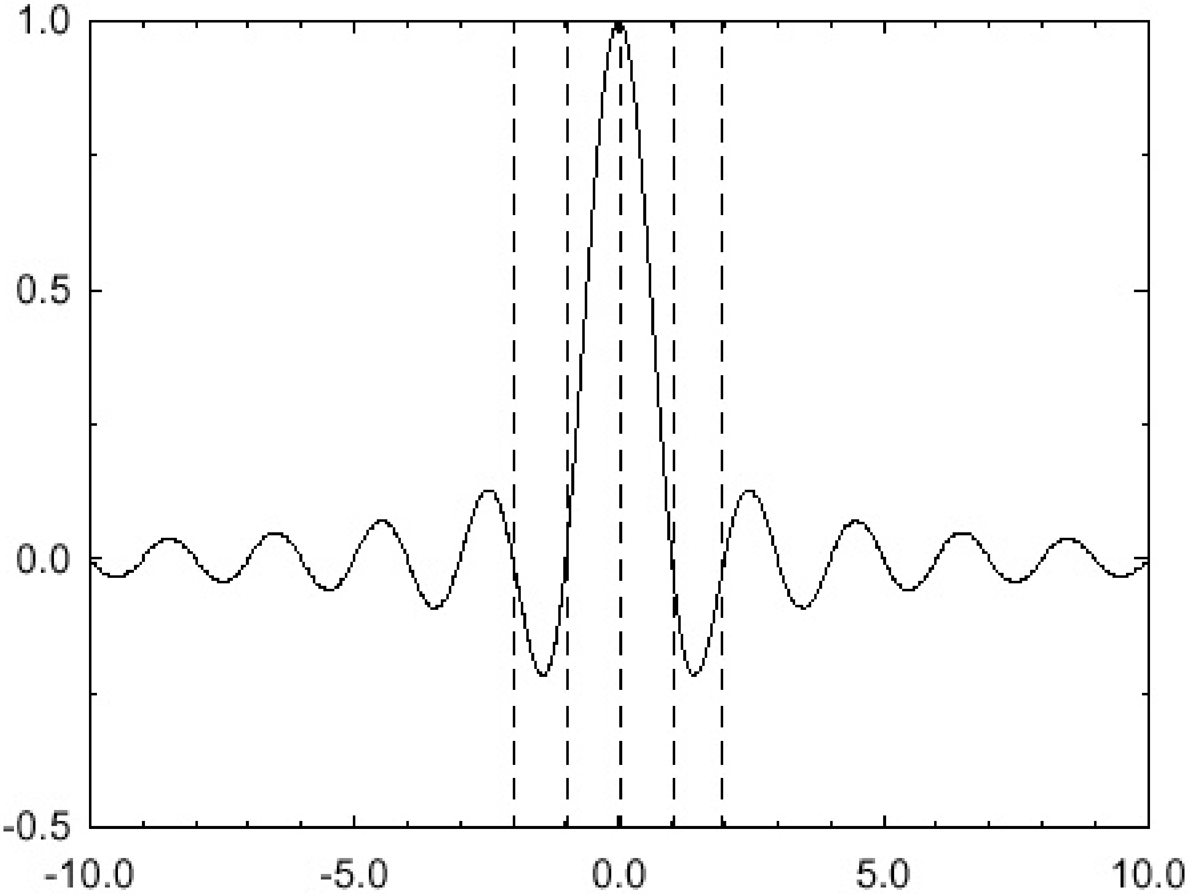


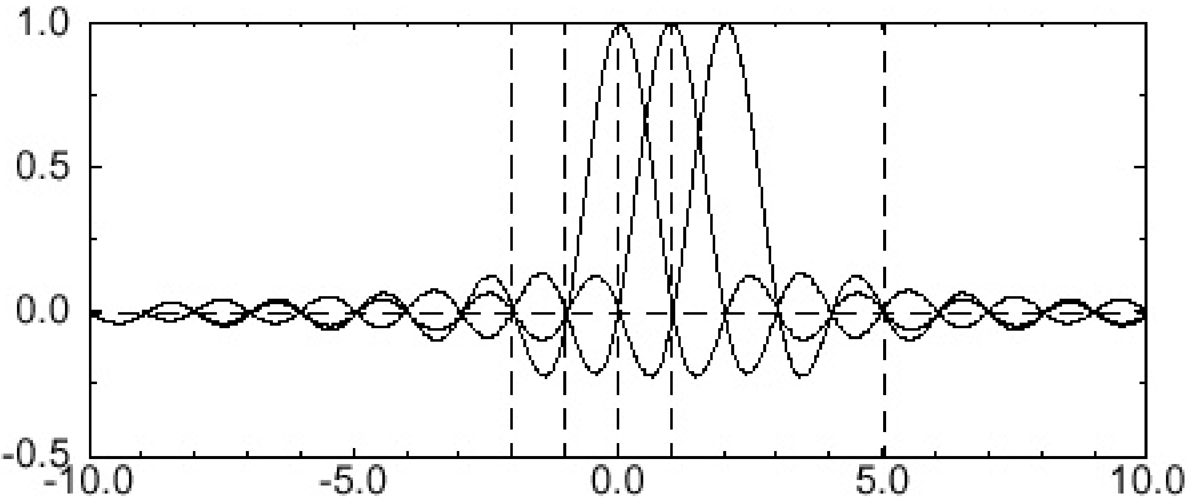
RECONSTRUCTION VIA SINC(X) INTERPOLATION

Recall our equation for reconstruction:

$$x_a(t) = \sum_{n=-\infty}^{\infty} x_a(nT) \frac{\sin((\pi/T)(t-nT))}{(\pi/T)(t-nT)}$$

This can be viewed as an interpolation process using shifted and delayed Sinc(x) functions:

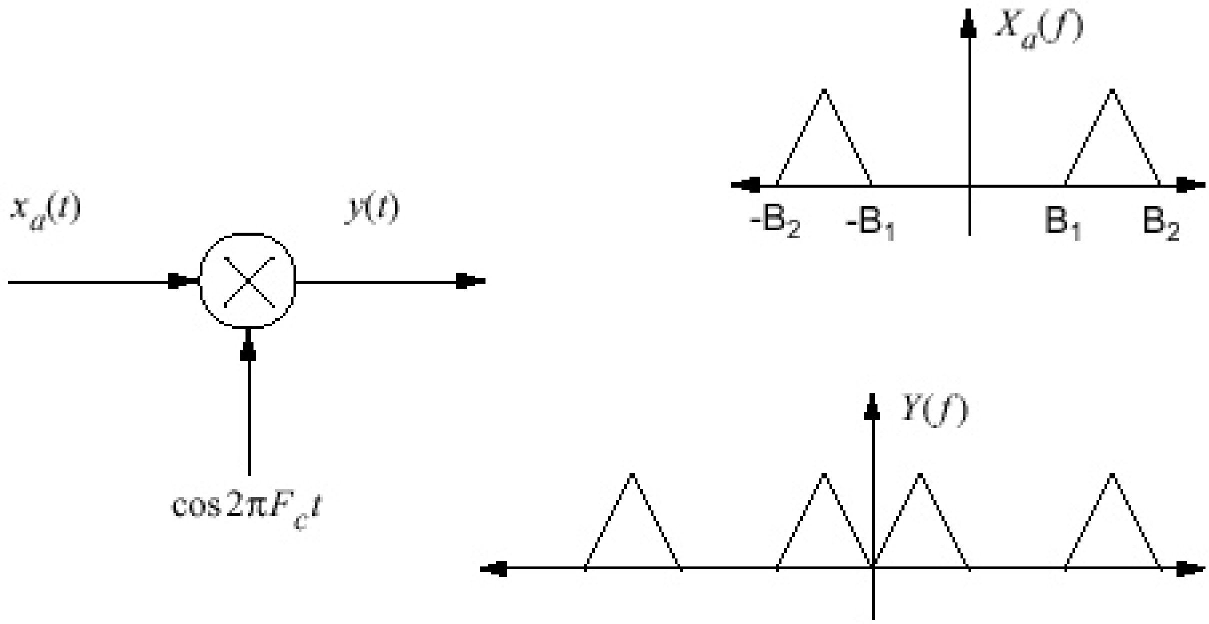




Note that these Sinc functions are exactly zero at the original sample instances.

AN INTUITIVE EXPLANATION OF THE SAMPLING THEOREM FOR BANDLIMITED SIGNALS

Consider the following system:



We can sample a bandpass signal at a frequency lower than its "Nyquist rate" by converting it to a lowpass signal.

In general, we suspect we can directly sample the signal, but we to select a sample frequency such that folding does not cause aliasing.

A general guideline is:

$$2B \le F_s \le 4B$$

A more rigorous equation is:

$$F_s = 2B\frac{r'}{r}$$

where

$$r' = \frac{F_c + B/2}{B}$$

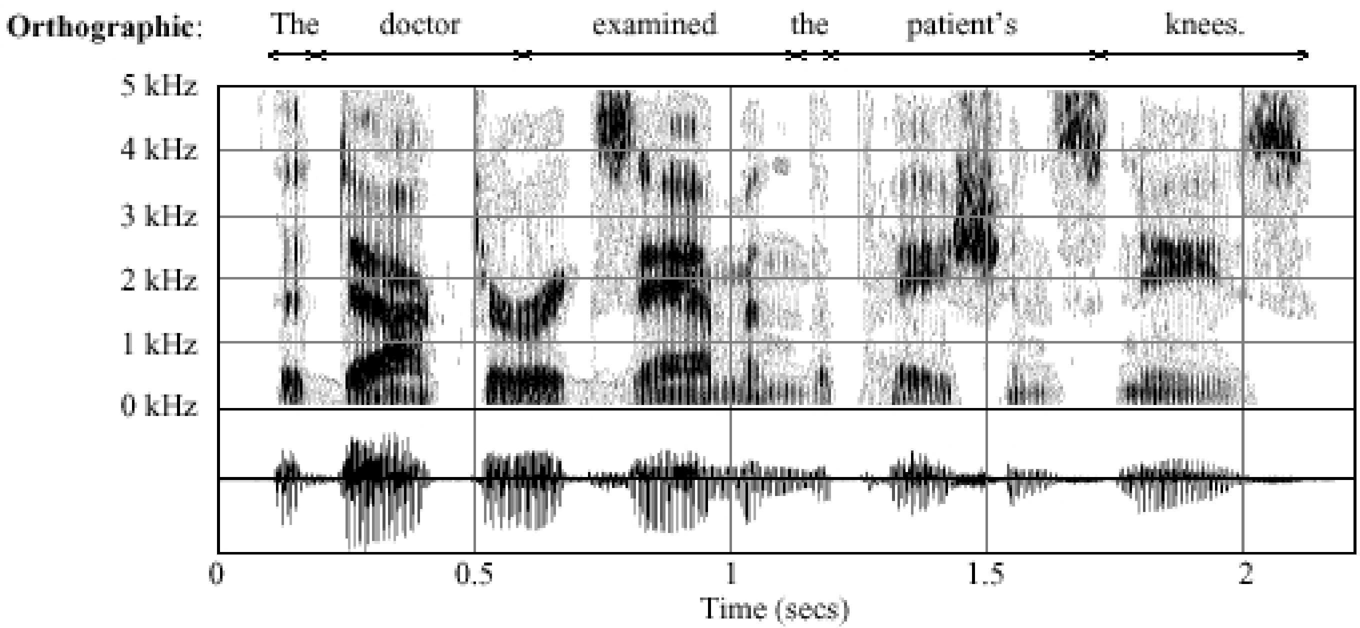
and

$$r = \lfloor r' \rfloor$$ (greatest integer less than or equal to r)

$$F_c = \frac{B_1 + B_2}{2}$$

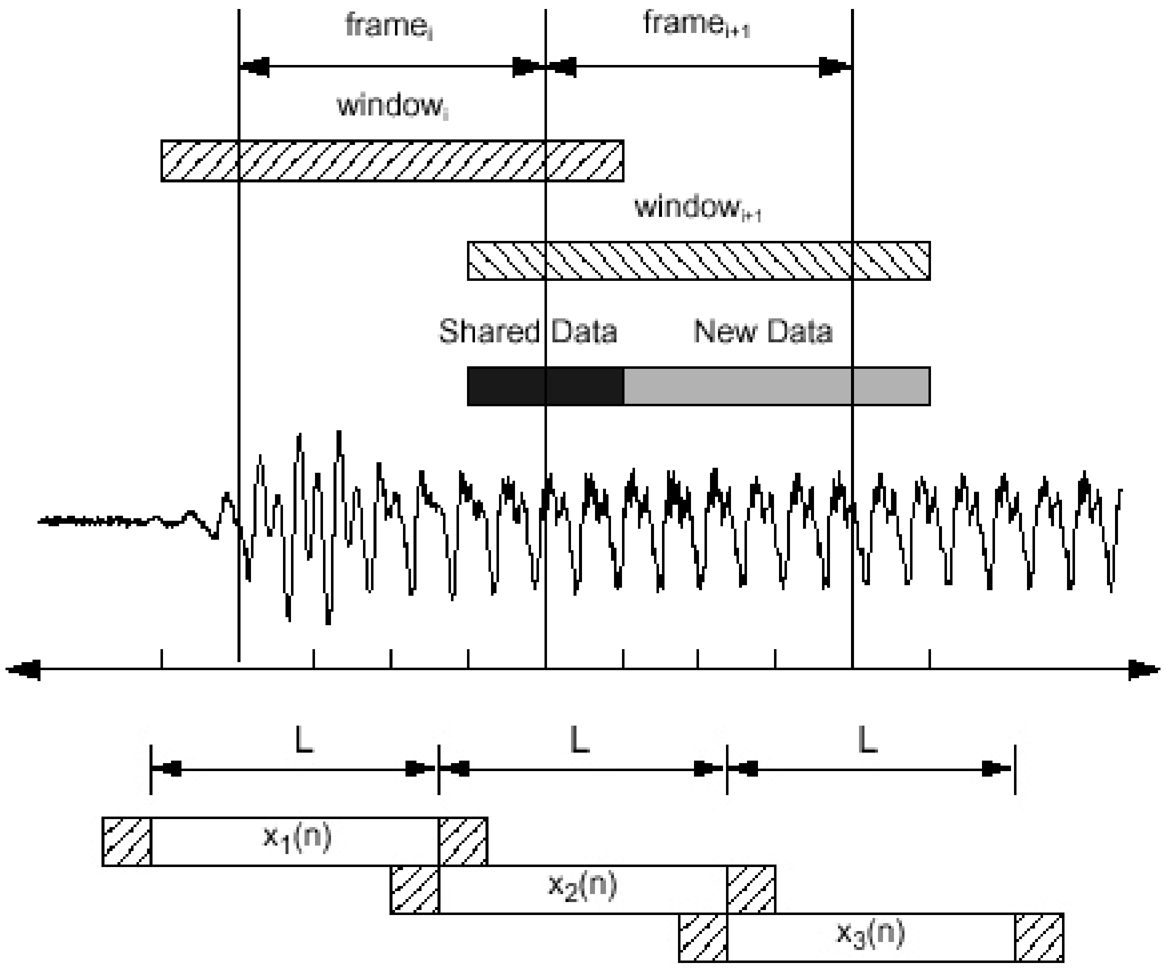
TYPICAL SAMPLING FREQUENCIES IN SPEECH RECOGNITION

- **8 kHz**: Popular in digital telephony. Provides coverage of first three formants for most speakers and most sounds.
- **16 kHz**: Popular in speech research. Why?
- **6.67 kHz**: Why?
- **Sub 8 kHz Sampling**: Can aliasing be useful in speech recognition? Hint: Consumer electronics.



A FRAME-BASED ANALYSIS IS ESSENTIAL

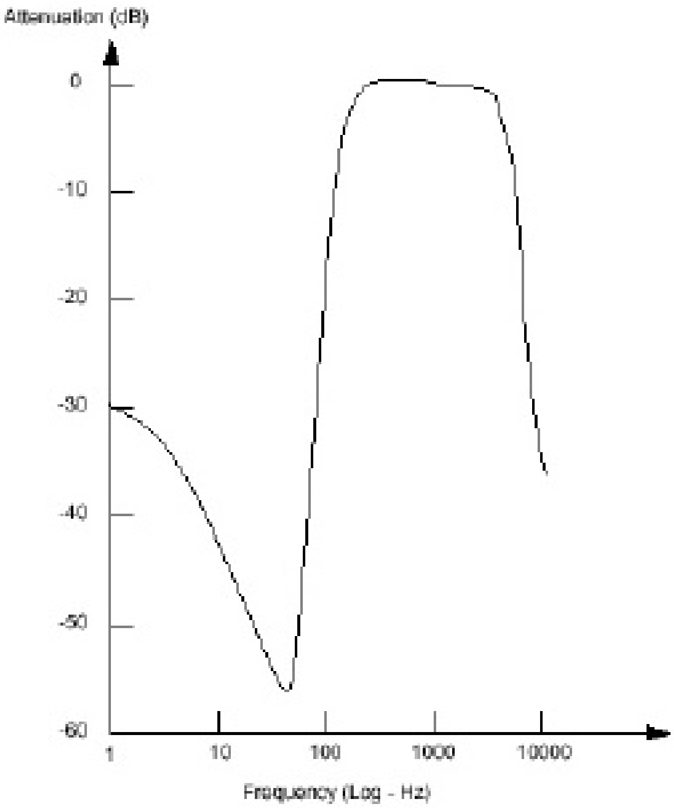
- Consider the problem of performing a piecewise linear analysis of a signal:



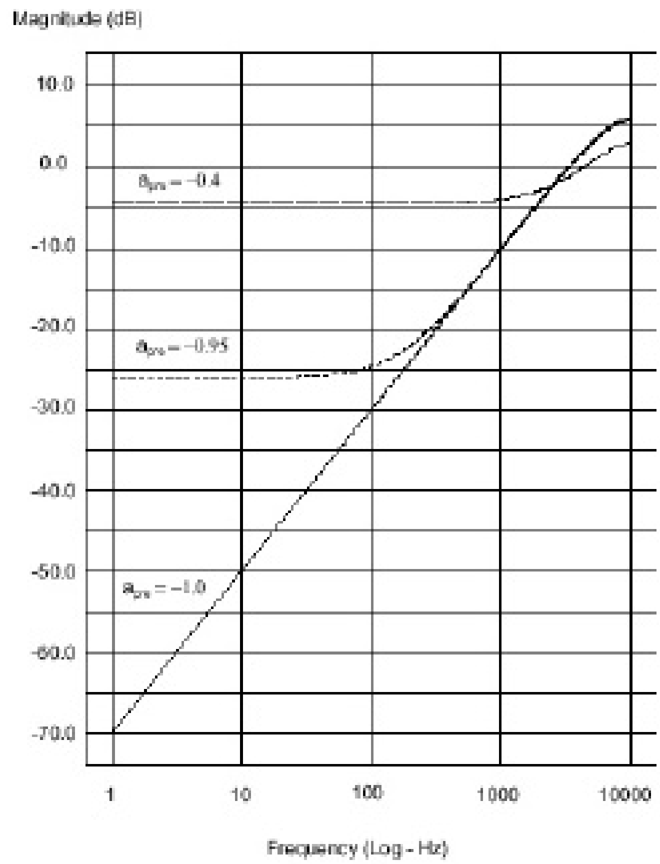
- This is most often implemented in hardware using a circular buffer.
- If we assume the signal is piecewise stationary, we can analyze the signal using a sliding window approach. Two key parameters are:
  - **Frame Duration**: how often we perform the analysis.
  - **Window Duration**: how many samples we use for the analysis.
- Recall we introduced similar parameters for the spectrogram. Typical values are a 10 ms frame duration and 25 ms window duration. Why?
- Important questions:
  - How does the window duration impact the spectral resolution?
  - Why so much overlap?
  - Why do we use a 10 ms frame duration?

SIGNAL CONDITIONING COMPENSATES FOR MICROPHONE AND CHANNEL CHARACTERISTICS

Frequency Response of a CODEC



Preemphasis Filter



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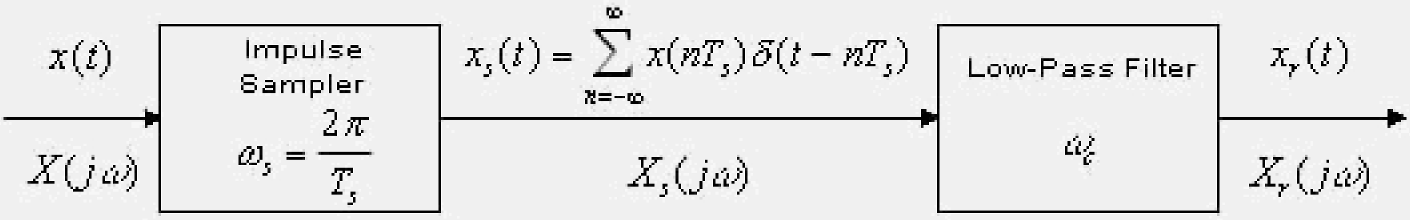

A continous-time signal *x(t)* is sampled at a frequency of ω_s *rad/sec.* to produce a sampled signal $x_s(t)$. We model $x_s(t)$ as an impulse train with the area of the *n*th impulse given by $x(nT_s)$. An ideal low-pass filter with cutoff frequency ω_c *rad/sec.* is used to obtain the reconstructed signal $x_r(t)$.

Suppose the highest-frequency component in *x(t)* is at frequency ω_m. Then the Sampling Theorem states that for $\omega_s > 2\omega_m$ there is no loss of information in sampling. In this case, choosing ω_c in the range $\omega_m < \omega_c < \omega_s - \omega_m$ gives $x_r(t) = x(t)$. These results can be understood by examining the Fourier transforms $X(j\omega)$, $X_s(j\omega)$, and $X_r(j\omega)$. If $\omega_s < 2\omega_m$ and/or ω_c is chosen poorly, then $x_r(t)$ might not resemble *x(t)*.

To explore sampling and reconstruction, select a signal or use the mouse to draw a signal *x(t)* in the window below. After a moment, the magnitude spectrum $|X(j\omega)|$ will appear. Then, enter a sampling frequency ω_s and click "Sample" to display the sampled signal and its magnitude spectrum. Finally, choose a cutoff frequency ω_c and click "Filter." The reconstructed signal and its magnitude spectrum will be shown.

[Fine Print.]

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Applet by Steve Crutchfield

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