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● Objectives:

- Basic properties of lossless tubes
- Resonant structure of the vocal tract
- Articulator positions (basic speech sounds) translate to predictable spectral signatures
- Digital filter-based models of the vocal tract (linear acoustics)
- Relationship of the parameters of these digital models to speech recognition.

Note that this lecture is based on material in this textbook:

J. Deller, et. al., *Discrete-Time Processing of Speech Signals*, MacMillan Publishing Co., ISBN: 0-7803-5386-2, 2000.



Introduction:

- 01: Organization
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Speech Signals:

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ECE 8463: FUNDAMENTALS OF SPEECH RECOGNITION

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URL: http://www.isip.msstate.edu/resources/courses/ece_8463

Modern speech understanding systems merge interdisciplinary technologies from Signal Processing, Pattern Recognition, Natural Language, and Linguistics into a unified statistical framework. These systems, which have applications in a wide range of signal processing problems, represent a revolution in Digital Signal Processing (DSP). Once a field dominated by vector-oriented processors and linear algebra-based mathematics, the current generation of DSP-based systems rely on sophisticated statistical models implemented using a complex software paradigm. Such systems are now capable of understanding continuous speech input for vocabularies of hundreds of thousands of words in operational environments.

In this course, we will explore the core components of modern statistically-based speech recognition systems. We will view speech recognition problem in terms of three tasks: signal modeling, network searching, and language understanding. We will conclude our discussion with an overview of state-of-the-art systems, and a review of available resources to support further research and technology development.

Tar files containing a compilation of all the notes are available. However, these files are large and will require a substantial amount of time to download. A tar file of the html version of the notes is available [here](#). These were generated using wget:

```
wget -np -k -m http://www.isip.msstate.edu/publications/courses/ece_8463/lectures/current
```

A pdf file containing the entire set of lecture notes is available [here](#). These were generated using Adobe Acrobat.

Questions or comments about the material presented here can be directed to help@isip.msstate.edu.

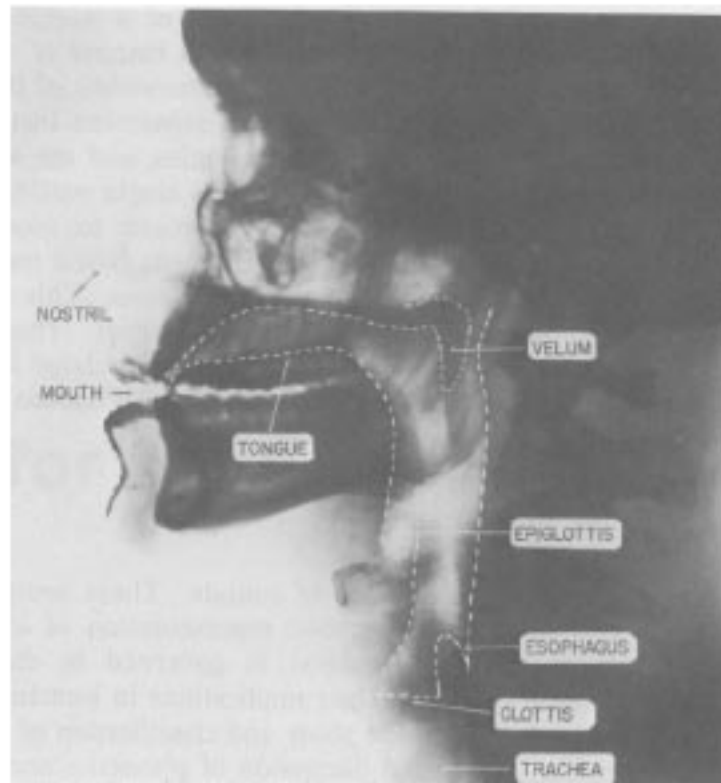
LECTURE 03: SOUND PROPAGATION

- Objectives:
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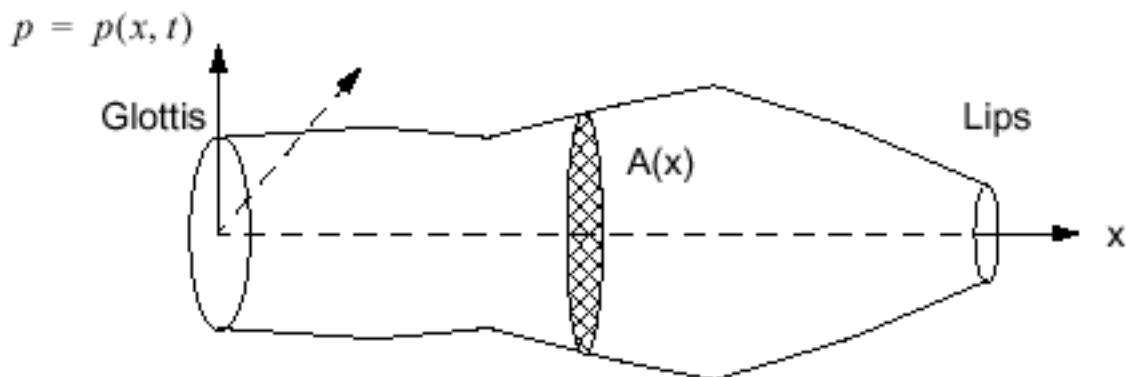
SOUND PROPAGATION



A detailed acoustic theory must consider the effects of the following:

- Time variation of the vocal tract shape
- Losses due to heat conduction and viscous friction at the vocal tract walls
- Softness of the vocal tract walls
- Radiation of sound at the lips
- Nasal coupling
- Excitation of sound in the vocal tract

Let us begin by considering a simple case of a lossless tube:



UNIFORM LOSSLESS TUBE

For frequencies that are long compared to the dimensions of the vocal tract (less than about 4000 Hz, which implies a wavelength of 8.5 cm), sound waves satisfy the following pair of equations:

$$\begin{aligned} \rho \frac{\partial(u/A)}{\partial t} + \text{grad}^m p &= 0 & -\frac{\partial p}{\partial x} &= \rho \frac{\partial(u/A)}{\partial t} \\ \frac{1}{\rho c^2} \frac{\partial p}{\partial t} + \frac{\partial A}{\partial t} + \text{div } u &= 0 & \text{or} & \\ -\frac{\partial u}{\partial x} &= \frac{1}{\rho c^2} \frac{\partial(pA)}{\partial t} + \frac{\partial A}{\partial t} \end{aligned}$$

where

$p = p(x, t)$ is the variation of the sound pressure in the tube

$u = u(x, t)$ is the variation in the volume velocity

ρ is the density of air in the tube (1.2 mg/cc)

c is the velocity of sound (35000 cm/s)

$A = A(x, t)$ is the area function (about 17.5 cm long)

Uniform Lossless Tube

If $A(x, t) = A$, then the above equations reduce to:

$$-\frac{\partial p}{\partial x} = \frac{\rho}{A} \frac{\partial u}{\partial t} \quad -\frac{\partial u}{\partial x} = \frac{A}{\rho c^2} \frac{\partial p}{\partial t}$$

The solution is a traveling wave:

$$\begin{aligned} u(x, t) &= u^+(t - x/c) - u^-(t + x/c) \\ p(x, t) &= \frac{\rho c}{A} [u^+(t - x/c) + u^-(t + x/c)] \end{aligned}$$

which is analogous to a transmission line:

$$-\frac{\partial v}{\partial x} = L \frac{\partial i}{\partial t} \quad -\frac{\partial i}{\partial x} = C \frac{\partial v}{\partial t}$$

What are the salient features of the lossless transmission line model?

RESONANT FREQUENCIES OF A LOSSLESS TUBE

where

<i>Acoustic Quantity</i>	<i>Analogous Electric Quantity</i>
p - pressure	v - voltage
u - volume velocity	i - current
ρ/A - acoustic inductance	L - inductance
$A/(\rho c^2)$ - acoustic capacitance	C - capacitance

The sinusoidal steady state solutions are:

$$p(x, t) = jZ_0 \frac{\sin[\Omega(l-x)/c]}{\cos[\Omega l/c]} U_G(\Omega) e^{j\Omega t}$$

$$u(x, t) = \frac{\cos[\Omega(l-x)/c]}{\cos[\Omega l/c]} U_G(\Omega) e^{j\Omega t}$$

where $Z_0 = \frac{\rho c}{A}$ is the characteristic impedance.

The transfer function is given by:

$$\frac{U(l, \Omega)}{U(0, \Omega)} = \frac{1}{\cos(\Omega l/c)}$$

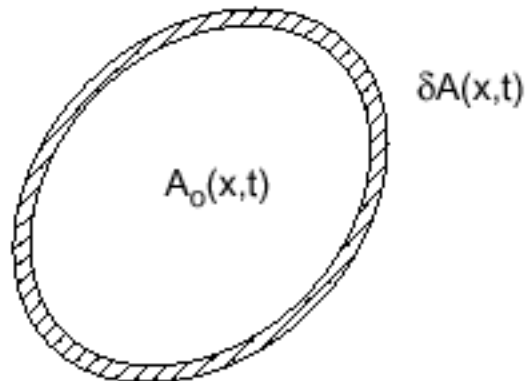
This function has poles located at every $\frac{(2n+1)\pi c}{2l}$. Note that these correspond to the frequencies at which the tube becomes a quarter wavelength: $\left(\frac{\Omega l}{c} = \frac{\pi}{2}\right) \Rightarrow \left(\Omega = \frac{c}{4l}\right)$.



Is this model realistic?

EFFECTS OF LOSSES

What do we predict the effects of yielding walls to be?



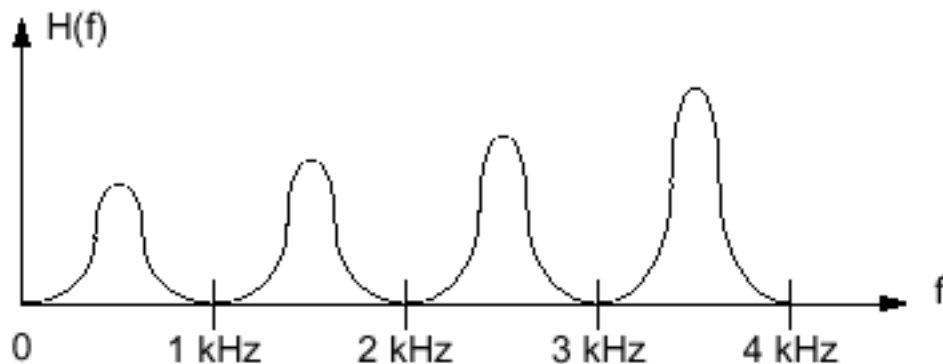
Use perturbation analysis:

$$A(x, t) = A_0(x, t) + \delta A(x, t)$$

We can develop a model that relates $\delta A(x,t)$ to pressure:

$$\frac{m_w d^2(\delta A)}{dt^2} + b_w \frac{d(\delta A)}{dt} + k_w(\delta A) = p(x, t)$$

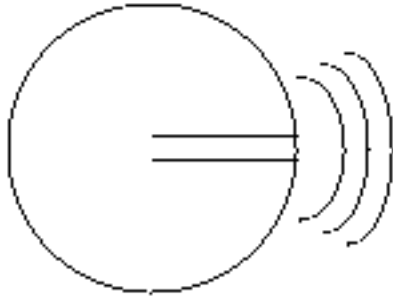
and solve for the new transfer function. But we can easily predict the effect of this:



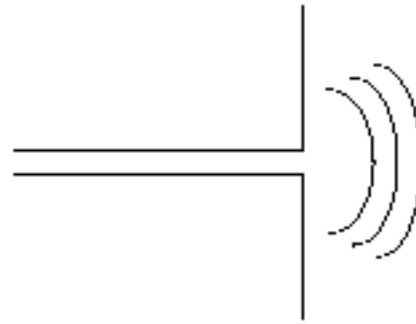
What would you expect to be the effect of friction and thermal losses?

LIP RADIATION

How is the sound pressure wave within the vocal tract coupled into the air?



Radiation from a spherical baffle



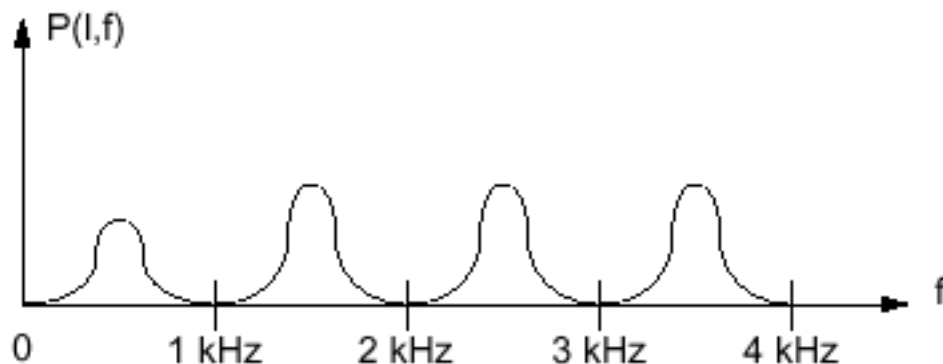
Radiation from an infinite plane baffle

Net effect is to place a complex load on the system:

$$Z_L(\Omega) = \frac{j\Omega L_r R_r}{R_r + j\Omega L_r} \quad \text{and} \quad P(l, \Omega) = Z_L(\Omega)U(l, \Omega)$$

where $R_r = 128/9\pi^2$ and $L_r = 8a/3\pi c$, and a is the radius of the opening.

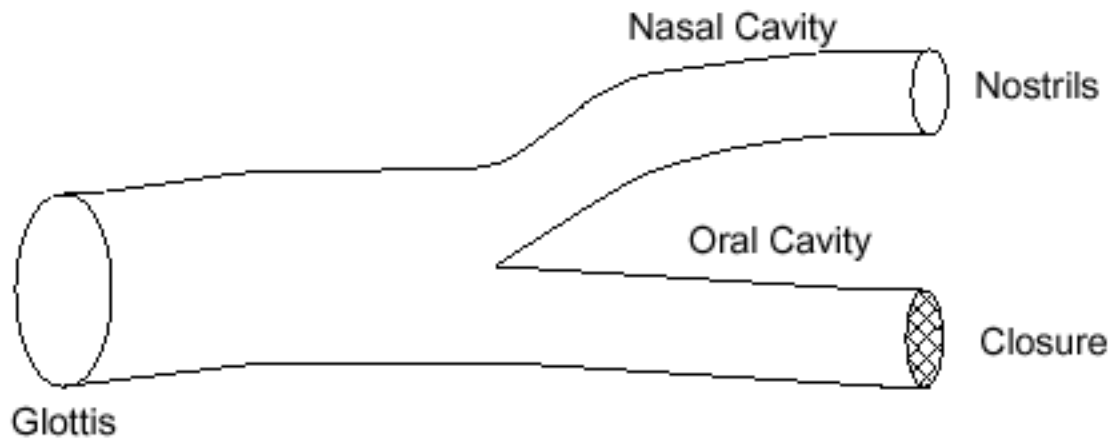
This impedance acts as a short circuit at low frequencies, and an imaginary impedance at high frequencies. The next effect on the volume velocity is to act as a highpass filter and to attenuate low frequencies. Lip radiation introduces a zero in the spectrum at DC and broadens the bandwidths at higher frequencies.



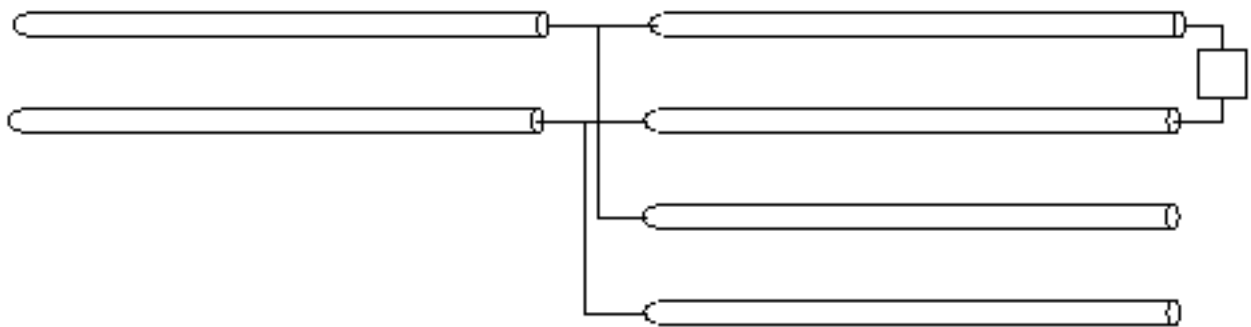
NASAL COUPLING

How is the sound pressure wave within the vocal tract coupled into the air?

We also must worry about the nasal cavity, especially for labial sounds for which the mouth is closed during sound production.



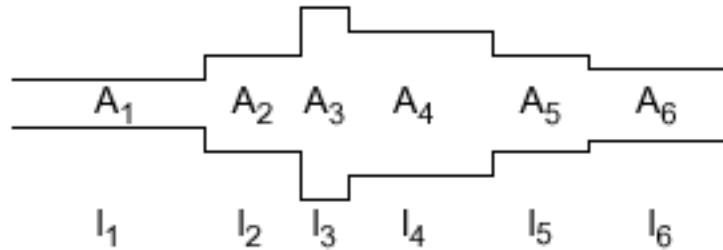
This is the equivalent of placing a transmission line in parallel with the vocal tract (oral cavity). What will the effect be?



The net effect is to produce a zero in the spectrum at about 1 kHz. As a result, nasal sounds (such as "m" and "n" in American English) have very little high frequency energy.

**PIECEWISE LINEAR APPROXIMATIONS
FOR THE VOCAL TRACT**

Consider the following approximation to the vocal tract area function:



Recall,

$$p_k(x, t) = \frac{\rho c}{A_k} [u_k^+(t - x/c) + u_k^-(t + x/c)]$$

$$u(x, t) = u_k^+(t - x/c) - u_k^-(t + x/c)$$

For the k^{th} section, if we apply the boundary conditions:

$$p_k(l_k, t) = p_{k+1}(0, t)$$

$$u_k(l_k, t) = u_{k+1}(0, t)$$

We can combine these two equations to show:

$$u_{k+1}^+(t) = \left[\frac{2A_{k+1}}{A_{k+1} + A_k} \right] u_k^+(t - \tau_k) + \left[\frac{A_{k+1} - A_k}{A_{k+1} + A_k} \right] u_{k+1}^-(t)$$

where $\tau_k = l_k/c$.

We can define a reflection coefficient for the k^{th} junction:

$$r_k = \frac{u_{k+1}^+(t)}{u_{k+1}^-(t)} = \frac{A_{k+1} - A_k}{A_{k+1} + A_k}$$

It is easy to show that the reflection coefficients are bounded: $-1 \leq r_k \leq 1$.

The velocity can be expressed in terms of the reflection coefficients:

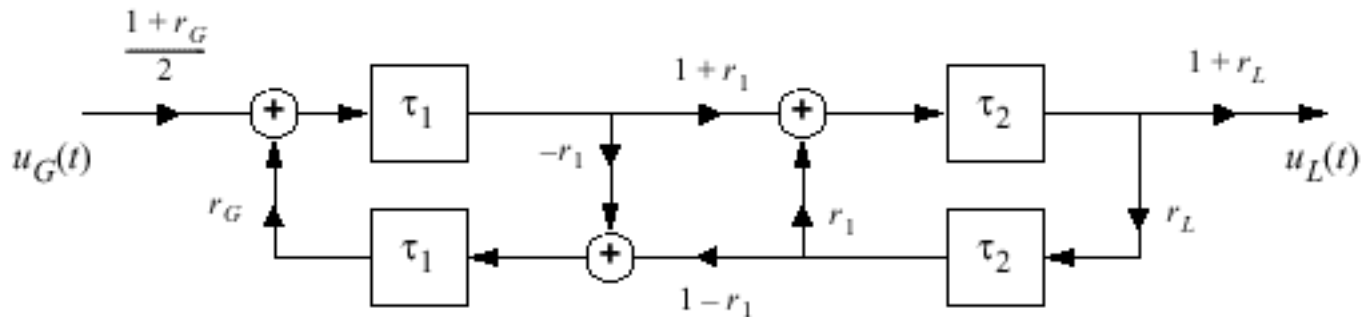
$$u_{k+1}^+(t) = (1 + r_k)u_k^+(t - \tau_k) + r_k u_{k+1}^-(t)$$

$$u_k^-(t + \tau_k) = (-r_k)u_k^+(t - \tau_k) + (1 - r_k)u_{k+1}^-(t)$$

Ultimately, we will relate $\{r_k\}$ to a discrete model of the velocity profile.

ACOUSTIC EXCITATION MODELS

Consider a two tube approximation to the vocal tract:

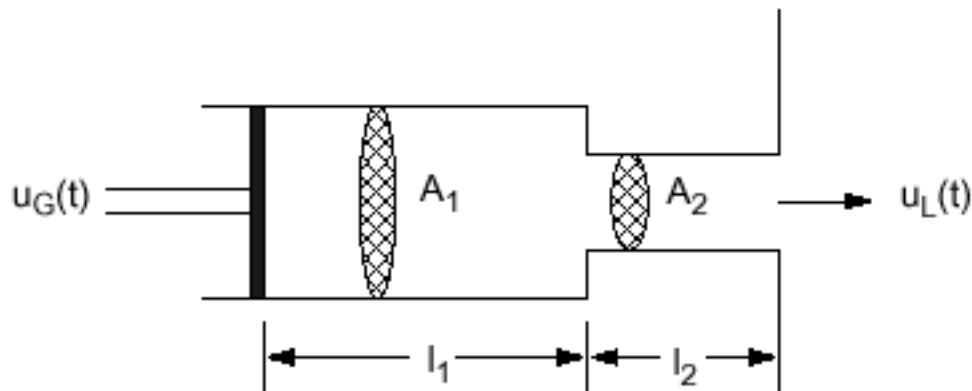


The frequency response of this system is:

$$V_a(\Omega) = \frac{U_L(\Omega)}{U_G(\Omega)} = \frac{0.5(1+r_G)(1+r_L)e^{-j\Omega(\tau_1+\tau_2)}}{1+r_1r_Ge^{-j\Omega 2\tau_1} + r_1r_Le^{-j\Omega 2\tau_2} + r_Lr_Ge^{-j\Omega 2(\tau_1+\tau_2)}}$$

What does this tell us about the frequency response?

If we consider the case $r_G = r_L = 1$:



For this system, the poles are located at values that satisfy the equation:

$$\frac{A_1}{A_2} \tan(\Omega\tau_2) = \cot(\Omega\tau_1)$$

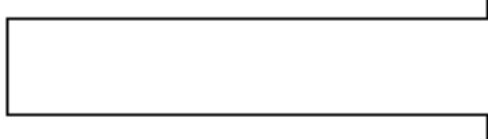
How does this compare to a single lossless tube?

Poles must be found through numerical analysis - nonlinear equation.

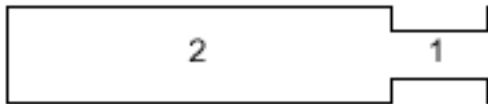
TWO TUBE MODELS

Resonator Geometry

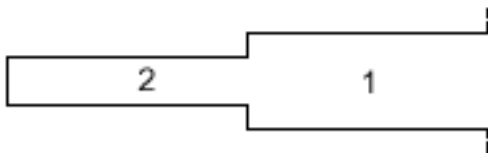
$L = 17.6 \text{ cm}$



$$L_2/L_1 = 8 \quad A_2/A_1 = 8$$



$$L_2/L_1 = 1.2 \quad A_2/A_1 = 1/8$$



$$L_2/L_1 = 1.0 \quad A_2/A_1 = 8$$



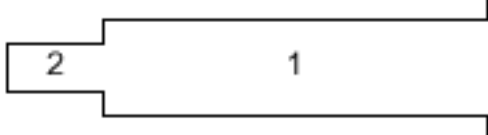
$$L_1 + L_2 = 17.6 \text{ cm}$$

$$L_2/L_1 = 1.5 \quad A_2/A_1 = 8$$



$$L_1 + L_2 = 14.5 \text{ cm}$$

$$L_2/L_1 = 1/3 \quad A_2/A_1 = 1/8$$



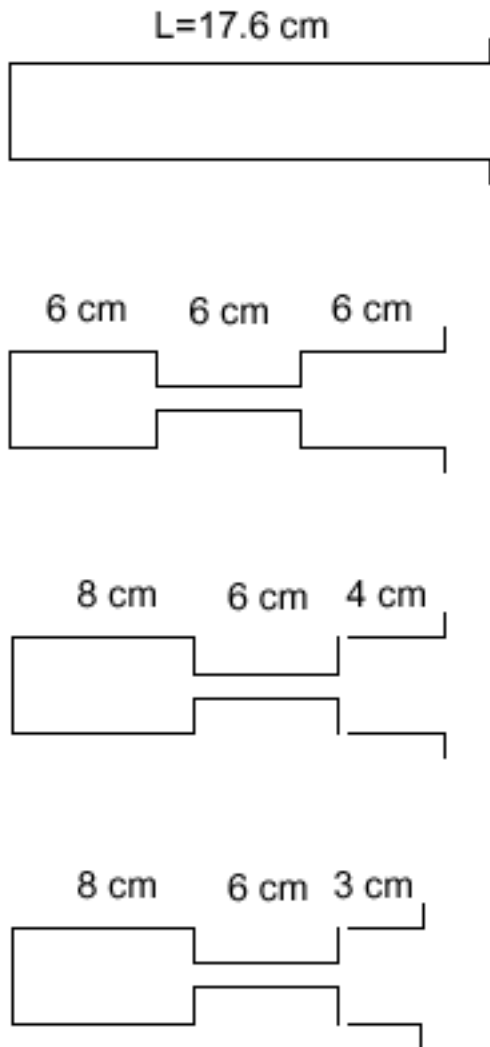
$$L_1 + L_2 = 17.6 \text{ cm}$$

Formant Patterns

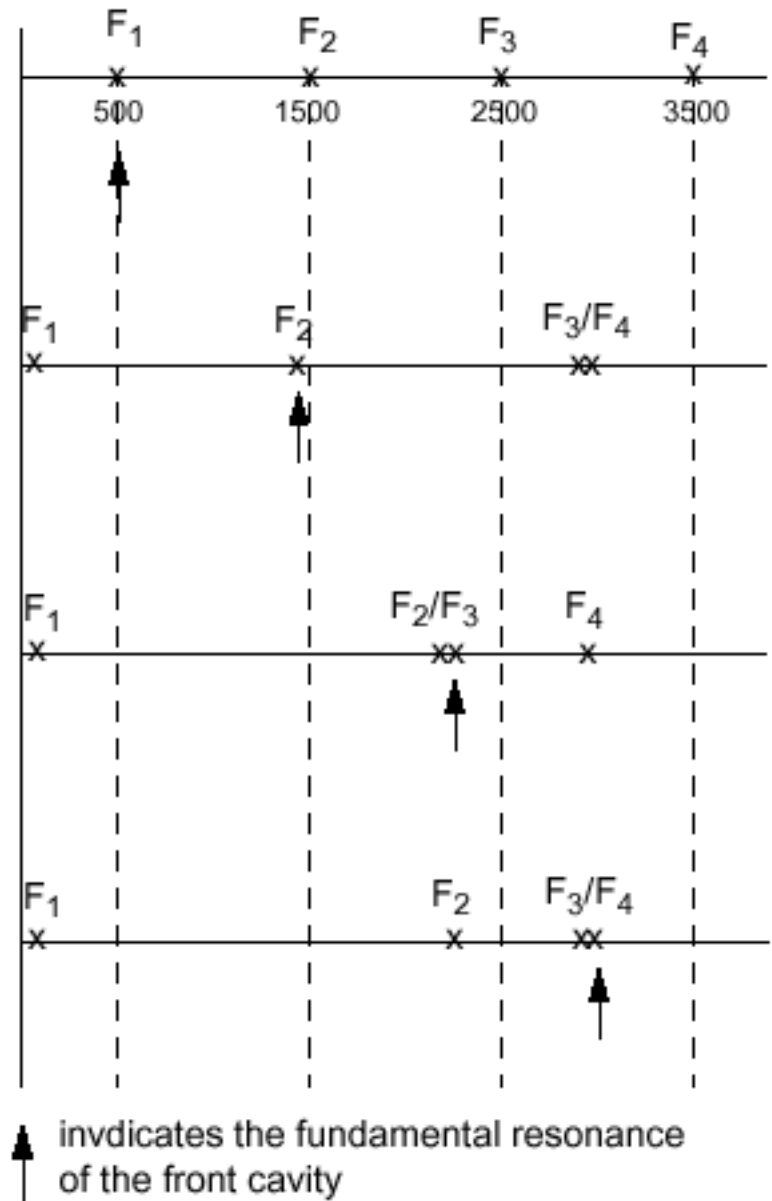
F_1 x 500	F_2 x 1500	F_3 x 2500	F_4 x 3900
F_1 x 320	F_2 x 1200	F_3 x 2300	F_4 x 3430
F_1 x 780	F_2 x 1240	F_3 x 2720	F_4 x 3350
F_1 x 220	F_2 x 1800	F_3 x 2230	F_4 x 3800
F_1 x 260	F_2 x 1990	F_3 x 3050	F_4 x 4130
F_1 x 630	F_2 x 1770	F_3 x 2280	F_4 x 3440

THREE TUBE MODELS

Resonator Geometry



Formant Patterns



TRANSFER FUNCTION OF THE LOSSLESS TUBE MODEL

Recall, $V(\Omega) = \frac{U_L(\Omega)}{U_G(\Omega)}$. In the discrete domain, we can write: $V(z) = \frac{U_L(z)}{U_G(z)}$.

Following our derivation of the wave equation, we can express the transfer function for a lossless tube as follows:

$$U_k = Q_k U_{k-1}$$

where

$$U_k = \begin{bmatrix} U_k^+(z) \\ U_k^-(z) \end{bmatrix} \quad \text{and} \quad Q_k = \begin{bmatrix} \frac{z^{1/2}}{1+r_k} & \frac{-r_k z^{1/2}}{1+r_k} \\ \frac{-r_k z^{1/2}}{1+r_k} & \frac{z^{-1/2}}{1+r_k} \end{bmatrix}$$

The combined transfer function is a product of these matrices. The net result is a transfer function that can be expressed as:

$$V(z) = \frac{0.5(1+r_G) \prod_{k=1}^N (1+r_k) z^{-N/2}}{D(z)}$$

where

$$D(z) = \begin{bmatrix} 1 & -r_G \end{bmatrix} \begin{bmatrix} 1 & -r_1 \\ -r_1 z^{-1} & z^{-1} \end{bmatrix} \cdots \begin{bmatrix} 1 & -r_N \\ -r_N z^{-N} & z^{-1} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

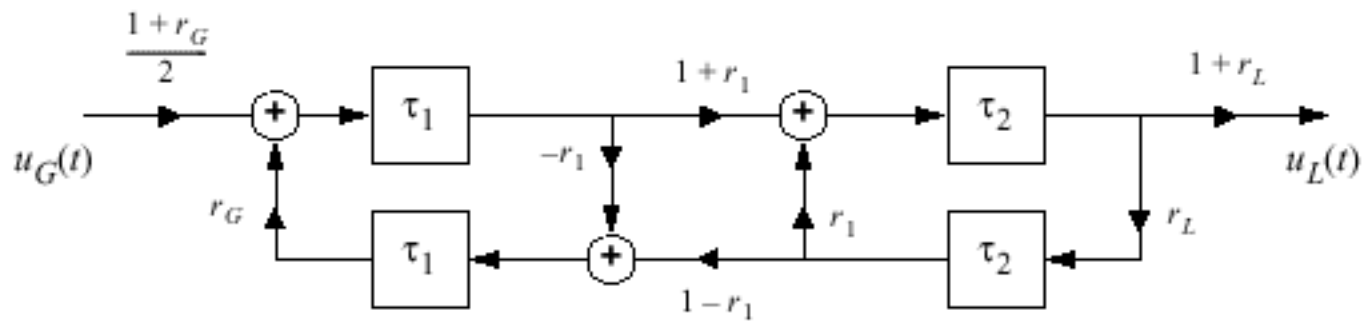
We can write $D(z)$ in a simpler form:

$$D(z) = 1 - \sum_{k=1}^N \alpha_k z^{-k}$$

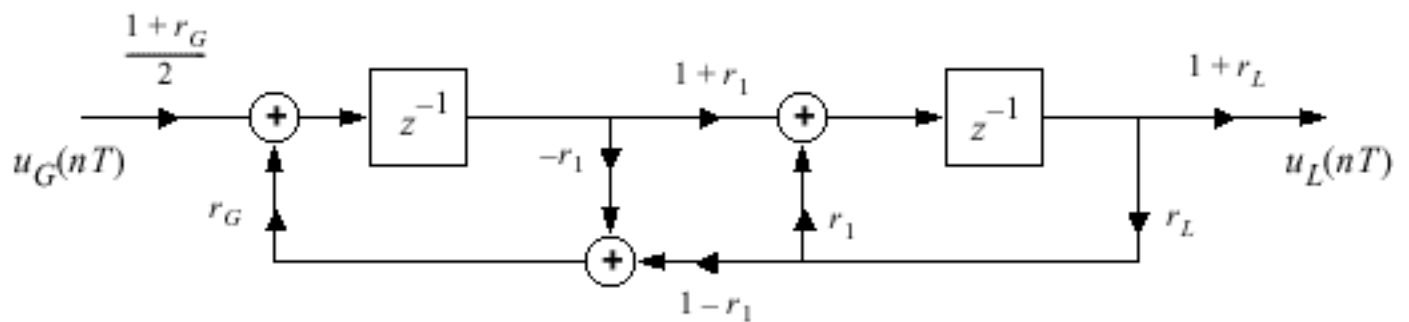
Why is this important?

DIGITAL SPEECH PRODUCTION MODELS

Recall our concatenated lossless tube model:



We can approximate this as a digital filter using the sampling theorem:



The transfer function of an N-tube model is:

$$V(z) = \frac{0.5(1+r_G) \prod_{k=1}^N (1+r_k) z^{-N/2}}{D(z)}$$

where

$$D(z) = \begin{bmatrix} 1 & -r_G \end{bmatrix} \begin{bmatrix} 1 & -r_1 \\ -r_1 z^{-1} & z^{-1} \end{bmatrix} \cdots \begin{bmatrix} 1 & -r_N \\ -r_N z^{-N} & z^{-1} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

We can compute $D(z)$ recursively:

$$D_0(z) = 1$$

$$D_k(z) = D_{k-1}(z) + r_k z^{-k} D_{k-1}(z^{-1}) \quad k = 1, 2, \dots, N$$

$$D(z) = D_N(z)$$

ALTERNATE DIGITAL FILTER IMPLEMENTATIONS USING DIGITAL RESONATORS

Note that for $D(z)$ to have real coefficients, zeros must occur in complex conjugate pairs. We can transform zeros in the Laplace domain:

$$s_k, s_k^* = -\sigma \pm j2\pi F_k$$

The corresponding complex conjugate poles in the discrete-domain are:

$$\begin{aligned} z_k, z_k^* &= e^{-\sigma_k T} e^{\pm j2\pi F_k T} \\ &= e^{-\sigma_k T} \cos(2\pi F_k T) \pm j e^{-\sigma_k T} \sin(2\pi F_k T) \end{aligned}$$

Note that magnitude of the pole in the z -plane is related to the bandwidth.

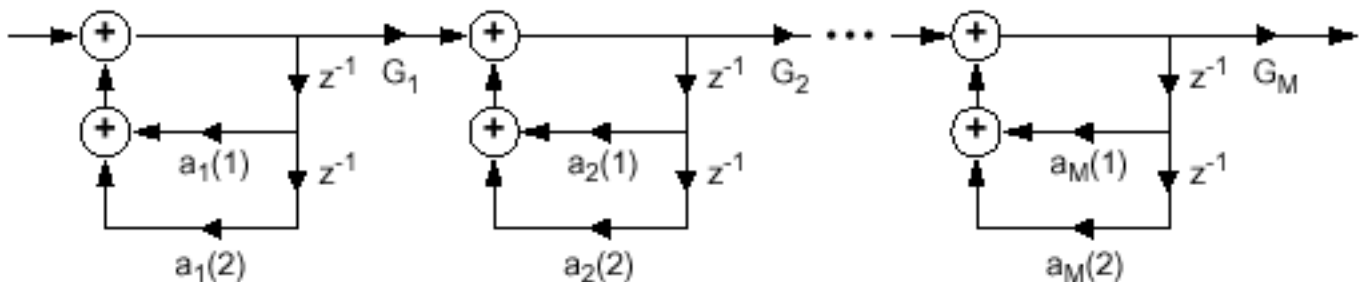
We can write a transfer function as a product of these poles:

$$V(z) = \prod_{k=1}^M V_k(z)$$

where

$$V_k(z) = \frac{(1 - 2|z_k| \cos(2\pi F_k T) + |z_k|^2)}{(1 - 2|z_k| \cos(2\pi F_k T)z^{-1} + |z_k|^2 z^{-2})}$$

This is an all-pole filter. It can be realized using a number of structures:
Under what conditions is this filter stable?



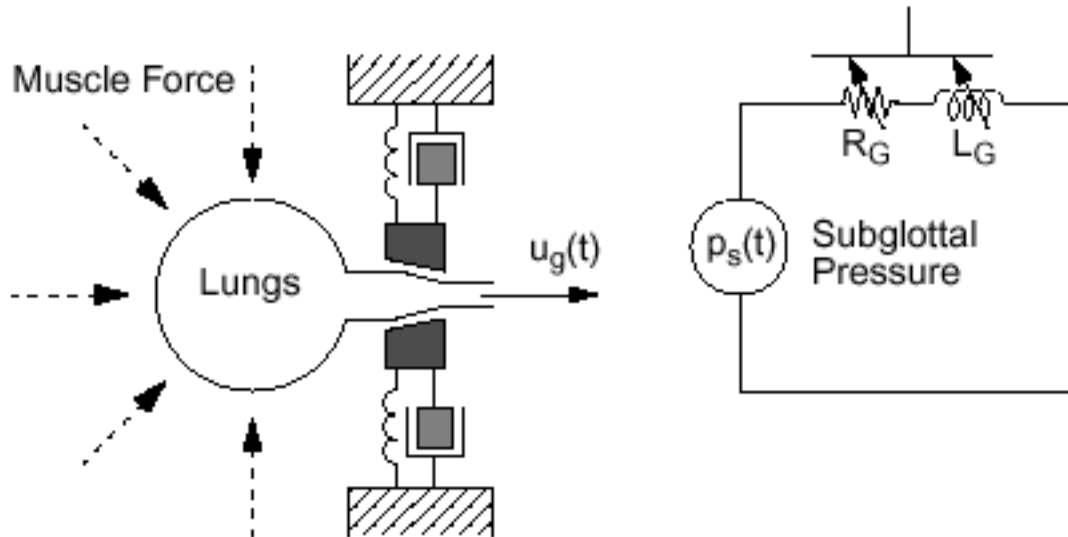
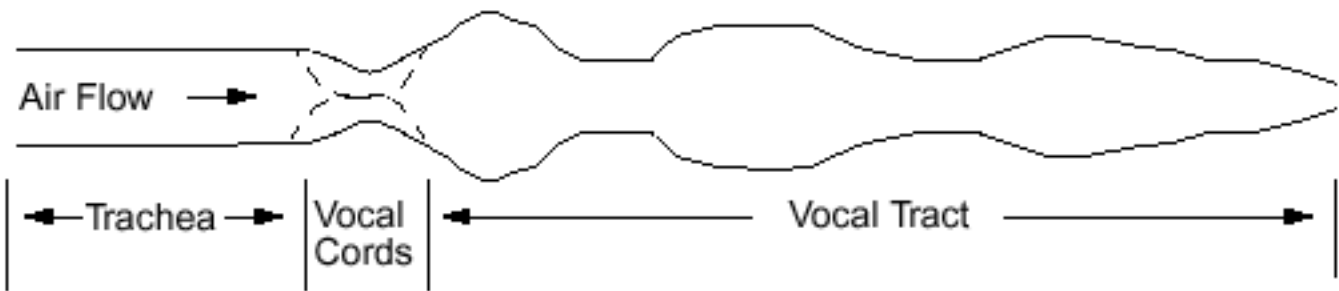
where,

$$V_k(z) = \frac{G_M}{1 - a_k(1)z^{-1} - a_k(2)z^{-2}}$$

$$a_k(1) = 2|z_k| \cos(2\pi F_k T) \quad a_k(2) = -|z_k|^2 \quad G_k = 1 - 2|z_k| \cos(2\pi F_k T) + |z_k|^2$$

EXCITATION MODELS

How do we couple energy into the vocal tract?



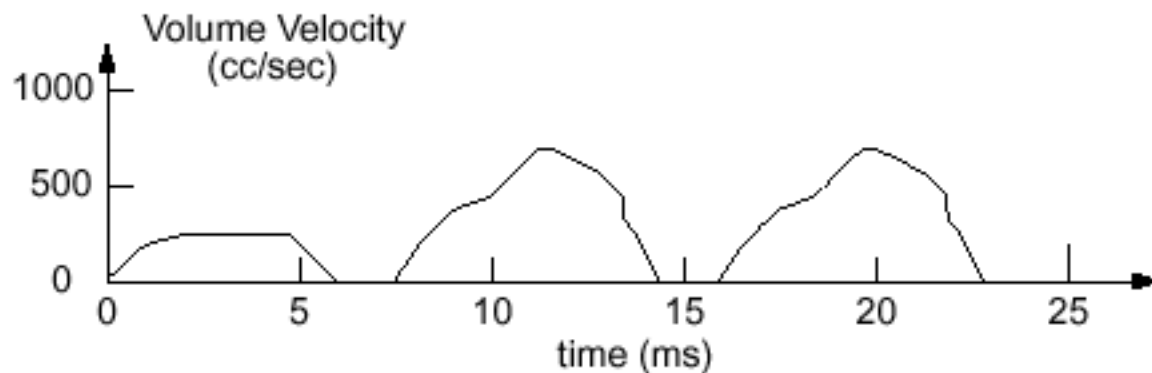
The glottal impedance can be approximated by:

$$Z_G = R_G + j\Omega L_G$$

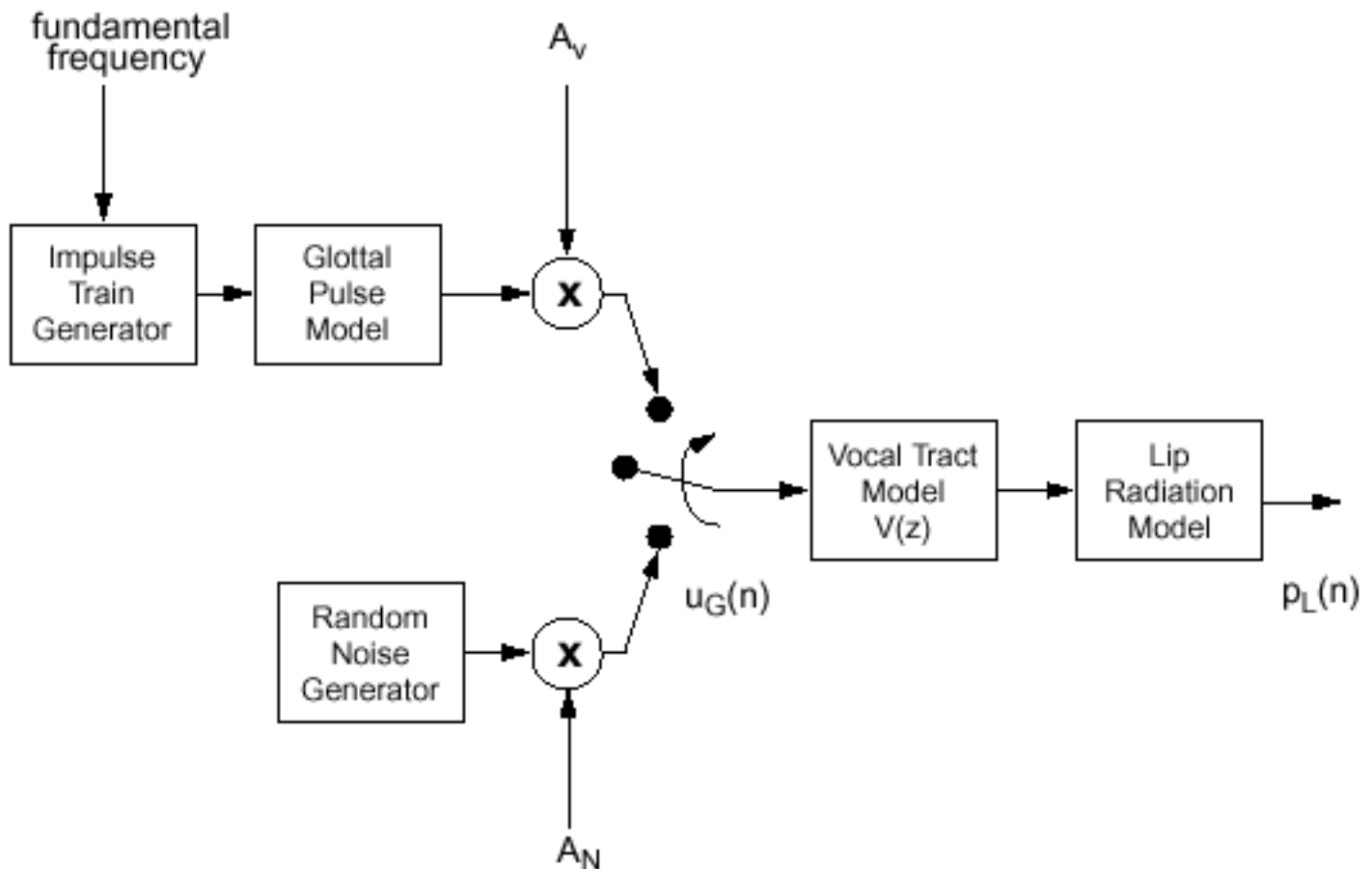
The boundary condition for the volume velocity is:

$$U(0, \Omega) = U_G(\Omega) - P(0, \Omega)/Z_G(\Omega)$$

For voiced sounds, the glottal volume velocity looks something like this:



THE VOCODER (COMPLETE) DIGITAL MODEL



Notes:

- Sample frequency is typically 8 kHz to 16 kHz
- Frame duration is typically 10 msec to 20 msec
- Window duration is typically 30 msec
- Fundamental frequency ranges from 50 Hz to 500 Hz
- Three resonant frequencies are usually found within 4 kHz bandwidth
- Some sounds, such as sibilants ("s") have extremely high bandwidths

Questions:

- What does the overall spectrum look like?
- What happened to the nasal cavity?
- What is the form of $V(z)$?

Lecture 2

Sound Waves in a Tube

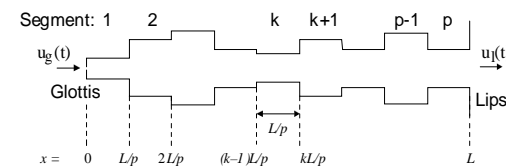
- Derive a theoretical model of how sound waves are affected by the vocal tract
- Describe a model for lip radiation
- Describe a model for the pulsating glottal waveform during voiced speech
- Assemble the components of a simple speech synthesiser

Appendix (not examinable)

- The physics of 1-dimensional sound waves

Multi-Tube Model of Vocal Tract

We model the vocal tract as a tube that has p segments:



u_g and u_l are the volume flows of air at the glottis and lips respectively (measured in litres per second).

Vocal tract is of length L (typically 15-17 cm in adults)

Length of each segment is the distance sound travels in half a sample period = $0.5cT$: 1.5 cm @ 11 kHz

- c = speed of sound in air

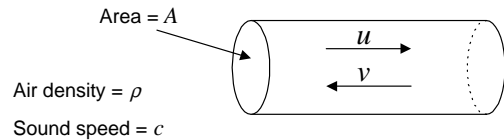
$$\approx 20\sqrt{\text{Absolute Temperature}} \approx 340 \text{ m/s}$$

- T = sample period = $1/f_{\text{samp}}$

Number of tube segments needed = $2L/cT \approx 0.001 f_{\text{samp}}$

Sound Waves in a Tube

Acoustic signal is the superposition of two waves: u in the forward direction and v in the reverse direction:



$$\text{Total volume flow} = u - v$$

$$\text{Total acoustic pressure} = (u + v) \times \rho c / A$$

Exactly analogous to transmission lines:

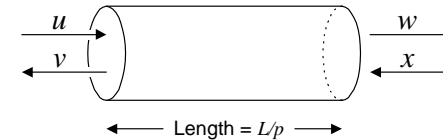
- Volume flow \approx Current, Pressure \approx Voltage
- Acoustic Impedance of tube = $\rho c / A$

Assumptions:

- Sound waves are 1-dimensional: true for frequencies < 3 kHz whose wavelengths are long compared to the tube width
- No frictional or wall-vibration energy losses

See appendix for a non-examinable derivation.

Segment Delays



Time for sound to travel along segment = L/cp

$$\text{Hence: } v(t) = x\left(t - \frac{L}{cp}\right) \quad \text{and} \quad u(t) = w\left(t + \frac{L}{cp}\right)$$

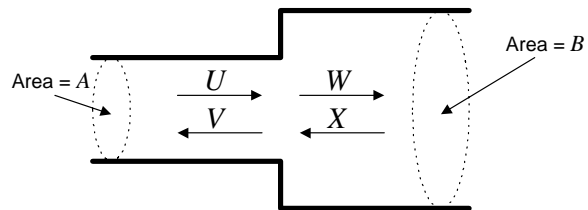
Segment length chosen to correspond to half a sample period. If we take z-transforms, this time delay corresponds to multiplying by $z^{-1/2}$:

$$V(z) = z^{-1/2} X(z) \quad \text{and} \quad U(z) = z^{+1/2} W(z)$$

In matrix form:

$$\begin{pmatrix} U \\ V \end{pmatrix} = \begin{pmatrix} z^{+1/2} & 0 \\ 0 & z^{-1/2} \end{pmatrix} \begin{pmatrix} W \\ X \end{pmatrix} = z^{+1/2} \begin{pmatrix} 1 & 0 \\ 0 & z^{-1} \end{pmatrix} \begin{pmatrix} W \\ X \end{pmatrix}$$

Segment Junction



Flow Continuity: $(U - V) = (W - X)$

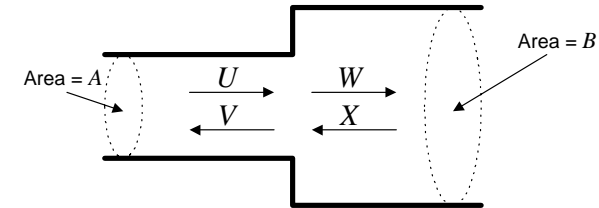
Pressure Continuity: $\frac{\rho c}{A}(U + V) = \frac{\rho c}{B}(W + X)$

In matrix form:
$$\begin{pmatrix} 1 & -1 \\ B & B \end{pmatrix} \begin{pmatrix} U \\ V \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ A & A \end{pmatrix} \begin{pmatrix} W \\ X \end{pmatrix}$$

Hence:

$$\begin{aligned} \begin{pmatrix} U \\ V \end{pmatrix} &= \frac{1}{2B} \begin{pmatrix} B & 1 \\ -B & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ A & A \end{pmatrix} \begin{pmatrix} W \\ X \end{pmatrix} \\ &= \frac{1}{2B} \begin{pmatrix} A+B & A-B \\ A-B & A+B \end{pmatrix} \begin{pmatrix} W \\ X \end{pmatrix} \end{aligned}$$

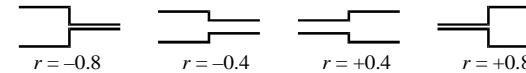
Reflection Coefficients



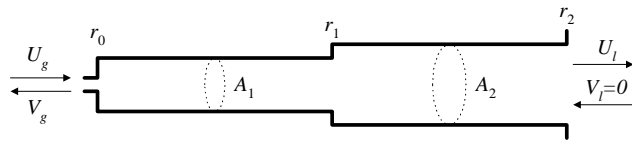
Define the reflection coefficient to be $r = \frac{B - A}{B + A}$

$$\begin{pmatrix} U \\ V \end{pmatrix} = \frac{1}{2B} \begin{pmatrix} A+B & A-B \\ A-B & A+B \end{pmatrix} \begin{pmatrix} W \\ X \end{pmatrix} = \frac{1}{1+r} \begin{pmatrix} 1 & -r \\ -r & 1 \end{pmatrix} \begin{pmatrix} W \\ X \end{pmatrix}$$

Reflection coefficients always lie in the range ± 1 :



2-Segment Vocal Tract



$$r_k = \frac{A_{k+1} - A_k}{A_{k+1} + A_k}$$

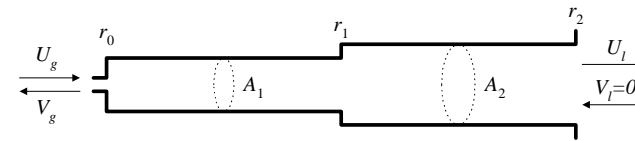
$$\frac{1}{1+r_2} \begin{pmatrix} 1 & -r_2 \\ -r_2 & 1 \end{pmatrix} \begin{pmatrix} U_l \\ 0 \end{pmatrix}$$

$$\frac{1}{1+r_1} \begin{pmatrix} 1 & -r_1 \\ -r_1 & 1 \end{pmatrix} \times z^{1/2} \begin{pmatrix} 1 & 0 \\ 0 & z^{-1} \end{pmatrix} \times$$

$$\begin{pmatrix} U_g \\ V_g \end{pmatrix} = \frac{1}{1+r_0} \begin{pmatrix} 1 & -r_0 \\ -r_0 & 1 \end{pmatrix} \times z^{1/2} \begin{pmatrix} 1 & 0 \\ 0 & z^{-1} \end{pmatrix} \times$$

- Assume $V_l = 0$: no sound reflected back into mouth
- Work backwards from lips towards glottis:
 - Junction: use the reflection matrix
 - Tube segment: use the delay matrix
- A_3 is large but not infinite: assumption of narrow tube breaks down at this point
- A_0 is approximately zero: area of glottis opening

Vocal Tract Transfer Function



Multiplying out the matrices gives:

$$\begin{pmatrix} U_g \\ V_g \end{pmatrix} = \frac{z^{+1}}{\prod_{k=0}^2 (1+r_k)} \begin{pmatrix} 1 + (r_0 r_1 + r_1 r_2) z^{-1} + r_0 r_2 z^{-2} \\ -r_0 - (r_1 + r_0 r_1 r_2) z^{-1} - r_2 z^{-2} \end{pmatrix} U_l$$

We can ignore V_g : it gets absorbed in the lungs.

The vocal tract transfer function is given by the ratio of U_l to U_g :

$$\begin{aligned} \frac{U_l}{U_g} &= \frac{\prod_{k=0}^2 (1+r_k) \times z^{-1}}{1 + (r_0 r_1 + r_1 r_2) z^{-1} + r_0 r_2 z^{-2}} \\ &= \frac{G z^{-1}}{1 + (r_0 r_1 + r_1 r_2) z^{-1} + r_0 r_2 z^{-2}} \\ &= \frac{G z^{-1}}{1 - a_1 z^{-1} - a_2 z^{-2}} \end{aligned}$$

p-segment Vocal Tract

Note that:
$$\frac{1}{1+r} \begin{pmatrix} 1 & -r \\ -r & 1 \end{pmatrix} \times z^{1/2} \begin{pmatrix} 1 & 0 \\ 0 & z^{-1} \end{pmatrix} = \frac{z^{1/2}}{1+r} \begin{pmatrix} 1 & -rz^{-1} \\ -r & z^{-1} \end{pmatrix}$$

Multiplying together all the matrices for a p -segment vocal tract gives:

$$\begin{pmatrix} U_g \\ V_g \end{pmatrix} = \frac{z^{1/2 p}}{\prod_{k=0}^p (1+r_k)} \prod_{k=0}^{p-1} \begin{pmatrix} 1 & -r_k z^{-1} \\ -r_k & z^{-1} \end{pmatrix} \times \begin{pmatrix} 1 \\ -r_p \end{pmatrix} U_l$$

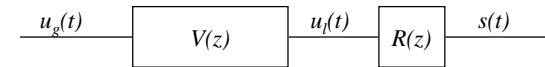
This results in a transfer function of the form:

$$V(z) = \frac{U_l}{U_g} = \frac{G z^{-1/2 p}}{1 - a_1 z^{-1} - a_2 z^{-2} - \dots - a_p z^{-p}}$$

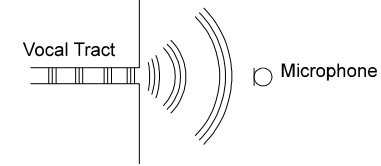
Where:

- G is a gain term
- $z^{-1/2 p}$ is the acoustic time delay along the vocal tract
- The denominator represents a p^{th} order all-pole filter

Lip Radiation



$R(z)$ is the transfer function between *airflow* at the lips and *pressure* at the microphone.



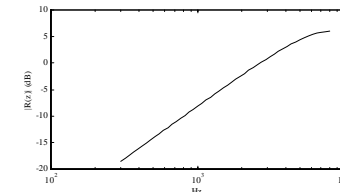
For a lip-opening area of A , acoustic theory predicts a 1st-order high-pass response with a corner frequency of:

$$\frac{c}{\sqrt{4A}} \text{ Hz} \approx 5 \text{ kHz}$$

For $f_{\text{samp}} < 20 \text{ kHz}$, a good approximation is:

$$R(z) = \frac{S(z)}{U_l(z)} = 1 - z^{-1}$$

$$\Rightarrow |R(z)| = 2 \sin\left(\frac{\omega T}{2}\right)$$

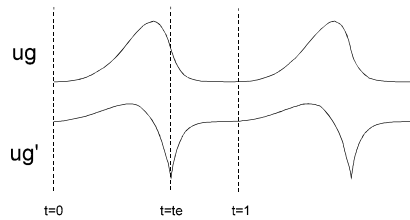


Spectrum of Glottal Waveform

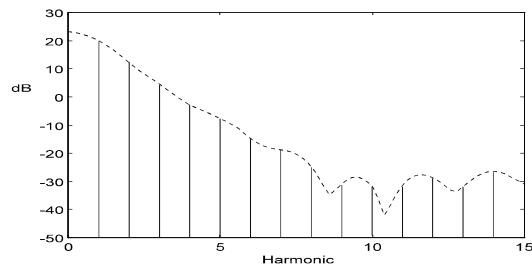
“LF Model” (Liljencrants & Fant)

$$u'_g(t) = \begin{cases} e^{at} \sin(bt) & 0 \leq t < t_e \\ c + de^{-ft} & t_e \leq t < 1 \end{cases}$$

with $u_g(0) = u_g(1) = 0$; $u_g(t)$ and $u'_g(t)$ continuous at t_e

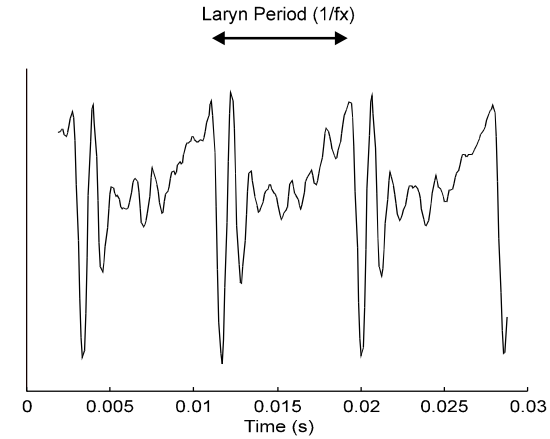


Line Spectrum of u_g (approx -12 dB/octave):



Vowel Waveform

Vowel /a/ from “part”



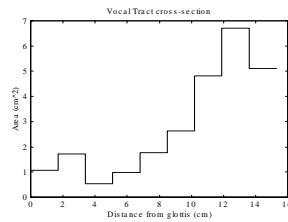
- Larynx Frequency ≈ 130 Hz
- First Vocal tract resonance (formant) ≈ 1 kHz

There is not necessarily any relation between the larynx frequency and the vocal tract resonances.

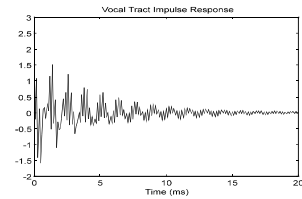
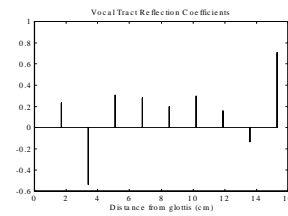
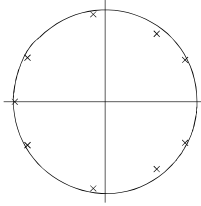
Resonances at a multiple of the larynx frequency will be louder (good for singers)

Vocal Tract Shape and Response

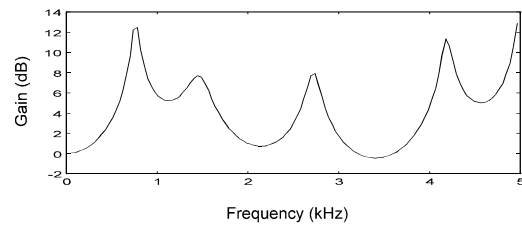
Example: /a/ vowel ("part")



Z-plane Pole Positions



Vocal Tract Filter Response



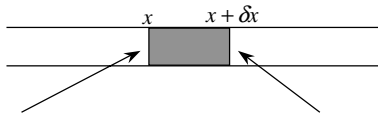
Appendix

Theoretical Derivation of Sound Waves

This section is non-examinable

1-Dimensional Sound Waves

Consider a small chunk of air in a tube with a uniform cross-sectional area A :



$$\text{Pressure} = p$$

$$\text{Pressure} = p + \delta x \frac{\partial p}{\partial x}$$

$$\text{Velocity} = v = \frac{u}{A}$$

$$\text{Velocity} = v + \delta v = \frac{1}{A} \left(u + \delta x \frac{\partial u}{\partial x} \right)$$

$$\Rightarrow \delta v = \frac{\delta x}{A} \frac{\partial u}{\partial x}$$

Volume of air chunk: $V = A \times \delta x$

Hence: $\frac{\partial V}{\partial t} = A \times \delta v = A \times \frac{\delta x}{A} \frac{\partial u}{\partial x} = \frac{V}{A} \times \frac{\partial u}{\partial x}$ ①

Net force on air chunk:

$$F = Ap - A \left(p + \delta x \frac{\partial p}{\partial x} \right) = -A \delta x \frac{\partial p}{\partial x}$$

Gas Laws

Ideal Gas Law :

We can express the pressure in terms of the density:

$$\begin{aligned} pV &= nRT & n &= \text{moles of air} = \text{molecules} \div (6 \times 10^{23}) \\ &= \frac{\rho V}{M} RT & R &= \text{gas constant} = 8.314 \text{ J / (K} \cdot \text{mol)} \\ & & T &= \text{Temperature (}^\circ\text{K)} \\ \Rightarrow p &= \rho \times \frac{RT}{M} & \rho &= \text{density} (\approx 1.225 \text{ kg / m}^3) \\ & & M &= \text{molecular weight of air} = 0.029 \text{ kg / mol} \\ & & \gamma &= \text{specific heat ratio of air} = 1.4 \end{aligned}$$

We define $c^2 = \frac{\gamma RT}{M} \approx (340 \text{ m / s})^2 \Rightarrow p\gamma = \rho c^2$ ②

c will turn out to be the speed of sound and depends only on T .

Adiabatic Gas Law: For pressure changes too rapid for heat conduction to occur (e.g. sound vibrations):

$$\frac{d}{dt}(pV^\gamma) = 0 \Rightarrow V^\gamma \frac{\partial p}{\partial t} + p\gamma V^{\gamma-1} \frac{\partial V}{\partial t} = 0$$

using ① and ② $\Rightarrow V^\gamma \frac{\partial p}{\partial t} = -\rho c^2 \times \frac{V^\gamma}{A} \times \frac{\partial u}{\partial x}$

$$\Rightarrow A \frac{\partial p}{\partial t} = -\rho c^2 \frac{\partial u}{\partial x}$$
 ③

Wave Equations

Mass x Acceleration = Force:

$$\rho V \times \frac{1}{A} \frac{\partial u}{\partial t} = -A \delta x \frac{\partial p}{\partial x} = -V \frac{\partial p}{\partial x} \Rightarrow \rho \frac{\partial u}{\partial t} = -A \frac{\partial p}{\partial x} \quad \textcircled{4}$$

Wave Equations:

Equations $\textcircled{3}$ and $\textcircled{4}$ are known as the *wave equations*:

$$\rho \frac{\partial u}{\partial t} = -A \frac{\partial p}{\partial x} \quad \text{and} \quad A \frac{\partial p}{\partial t} = -\rho c^2 \frac{\partial u}{\partial x}$$

Solution:

$$u(x, t) = u^+(t - x/c) - u^-(t + x/c)$$

$$p(x, t) = \frac{\rho c}{A} \times \{u^+(t - x/c) + u^-(t + x/c)\}$$

It is easily verified that this solution satisfies the wave equations for any differentiable functions u^+ and u^- .

The two functions u^+ and u^- represent waves travelling in +ve and -ve x directions at velocity c . The actual values of the waves are determined by the boundary conditions at the end of the tube section.

The equations are the same as for a transmission line with $u \approx$ current, $p \approx$ voltage and $\rho c/A \approx$ impedance.

University of California
Berkeley

College of Engineering
Department of Electrical Engineering
and Computer Sciences

Professors : N.Morgan / B.Gold
EE225D

Spring, 1999

Acoustic Tube Models

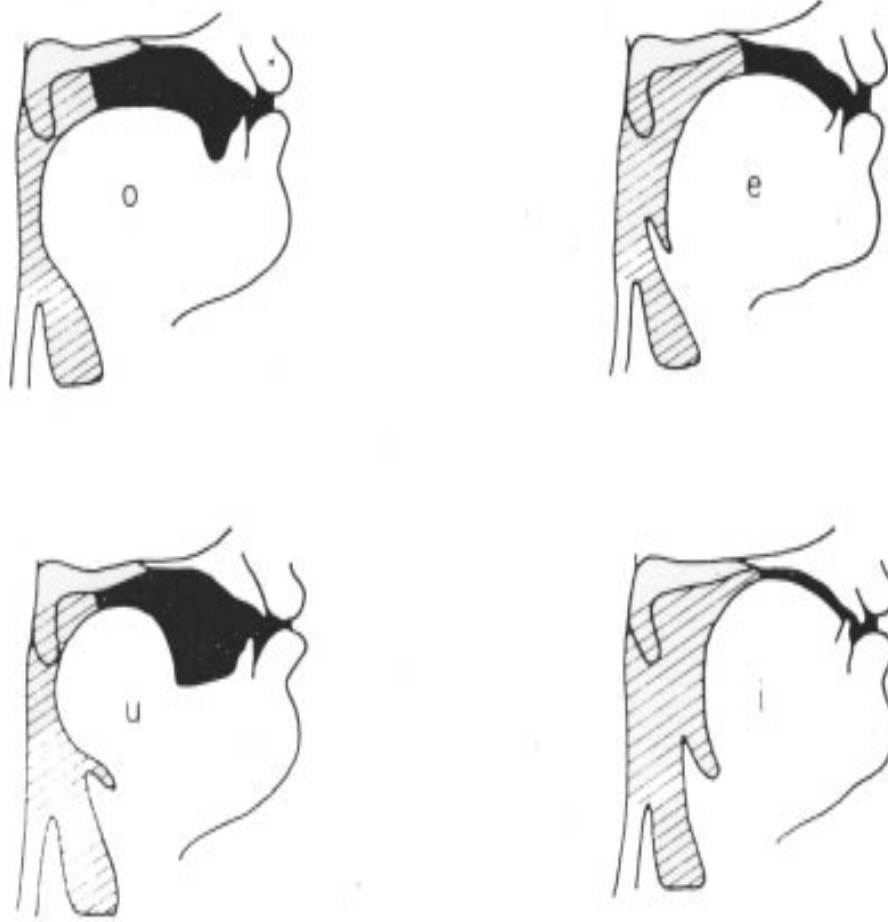
Lecture 13

Introduction :

Acoustic Tube Models of English Phonemes → 2 tube model.

Assumptions :

- Lossless tubes
- Plane waves
- Rigid walls
- Friction
- Thermal effect



Vocal tract area for four vowel sounds

Vocal tract areas for four vowel sounds.

i - Tongue is High.

e - Tongue is a little Lower.

u - Tongue is very Low.

o - Tongue is somewhat low.

1. Tube response vs. area function.
2. Discrete-time-space version.
3. Example - 2 tube representation of vowels.

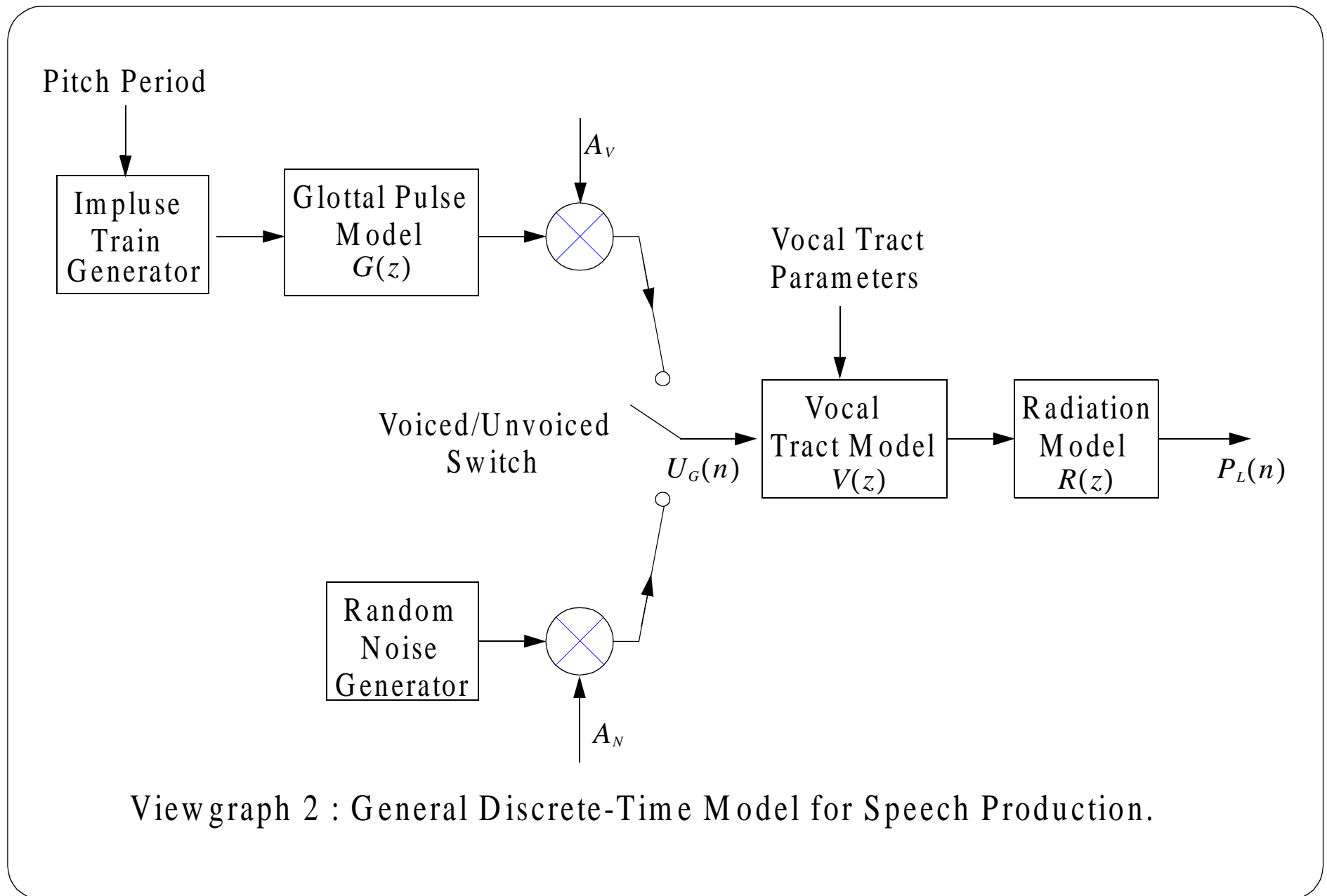
Problem for Today :

Develop a 2 tube model to derive a frequency response that approximates some vowels.

By solving a complicated wave equation, the frequency response can be found.

Look up equation in R & S.

$$\begin{aligned} -\frac{\partial p}{\partial x} &= \rho \frac{\partial}{\partial t}(u/A) \\ -\frac{\partial u}{\partial t} &= \frac{1}{\rho c^2} \frac{\partial}{\partial t}(pA) + \frac{\partial A}{\partial t} \end{aligned}$$



Assumption in this Model :

Vocal Tract Model - Time varying

Radiation Model - May be time varying

Glottal Pulse Model - Usually considered independent of vocal tract model, but later we'll examine this wave closely

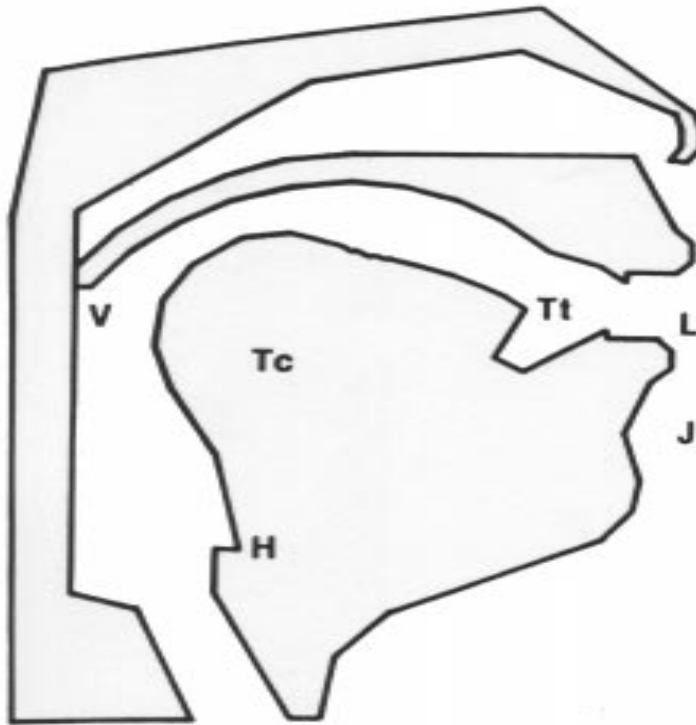
$$u(x, t) = u^+ \left(t - \frac{x}{C} \right) - u^- \left(t + \frac{x}{C} \right)$$

$$p(x, t) = Z_o \left[u^+ \left(t - \frac{x}{C} \right) + u^- \left(t + \frac{x}{C} \right) \right]$$

$p(l, t) = 0$: open tube

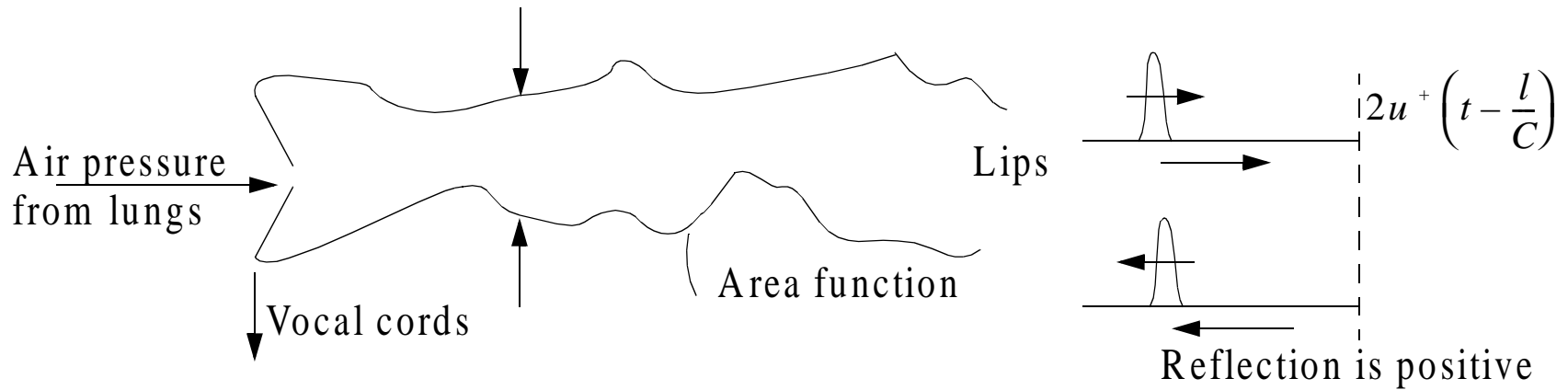
$$u^+ \left(t - \frac{l}{C} \right) = -u^- \left(t + \frac{l}{C} \right)$$

$$u(l, t) = 2u^+ \left(t - \frac{l}{C} \right)$$



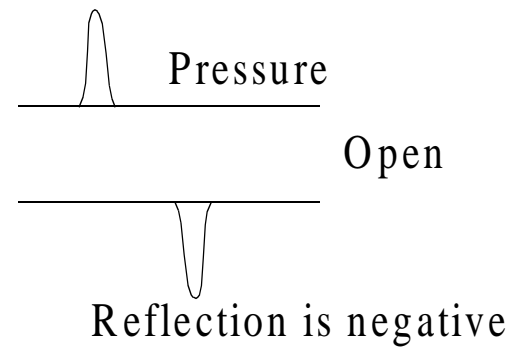
Model of Vocal Tract

- H = HYOID POSITION**
- J = ANGLE OF JAW OPENING**
- L = LIP PROTRUSION AND ELEVATION**
- Tc = TONGUE CENTER**
- Tt = POSITION OF TONGUE TIP**
- V = VELUM OPENING**



Closed Tube $u(l, t) = 0$

$$\text{so } u^+ \left(t - \frac{l}{C} \right) = -u^- \left(t + \frac{l}{C} \right)$$



- Given Area Function we can compute Spectrum

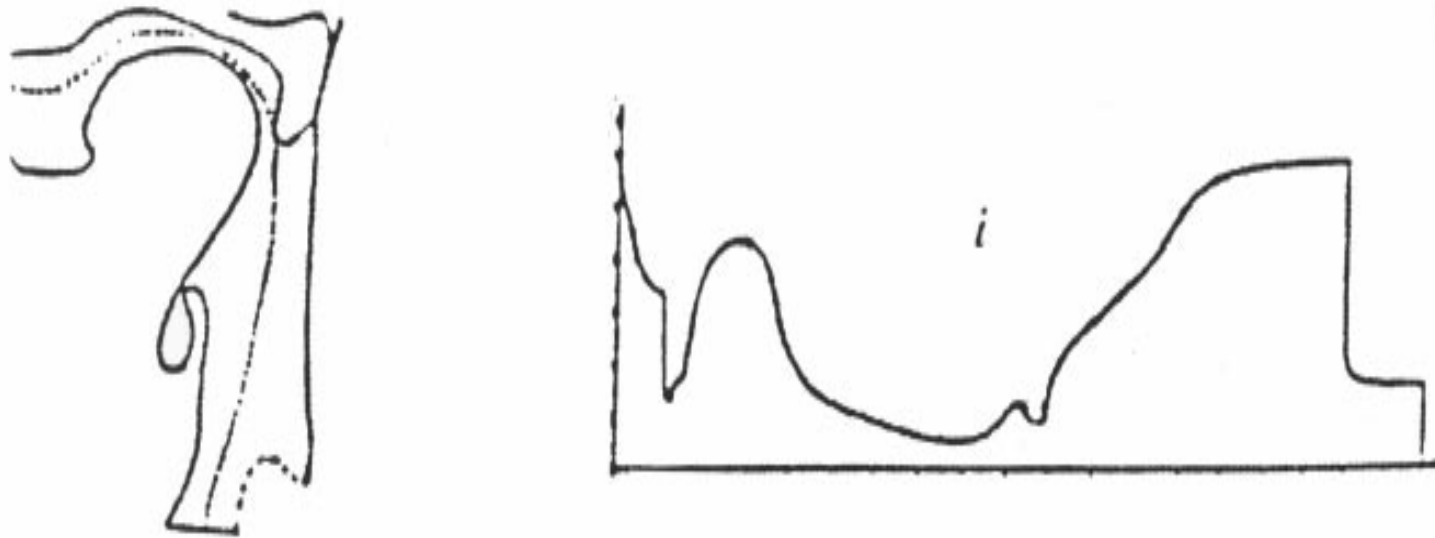


Figure 11.1: X-ray tracing and area function for phoneme /i/

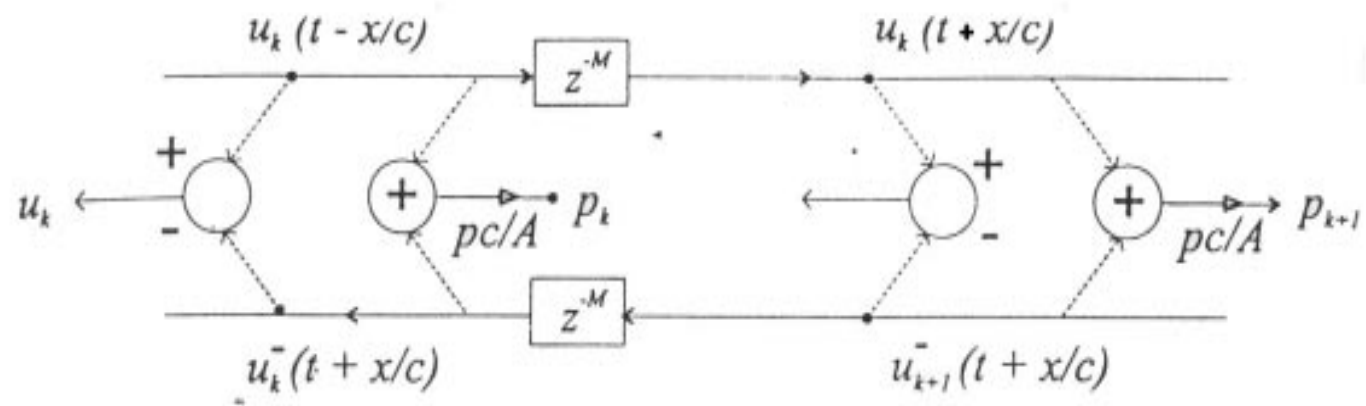


Figure 11.2: Single section of digital wave guide

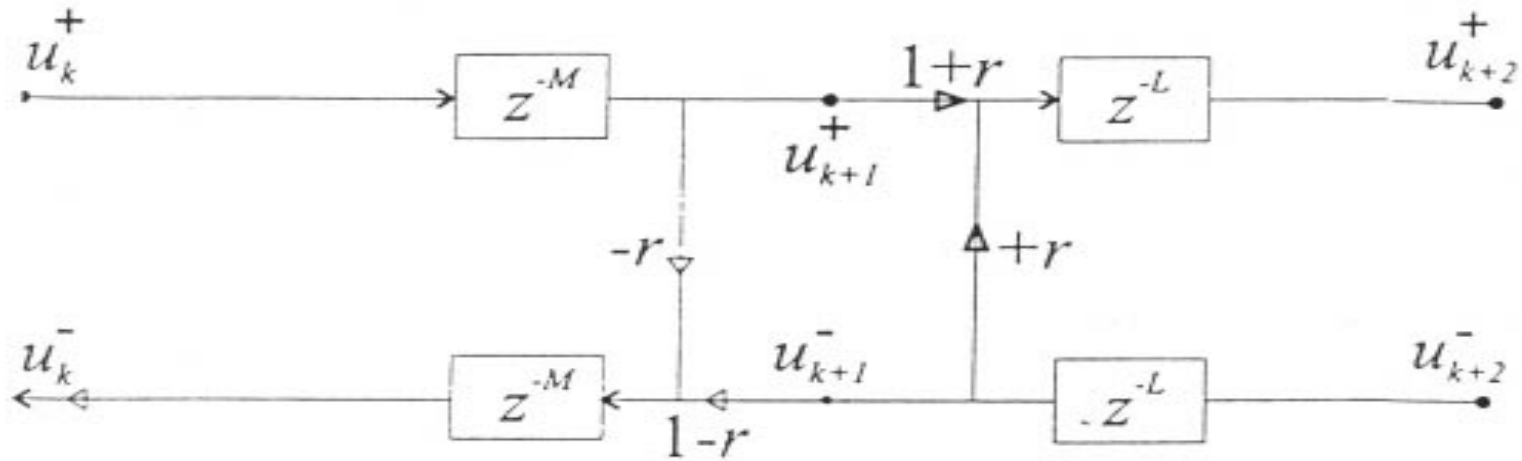


Figure 11.3: Two section digital wave guide

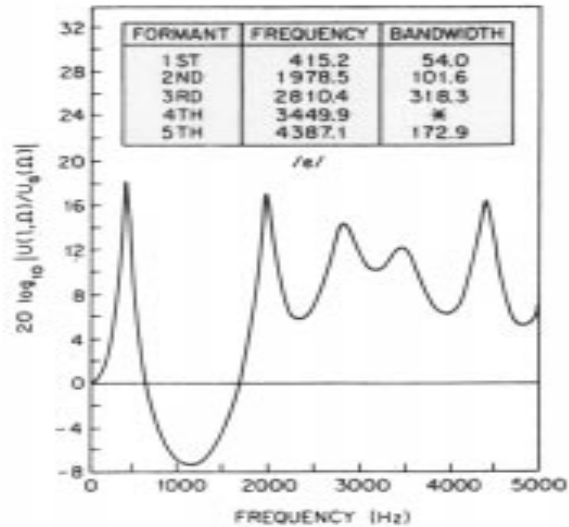
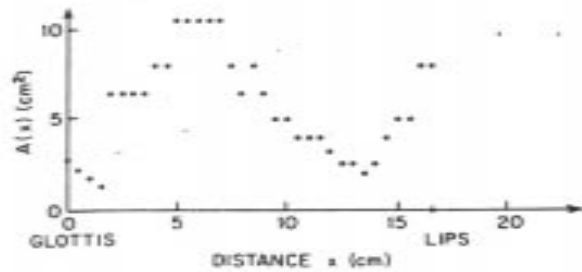


Figure 3.24 Area function and frequency response for the Russian vowel /e/

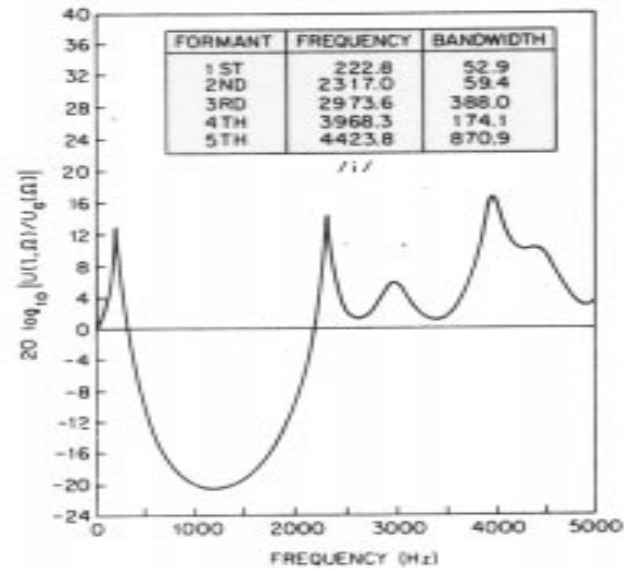
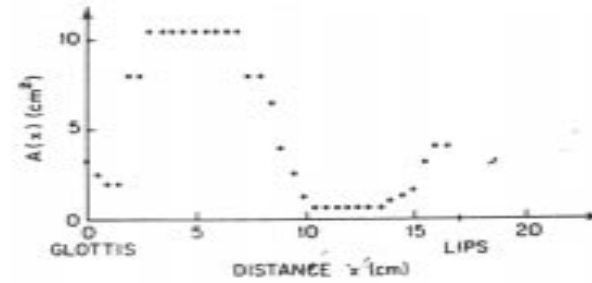
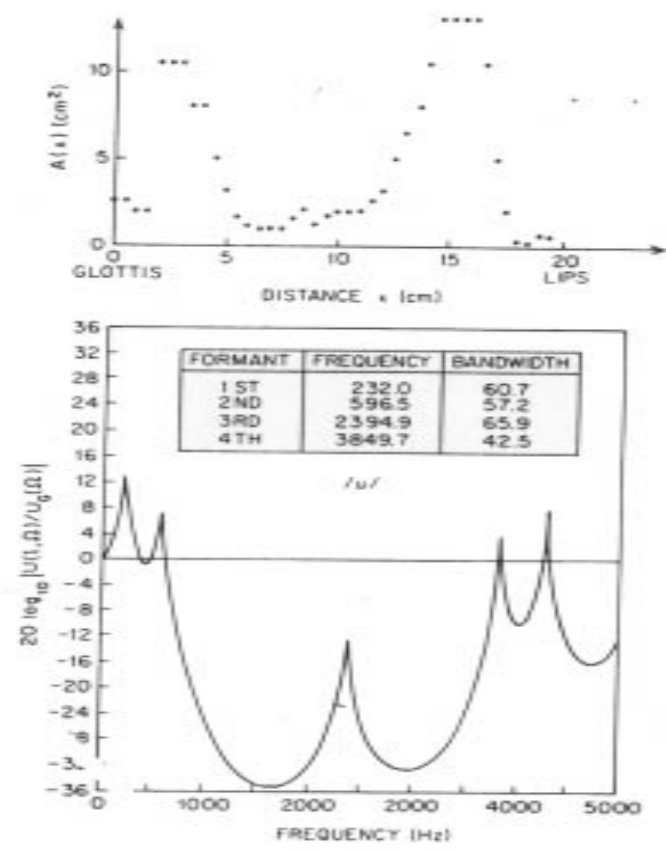
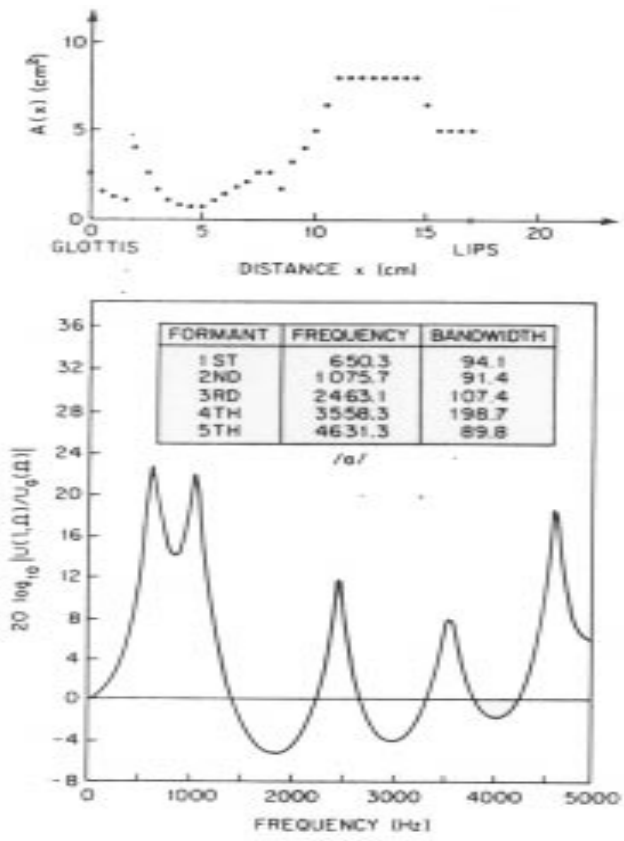
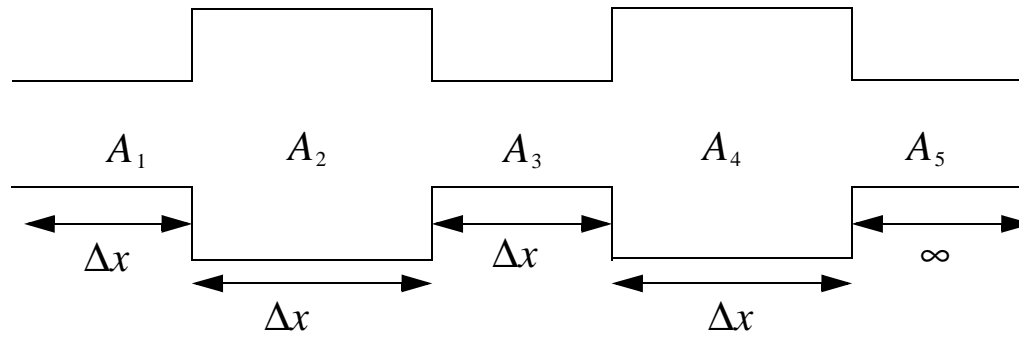


Figure 3.25 Area function and frequency response for the Russian vowel /i/

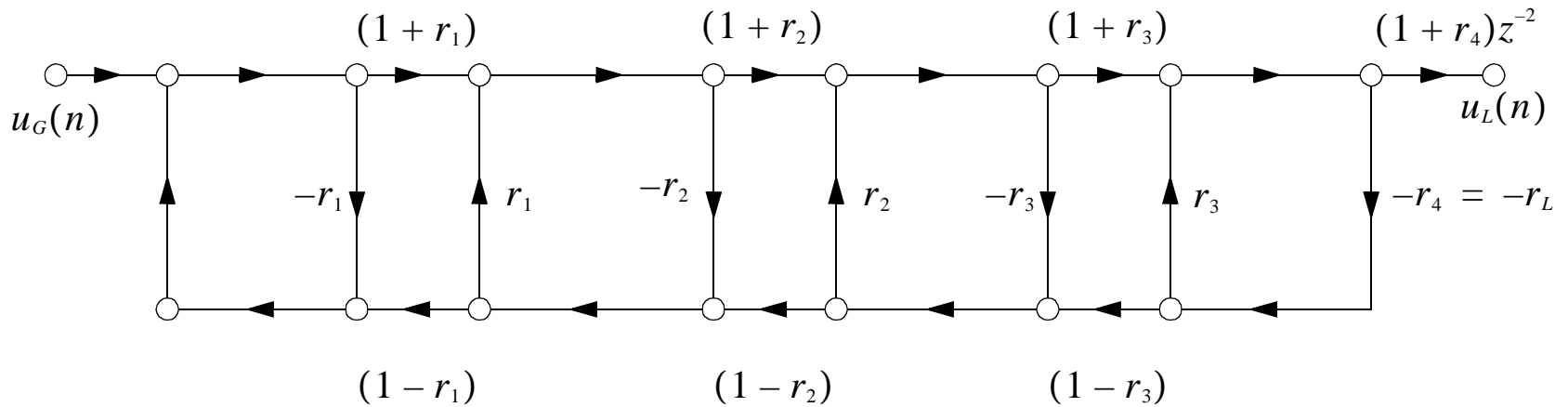


More Complex Tube Structure

a)



b)



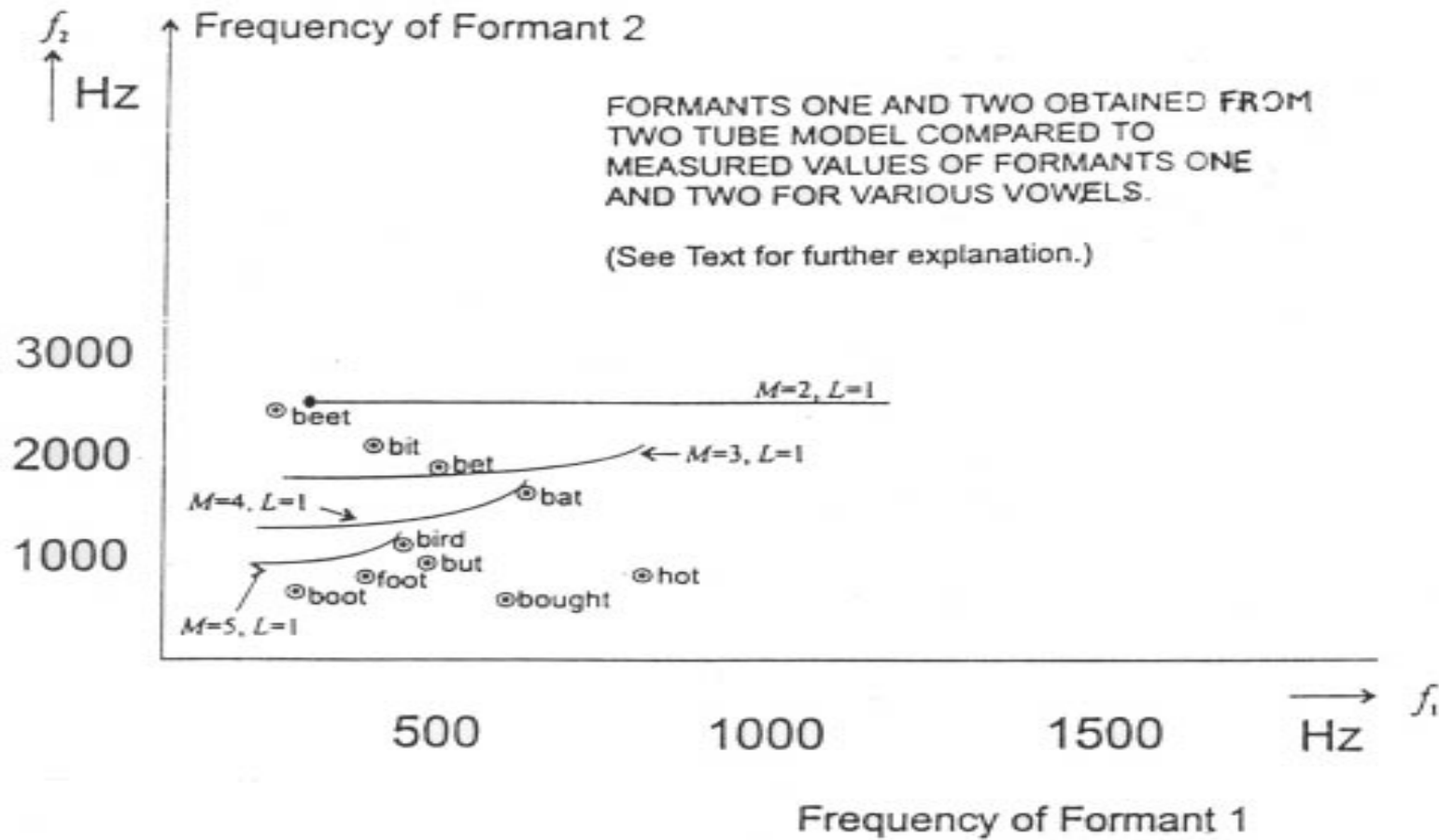


Figure 11.4 Formants 1 and 2 obtained from two tube model



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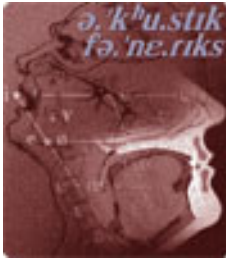
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Speech Production and Perception I

A New Multimedia Course on CD-ROM - \$75

- Developed over three years by teachers of speech science, experts in phonetics and speech research, working with a team of experienced writers, artists and programmers
- Designed to enable students to acquire an intuitive understanding of the correspondence between sound, spectrum and articulation.
- Interactive use of the computer for dynamic, experience-based learning with self- or teacher-guided instruction.
- Uses your Windows-compatible audio card
- Runs on Windows® 95/98/NT and Windows® 3.1
- In use at more than 150 universities and institutions internationally
- [Volume discounts](#) are available

Speech Production and Perception I: An Interactive Multimedia Course provides a non-mathematical introduction to the basic concepts of acoustic phonetics and speech science. The course cultivates genuine understanding of these concepts through personal interaction and experience, using hundreds of interactive models and simulations. Course development was carried out with major support from the National Institutes of Health (MH51970-SBIR).

The course has been created for undergraduate students studying Speech and Hearing Science, Communication Disorders, Linguistics, and Phonetics. Computer Science and Electrical Engineering

students interested in speech transmission and processing will also find the course stimulating and useful. Its cost is about the same as that of a good textbook.

Test-teaching at universities and colleges in the United States and Europe has shown that the course is an effective adjunct to lecture- and demonstration-based teaching, as well as a resource for independent learning by students.

The course incorporates its own state-of-the-art graphic interface, including custom-designed high-speed digital signal processing and full color visual displays. The combination of efficient, innovative code and a sophisticated user interface allows students easy access to the full range of course activities and resources.

The present course consists of units on:

- Spectrograms
- Vowel Acoustics
- Consonant Acoustics
- Speech Perception
- Vowel Perception

In addition, students have access to:

- A Library with IPA consonant and vowel charts. The library also contains more than 100 new digitally recorded examples of the consonants and vowels in the charts, with spectrograms of each one, an interactive glossary with definitions of more than 100 technical words and phrases, and cross-references to textbooks;
- A Lab in which they can make and compare wide- and narrow-band spectrograms of their own utterances, the speech of others, or any other sounds they wish to record.

The course contains more than 200 interactive demonstrations and a dozen interactive exercises on CD-ROM, as well as separate student worksheets, an Installation Guide, and a User's Guide. Typical interactive demonstrations include adjustable filtering of synthetic voicing sources; plotting the vowel spaces of adult and child speakers; identification and discrimination experiments with speech and non-speech stimuli; creating and analyzing conventional and 3-dimensional spectrograms; and, examining animated vocal tracts synchronized with audio playback and spectrogram displays.

A student workbook with more than 40 pages of questions complements the interactive exercises. The worksheets for the course provide students with a permanent written record of the material covered, and the instructor with a convenient way of evaluating the student's comprehension of the course content. The worksheets are keyed to the topics in the course; each worksheet contains from 5 to 15 questions, generally requiring paragraph-length answers. The questions cover the course material at various levels, from what might be expected of students in introductory courses to questions suitable for consideration by advanced graduate students.

An Instructor's Pack, including a Teacher's Guide with sample answers to worksheet problems, is also available.

