ECE7000 - Homework 1

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Problem 1

To calculate the observation series for the Henon map, the **computehenon** function was used. The following images show the time-delay reconstructed phase space (RPS) for different time lags.

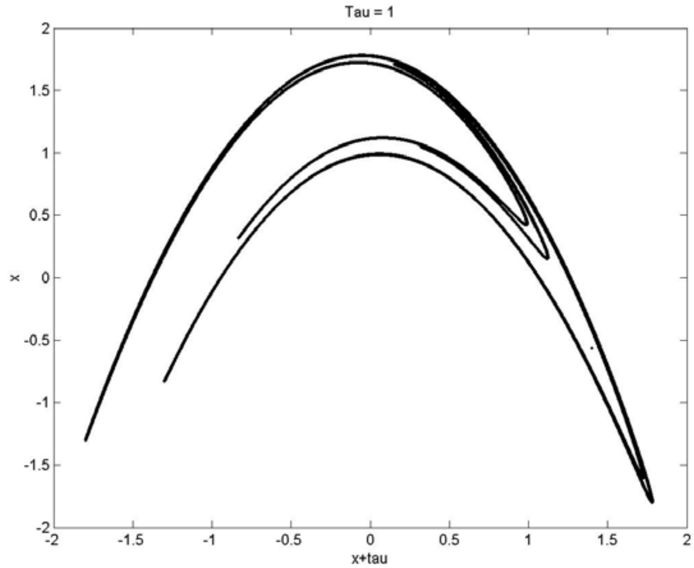


Figure 1.1: RPS Using tau=1

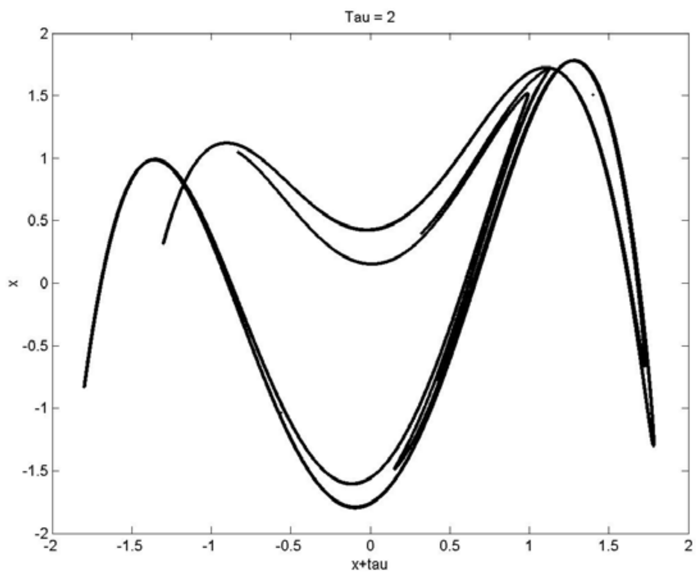


Figure 1.2: RPS Using tau=2

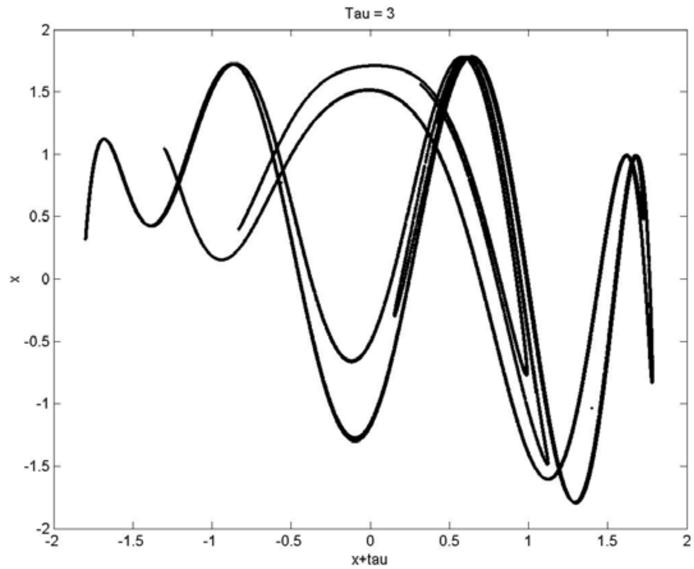


Figure 1.3: RPS Using tau=3

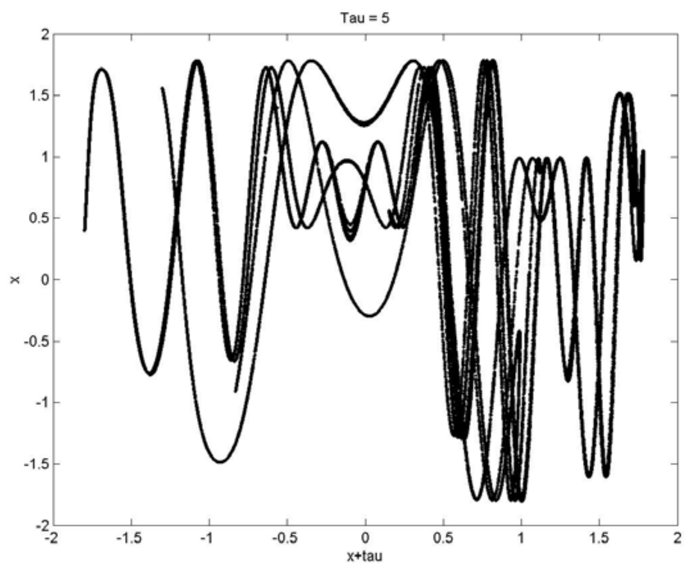


Figure 1.4: RPS Using tau=5

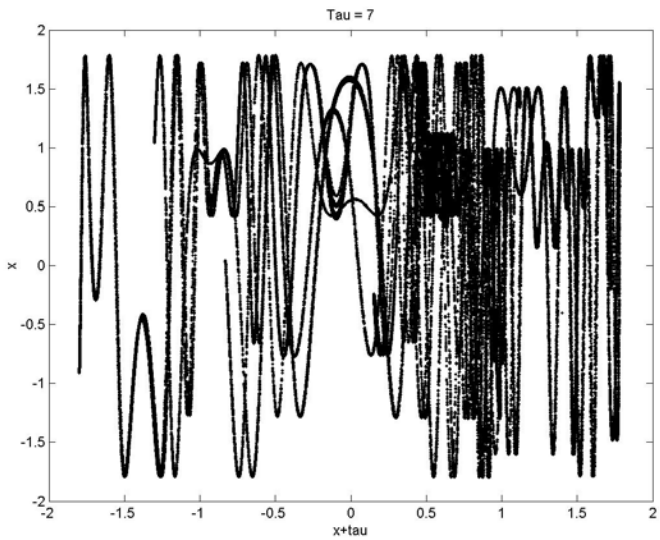


Figure 1.5: RPS Using tau=7

Using a time lag of 'tau=1' yields the clearest information. Time lags greater that 'tau=1' cause the RPS to become distorted. Distortion occurs when tau becomes large enough that x(n) and x(n+tau) become independent, and the RPS doesn't provide any useful information about the system.

Problem 2

The function **computelorenz** was written to compute the Lorenz time series. The variable *x* was chosen as the observable series. The function **delay_reconstruct** was written to reconstruct the phase space using time-delay embedding. (This function was also used to reconstruct the Henon map in Problem 1.) The figures below show the 2-D RPS for different time lags.

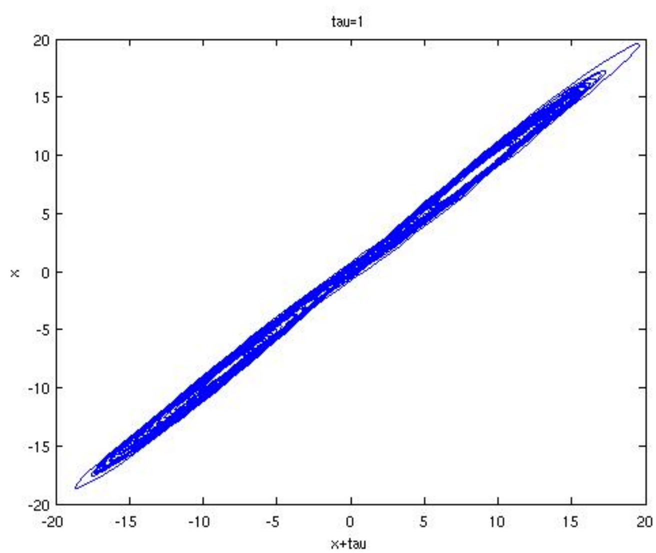


Figure 2.1: RPS Using tau=1

The original and time-delayed series are significantly correlated for tau=1 resulting in a 'line'. In other words, x(n) and x(n+tau) are too close together to reveal any useful information about the system.

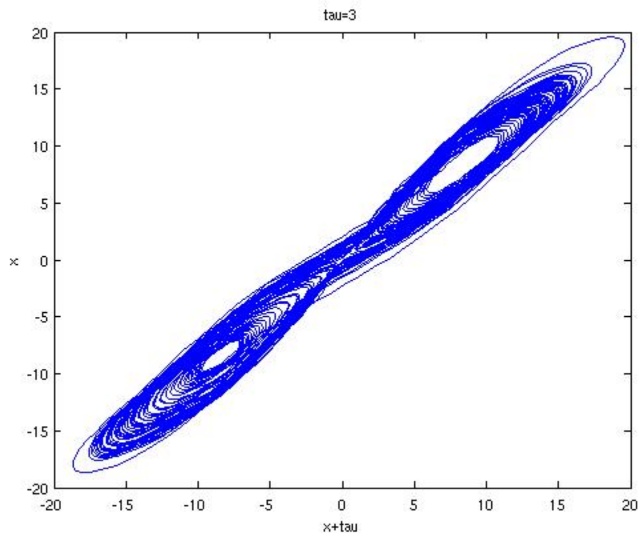


Figure 2.2: RPS Using tau=3

For a time delay of 3, the series are still correlated, but the shape of the trajectory is becoming clearer.

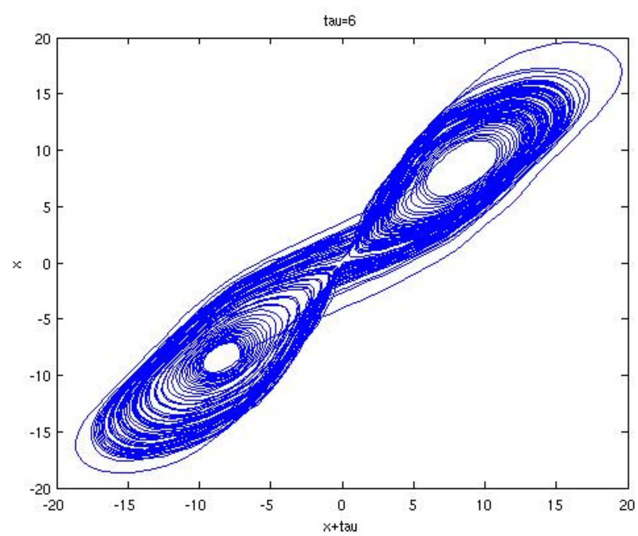


Figure 2.3: RPS Using tau=6

For a time delay of 6, the structure of the trajectory is clearly visible. The following two figures will illustrate how increasing the time delay much further will distort the RPS.

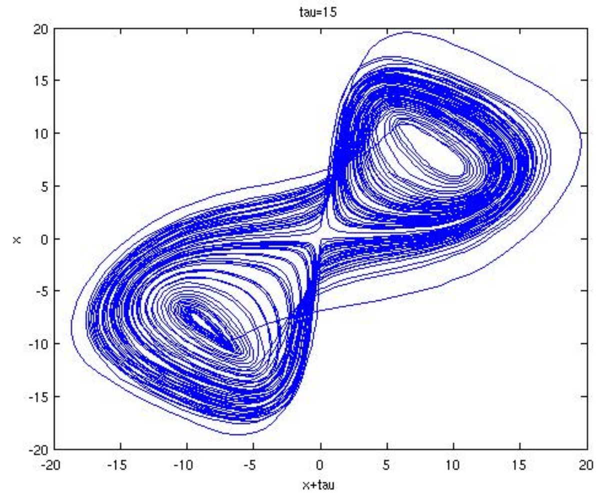


Figure 2.4: RPS Using tau=15

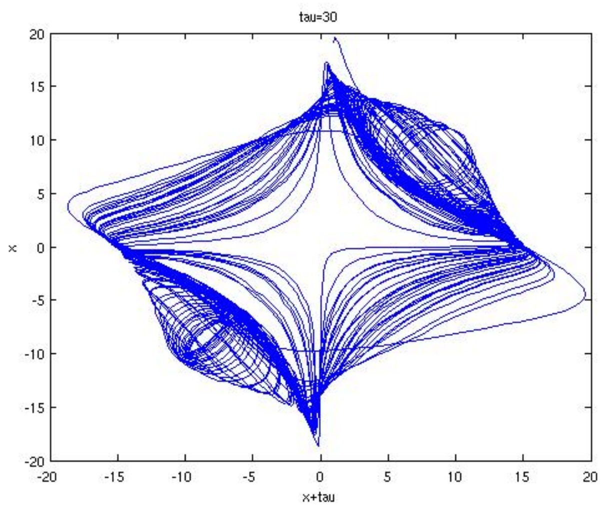


Figure 2.5: RPS Using tau=30

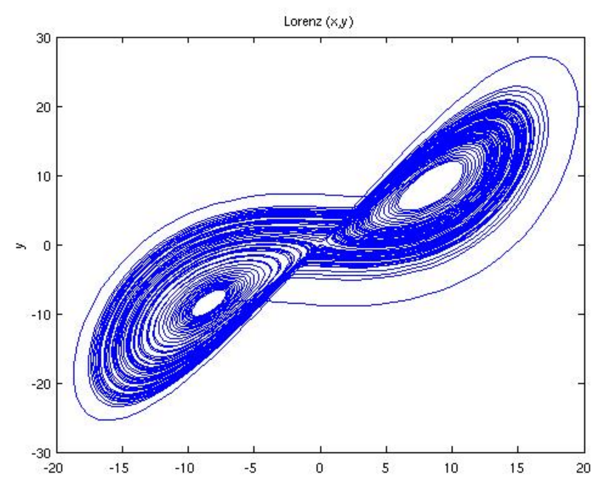


Figure 2.6: Actual 2-D Lorenz Trajectory

Based on Figure 2.3 and 2.6, the RPS using a time-delay of 6 is, visually, the closest representation of the actual phase space. The autocorrelation function for this time series is plotted below.0

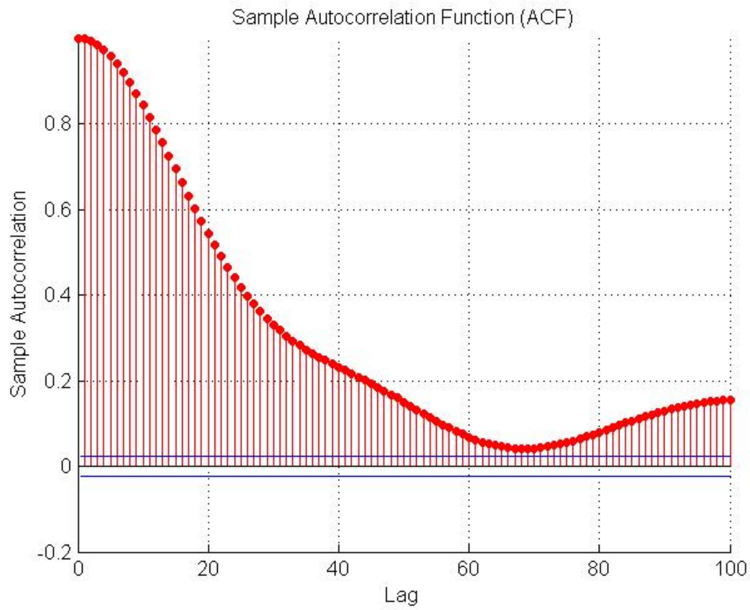


Figure 2.7: Autocorrelation Function

The first minimum of the autocorrelation function occurs around tau=67, however this is not the optimal choice for tau when reconstructing the phase space. The autocorrelation information between the original and time-delay series does not give us good information for choosing tau. This is explained more in Problem 4.

Problem 3

The function **poincare** was written to compute a Poincare map of a given 3-D trajectory and a plane. For this problem, the plane x=0 is used, and the map elements are recorded when the derivative with respect to x is negative.

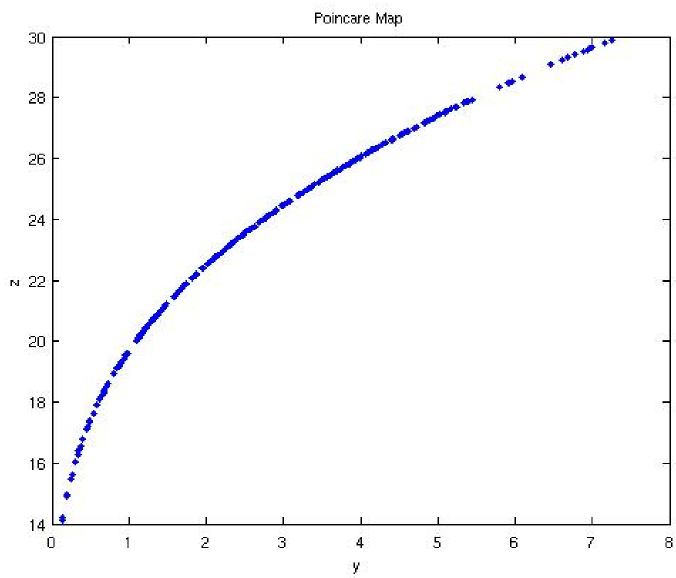
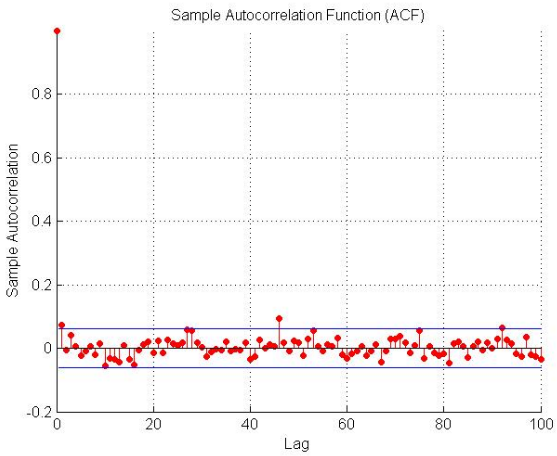


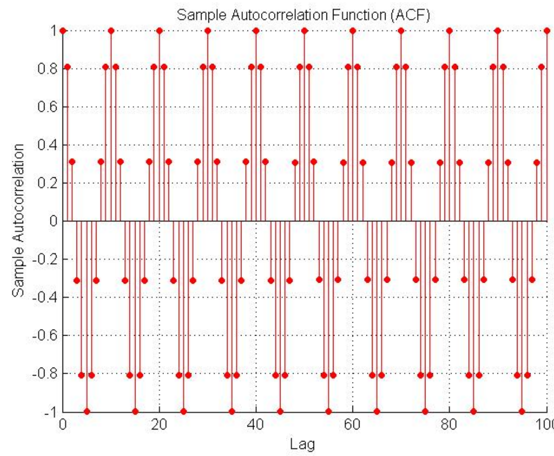
Figure 3.1: Poincare Map of Lorenz Attractor Using Plane x=0

Problem 4

Auto Mutual Information provides a quantity that measures the mutual dependence of a series with itself in a time-delay representation. While autocorrelation is able to detect similarities very well in periodic signals, AMI is better at measuring the similarity of signals that aren't periodic, but still have underlying dependencies. Using autocorrelation to determine the time-delay for embedding will usually yield undesirable results if the system is chaotic and doesn't have strong periodic characteristics. AMI, on the other hand, can measure the more subtle similarities and determine a better time delay. The figures below show the autocorrelation function for:

- sin(2*pi*100*t)
- Lorenz series (see Figure 2.7)
- Gaussian noise


Sample Autocorrelation Function (ACF)
Sample Autocorrelation
Lag
0.8
0.6
0.4
0.2
0
-0.2
0
20
40
60
80
100


Sample Autocorrelation Function (ACF)
Sample Autocorrelation
Lag
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
20
40
60
80
100