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Problem	Points	Score
1(a)	15	
1(b)	15	
1(c)	15	
2(a)	15	
2(b)	15	
3(a)	15	
3(b)	10	
Total	100	

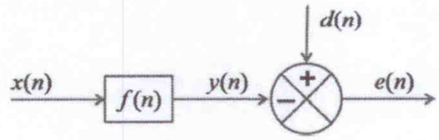
Notes:

(1) The exam is closed books and notes except for one double-sided sheet of notes.

(2) Please indicate clearly your answer to the problem.

(3) The details of your solutions are more important than the answers. Please explain your solutions clearly and include as many details as possible.

1. For the adaptive system shown to the right, assuming $f(n)$ is a linear, time-invariant moving average filter:



$$y(n) = \sum_{i=0}^{L-1} f(i)x(n-i)$$

(a) Derive the normal equations for the minimum least squares error estimate of the filter coefficients.

(b) Develop the concept of a linear prediction filter based on this model (explain how this model is modified to produce a linear prediction estimate of the filter coefficients).

(c) Derive the expression for the autocorrelation estimate of the linear prediction coefficients.

2. Modify the block diagram shown above to produce the basic LMS adaptive filter.

(a) Derive an expression for estimation of the filter coefficients using an iterative-in-time approach.

(b) Compare and contrast this to the approach in Prob. 1.

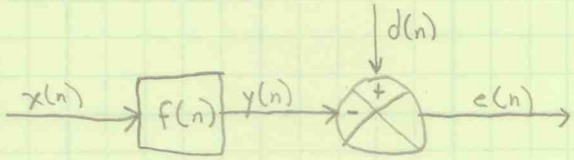
3. Suppose the input signal to the adaptive filter shown above is as follows: $x(n) = ax(n-1) + w(n)$. Assume $w(n)$ is zero-mean white Gaussian noise.

(a) Explain how successful the filter in no. 2 will be at correctly estimating the underlying parameters of this signal. Be as specific as possible and use terms such as the bias, variance, and convergence.

(b) Explain under what conditions we prefer approach no. 2 over approach no. 1. Again, be as specific as possible in your explanation, discussing issues such as, but not limited to, computational complexity.

1a

$$y(n) = \sum_{i=0}^{L-1} f(i)\,x(n-i)$$



$x(n) \rightarrow$ INPUT SIGNAL

$f(n) \rightarrow$ RESPONSE

$y(y) \rightarrow$ OUTPUT SIGNAL

$d(n) \rightarrow$ DESIRED SIGNAL

$e(n) \rightarrow$ ERROR SIGNAL

$$y(n) = \sum_{i=0}^{L-1} f(i)\,x(n-i)$$

$$e(n) = d(n) - y(n)$$

$$e(n) = d(n) - \sum_{i=0}^{L-1} f(i)\,x(n-i)$$

$$J = E\{e^2(n)\}$$

$$\frac{\partial J}{\partial f(j)} = \frac{\partial E\{e^2(n)\}}{\partial f(j)}$$

$$= 2E\left\{e(n)\frac{\partial e(n)}{\partial f(j)}\right\}$$

$$\frac{\partial e(n)}{\partial f(j)} = \frac{\partial}{\partial f(j)}\left(d(n) - \sum f(i)\,x(n-i)\right)$$

$$= -x(n-j)$$

(1a) Con't

$$\frac{\partial J}{\partial f(j)} = 2E\left\{e(n)\left(-x(n-j)\right)\right\}$$

$$e(n) = d(n) - \sum f(i)\,x(n-i)$$

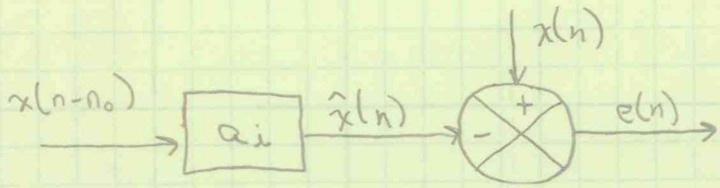
$$= -2E\left\{x(n-j)\,d(n)\right\} + 2E\left\{\sum f(i)\,x(n-i)\,x(n-j)\right\}$$

$$= \cancel{2}\left[\sum_i R(n-i,n-j)\,f(i) - g(n-j)\,d(n)\right] = 0$$

$$\boxed{g(n-j)\,d(n) = \sum_i R(n-i,n-j)\,f(i)}$$

(1b) THE LINEAR PREDICTION ESTIMATES ARE DERIVED FROM A COMBINATION OF ALL PREVIOUS VALUES

$$\hat{x}(n) = \sum a_i\,x(n-n_0-i)$$



$x(n) \rightarrow$ DESIRED SIGNAL

$e(n) \rightarrow$ ERROR SIGNAL

$\hat{x}(n) \rightarrow$ ESTIMATED SIGNAL

$a_i \rightarrow$ FILTER (PREDICTIVE)

$n_0 \rightarrow$ GROUP DELAY

1c

$$e(n) = x(n) - \hat{x}(n)$$

$$= x(n) - \sum a_i (x(n-i))$$

$$J = E\{e^2(n)\}$$

$$= E\left[\left(x(n) - \sum_i a_i (x(n-i))\right)^2\right]$$

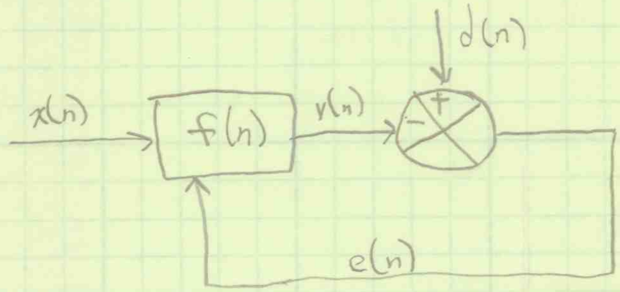
$$= E\{x^2(n)\} - 2\sum_i a_i \left[E\{x(n)x(n-i)\}\right]$$

$$+ E\left[\left(\sum_i a_i (x(n-i))\right)^2\right]$$

SUB $r(0)$ FOR $x(n)$

$$J = r(0) - \sum_i a_i (r(i))$$

2a



$x(n) \rightarrow$ INPUT SIGNAL

$f(n) \rightarrow$ RESPONSE

$y(n) \rightarrow$ OUTPUT SIGNAL

$d(n) \rightarrow$ DESIRED SIGNAL

$e(n) \rightarrow$ ERROR SIGNAL

$$f_{n+1} = f_n - \alpha_n p_n$$

$\alpha \rightarrow$ STEP SIZE

$p \rightarrow$ DIRECTION

$$p_n = \nabla J_n = \frac{\partial E\left[e^2(n)\right]}{\partial f_n}$$

$$f_{n+1} = f_n - \alpha \nabla J_n$$

$$\nabla J = \cancel{2}(Rf - g) \qquad f^* = R^{-1} g$$

$R \rightarrow$ AUTOCORRELATION

$$f_{n+1} = f_n - \alpha (Rf - g)$$

2b THE LMS ADAPTIVE FILTER IS COMPUTATIONALLY CHEAPER THAN IS THE LEAST SQUARES - THE LMS FILTER DOES NOT REQUIRE ANY MATRIX INVERSIONS. ALTHOUGH THE LEAST SQUARES FILTER COULD BE CONTINUOUSLY UPDATED, THE NUMBER OF OPERATIONS REQUIRED TO DO THIS IS PROHIBITIVE. THE LEAST SQUARES FILTER IS BETTER USED AS A WINDOWED FILTER, WHERE THE FILTER COEFFICIENTS ARE UPDATED BASED ON A BLOCK OF DATA RATHER THAN A SINGLE SAMPLE. THE LMS FILTER CAN BE USED IN REAL-TIME ON A SAMPLE-BY-SAMPLE BASIS SINCE IN IS COMPUTATIONALLY SIMPLER THAN LEAST SQUARES.

THE LEAST SQUARES APPROACH ALSO REQUIRES THE DATA OVER THE UPDATE PERIOD (WINDOW) BE STATIONARY, WHERE THE LMS FILTER IS CONTINUALLY ADAPTIVE AND DOES NOT REQUIRE THE INPUT TO BE STATIONARY

3a) THE INTRODUCTION OF WHITE NOISE INTO THE LMS FILTER MAY, IN SOME CASES, CAUSE IT TO CONVERGE FASTER, BUT COULD INTRODUCE A BIAS. THE KEY TO THE EFFICIENT USE OF THE LMS ALGORITHM IS THE STEP SIZE (α). IF α IS TOO SMALL IT WILL TAKE TOO MANY STEPS (AND TOO MUCH TIME) TO ARRIVE AT THE OPTIMAL SOLUTION. IF α IS TOO LARGE, THE ALGORITHM COULD OVERSHOOT THE SOLUTION, WHICH WOULD ALSO EFFECT THE CONVERGENCE TIME, AND COULD ARTIFICIALLY INTRODUCE A BIAS AND INCREASE THE VARIANCE. α IS THE MOST IMPORTANT PARAMETER AND KEY TO THE PROPER OPERATION OF THE LMS FILTER.

3b) $\longrightarrow$ PLEASE SEE ANSWER TO 2b.