

The 1996 Mississippi State University Conference on

## **Digital Signal Processing**

What: EE 4773/6773 Project Presentations  
Where: Simrall Auditorium, Mississippi State University  
When: December 2, 1996 — 1:00 to 4:00 PM

### **SUMMARY**

The Department of Electrical and Computer Engineering invites you to attend a mini-conference on Digital Signal Processing, being given by students in EE 6773 — Introduction to Digital Signal Processing. Papers will be presented on:

- parallel implementations of fast Fourier transforms;
- real-time audible frequency detection and classification;
- analysis of forestry images for scenic content.

Students will present their semester-long projects at this conference. Each group will give a 12 minute presentation, followed by 18 minutes of discussion. After the talks, each group will be available for a live-input real-time demonstration of their project. These projects account for 50% of their course grade, so critical evaluations of the projects are welcome.



## Session Overview

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- 1:00 PM — 1:10 PM: J. Picone, Introduction
- 1:15 PM — 1:45 PM: Michael Balducci, Ajitha Choudary, and **Jon Hamaker**, “Comparative Analysis of FFT Algorithms In Sequential and Parallel Form”
- 1:45 PM — 2:15 PM: **David Gray**, Craig McKnight, and Stephen Wood, “Audible Frequency Detection and Classification”
- 2:15 PM — 2:45 PM: Yaquin Hong, **Nirmala Kalidindi**, and Liang Zheng, “An Algorithm To Determine The Scenic Quality Of Images“
- 3:00 PM — 4:00 PM: Demonstrations in 434 Simrall

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Volume 2

**Digital Signal Processing**

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# Comparative Analysis of FFT Algorithms in Sequential and Parallel Form

by

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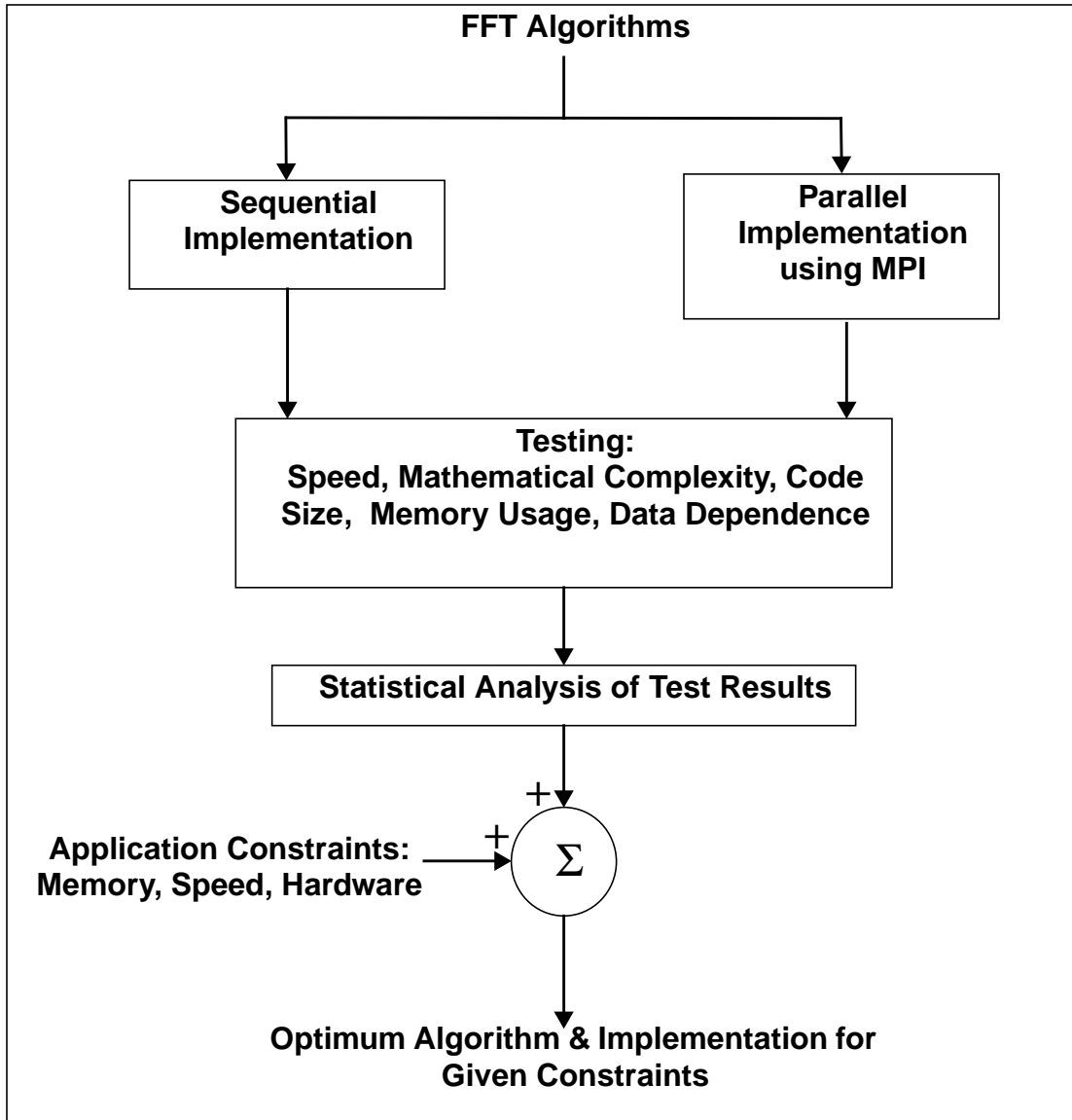
## ABSTRACT

- ➡ Motivation: Need for efficient FFT implementations which fit the specific constraints of an application.
- ➡ We merge a large number of FFT algorithms into a common, object-oriented framework.
- ➡ We have produced a public-domain collection of sequential and parallel implementations of a variety of FFT algorithms
- ➡ Performed statistical analysis of each algorithm based on speed, mathematical complexity, and memory usage.
- ➡ Future Goals: Develop routines which use our statistical results to pick the FFT algorithm implementation which best matches a user's application and hardware constraints.



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# Strategy



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## Theory

### Discrete Fourier Transform (DFT)

☞ Yields frequency spectrum, given N time samples of a signal

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn} \quad k = 0, 1, \dots, N-1$$

$$W_N = e^{-\frac{j2\pi}{N}}$$

**Q:** We have the DFT...why do we need the FFT.

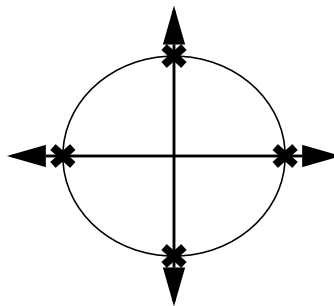
**A:** Complexity reduction...we reduce the number of operations from  $O(N^2)$  to  $O(N * \log_2 N)$ .

**For a 1024 point transform we reduce our operations from  $\sim 10^6$  to  $\sim 10^3$  - a 98% reduction!!**

### The Fast Fourier Transforms (FFTs)

☞ FFTs use symmetry and periodicity of  $W_N$  to eliminate redundancy

$$\begin{aligned} N &= 4 \\ k &= 0, 1, 2, 3 \\ n &= 0, 1, 2, 3 \end{aligned}$$



Only four unique values!

☞ Values of  $W_N$  can be pre-calculated to attain higher speeds.

☞ What's the catch?...We **trade speed for memory (money)**



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### Radix Algorithms

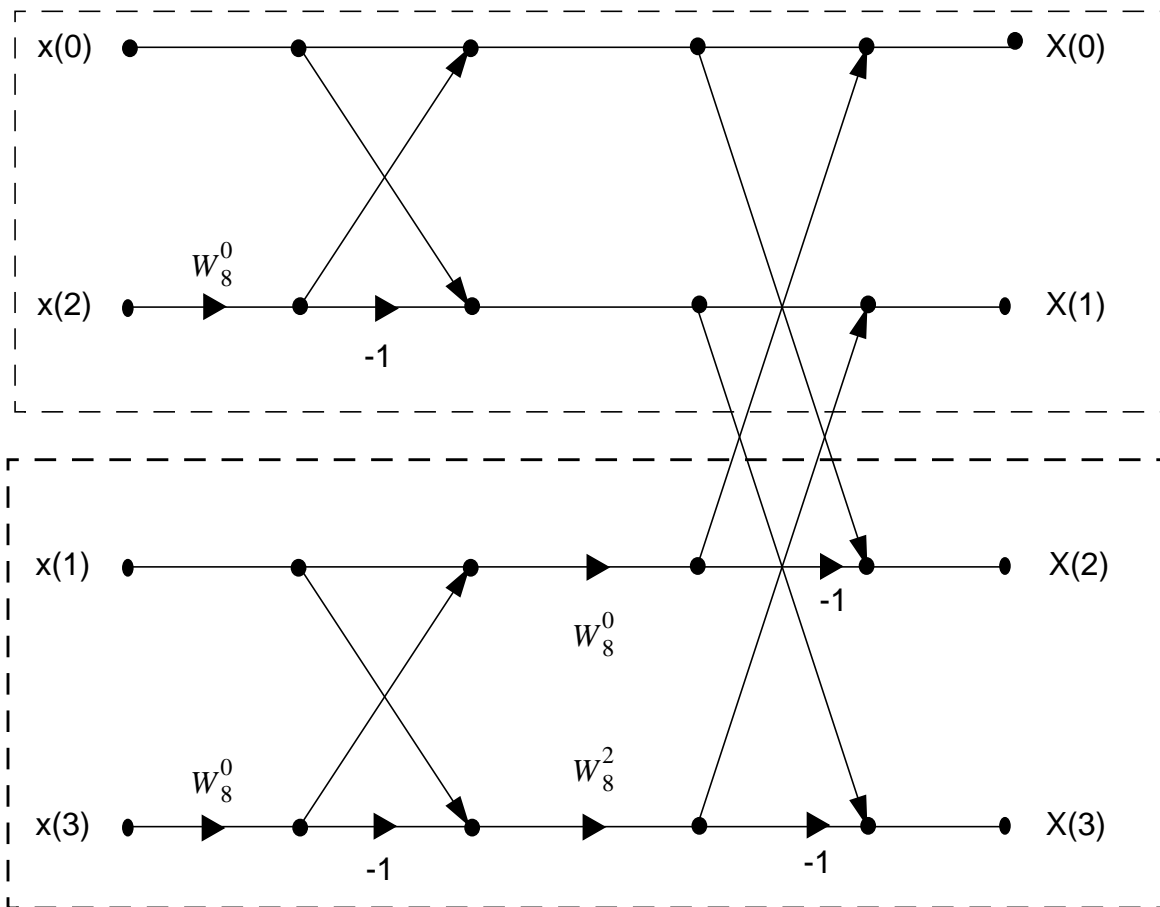
☞ If we limit our data size to be of the form  $R^V$  we can gain efficiency

☞ Most popular form of Radix algorithms are Radix-2 and Radix-4

$$X(k) = \sum_{n=0}^{N-1} x(n)W_N^{kn} = \sum_{n \text{ even}} x(n)W_N^{kn} + \sum_{n \text{ odd}} x(n)W_N^{kn}$$

☞ We break the DFT into a sum of smaller DFTs to reduce complexity.

### Parallel Strategy for Radix Algorithms



Encloses operations which can be performed as a separate process.





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## Split-Radix Algorithm

- ☞ DFT can be broken into a single N/2-point and two N/4-point DFTs.
- ☞ Using the Radix-4 for the N/4 portions reduces the number of calculations - increasing efficiency.
- ☞ Split-Radix uses both Radix-2 and Radix-4

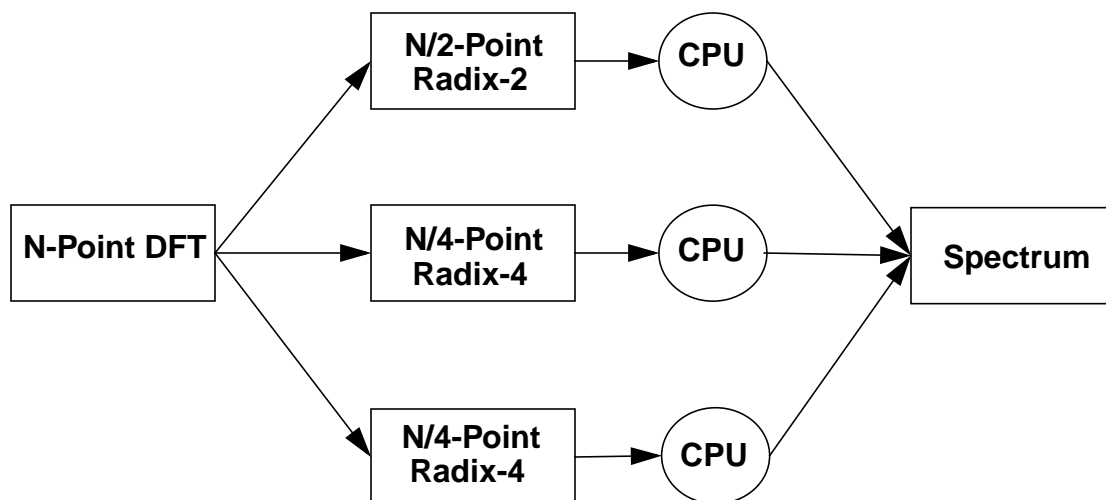
## Hartley Transform (FHT)

$$X(k) = \sum_{n=0}^{N-1} x(n) \left( \cos \frac{2\pi}{N} nk + \sin \frac{2\pi}{N} kn \right)$$

- ☞ **No complex arithmetic!**
- ☞ Can still use Radix-2, Radix-4, Split-Radix, etc.
- ☞ Conversion factors necessary to produce Fourier coefficients.

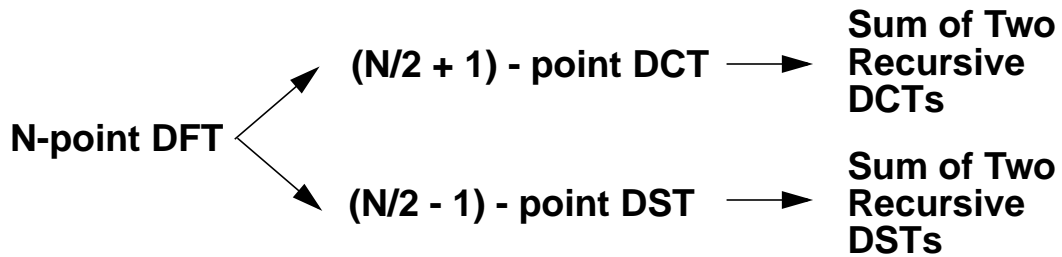
## Parallel Strategy for Split-Radix Algorithm and Split-Radix FHT

- ☞ Similar to Radix...use separate processor for independent butterflies

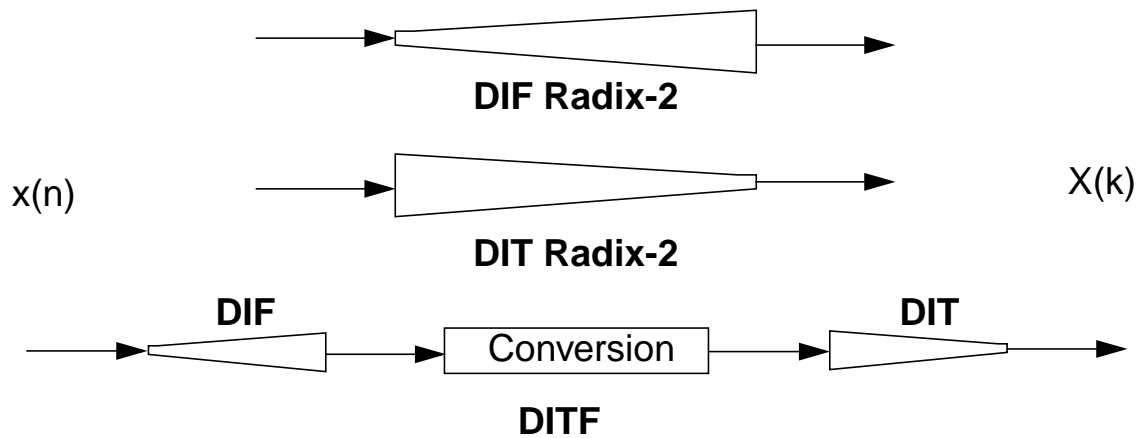


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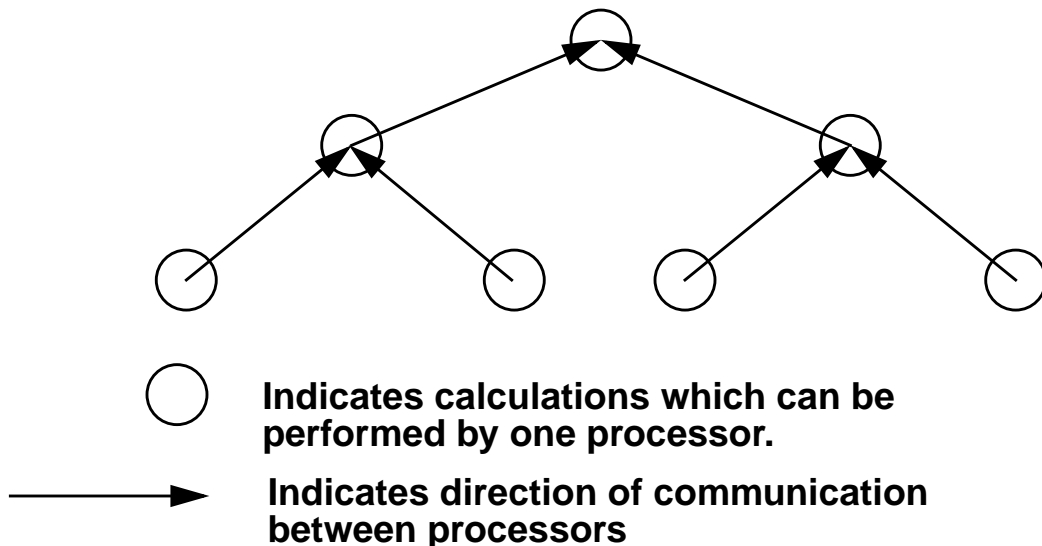
### Quick Fourier Transform (QFT)



### Decimation-in-Time-Frequency Algorithm (DITF)



### Parallel Strategy for Recursive QFT and DITF Algorithms



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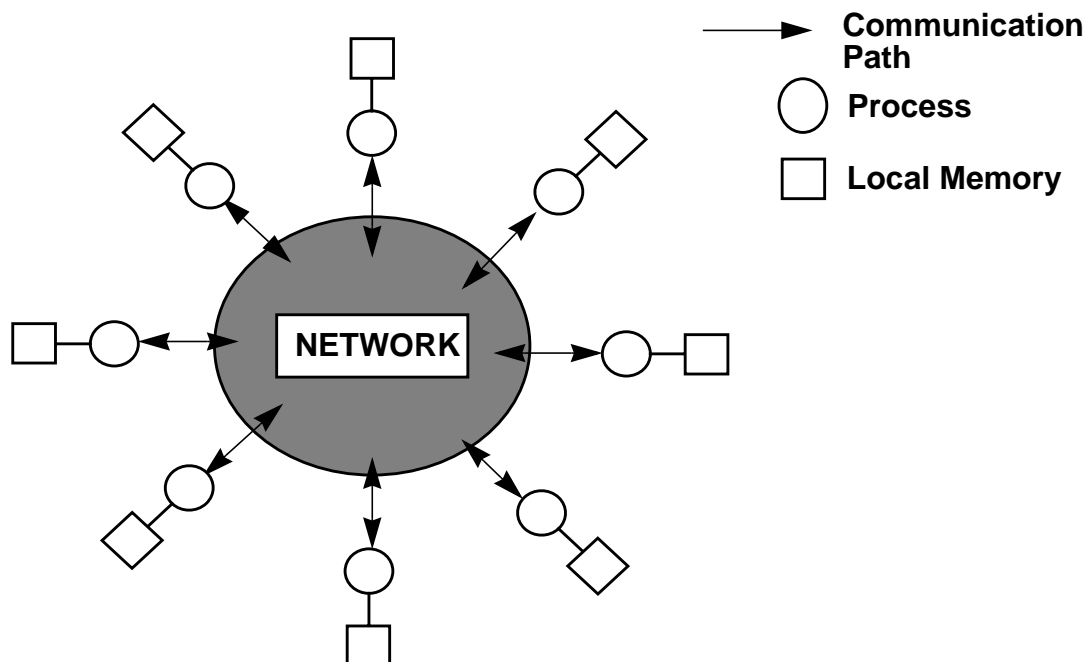
## Parallel Communications

**Q: How do we communicate between processors?**

**A: Message Passing Interface (MPI)**

### What is MPI?

- ➡ Portable set of Library functions
- ➡ Largely hardware independent
- ➡ Provides distributed computing
- ➡ Local memory and a shared communication network



### Why use MPI?

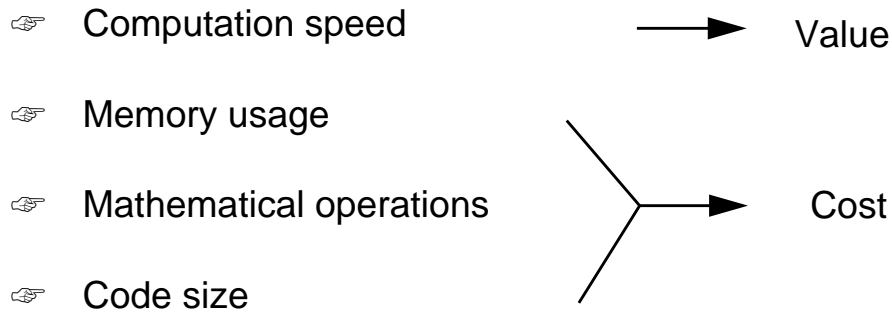
- ➡ Portability
- ➡ Allows collective communications
- ➡ Ease of use - do not need special knowledge of hardware



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## Evaluation Methodology

### Evaluation Criteria



A typical application will need the algorithm with the **fastest possible speed**, but which has the **lowest hardware requirements**.

We desire to determine the trade-off between speed and hardware for each algorithm.

### Iterative Approach

Statistics are taken as averages over a large number of iterations.

Why use an iterative method?

Eliminate transients caused by processor loading

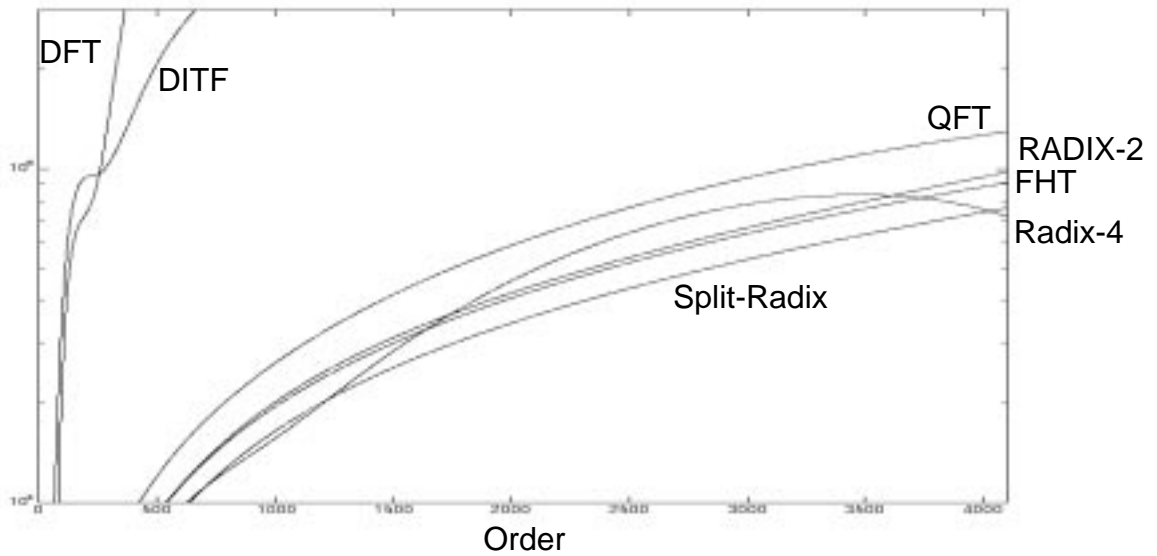
Convergence of time statistics



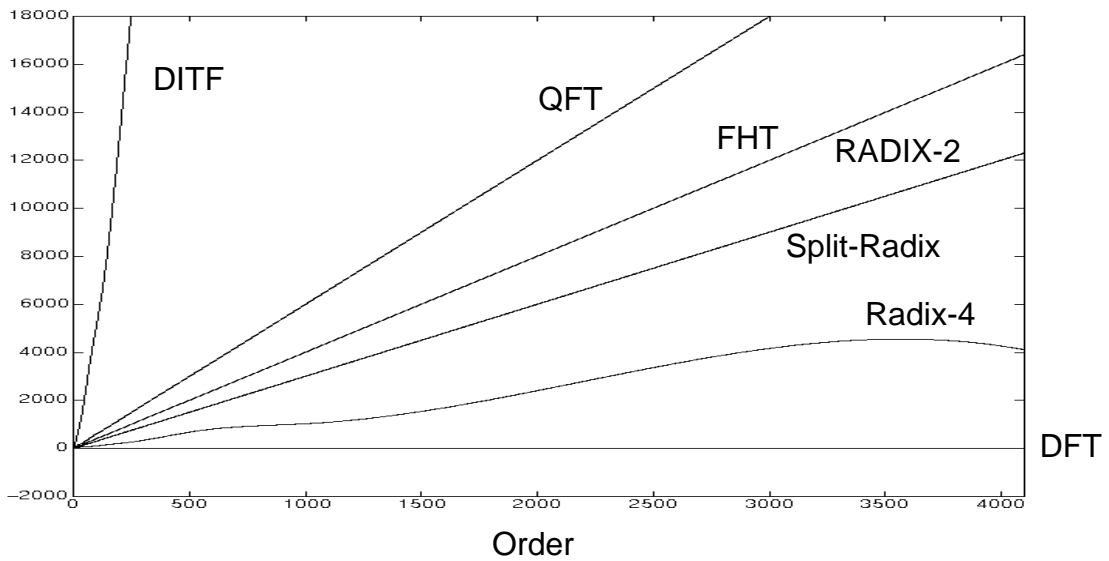
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## Results -> Sequential

Computation Time



Memory(Bytes)



Mathematical Complexity (1024-points)

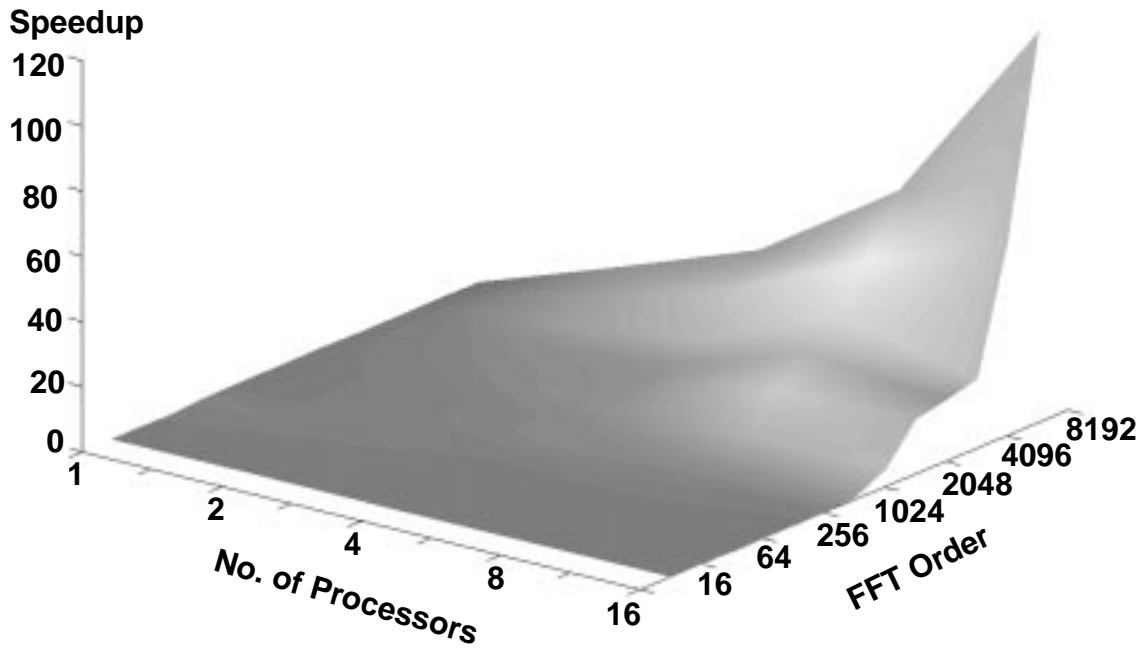
Rank	Algorithm	Total Operations
1	srfft	30764
2	fht	31176
3	qft	53628
4	rad4	56494
5	rad2	67581
6	ditf	112985
7	dft	5243904



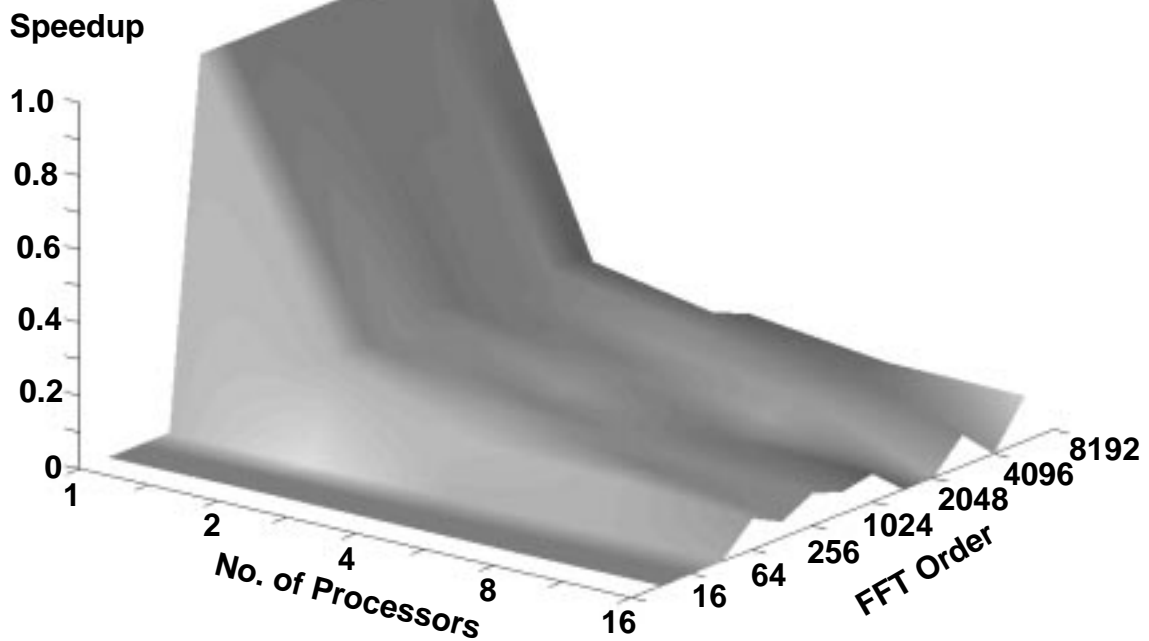
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# Results -> Parallel

## Best Case Results - Data Splitting



## Typical Results



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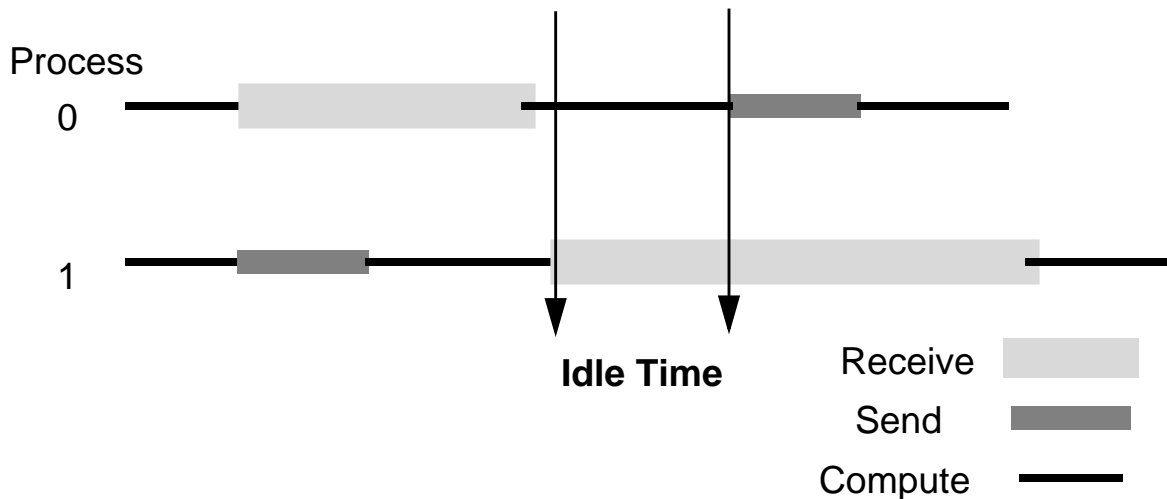
## The Bottom Line

➡ **Increasing the number of processors did not always increase the speed of the algorithms.**

➡ **Sequential code usually gave the best case results.**

➡ **WHY?**

- Not enough overlap in communication and computation



- Processors were idle while waiting for communication.
- Formed parallel code from tight sequential code.

➡ **How do we overcome these problems?**

- Redistribute the loading
- Rewrite routines to take better advantage of the symmetric algorithms



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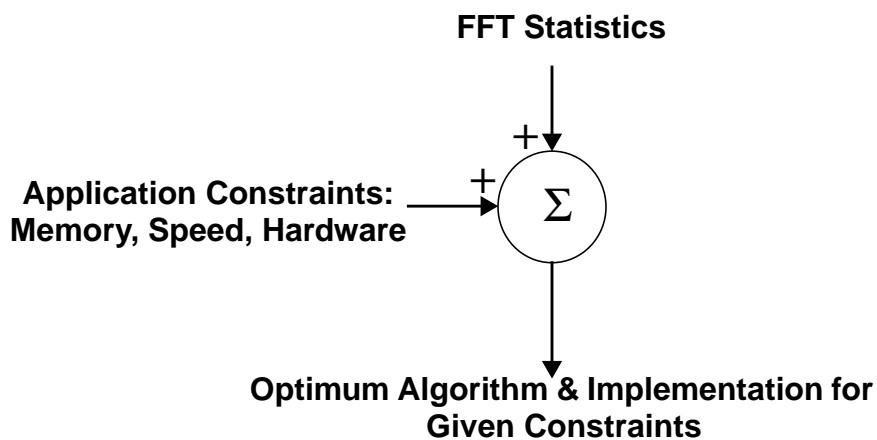
## Future Implications...

### ☞ What have we accomplished?

- First unified collection of parallel FFTs in the public domain.
- Compiled complexity figures which include the overhead involved in coding. (loop counters, intermediate operation, temporary variables, etc.)
- Have framework for application and constraint driven software.

### ☞ The Next Steps...

- Rewrite code with a parallel structure as opposed to converting sequential code
- Explore other parallel processing techniques (shared memory, threads, etc.)





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# Audible Frequency Detection and Classification

by

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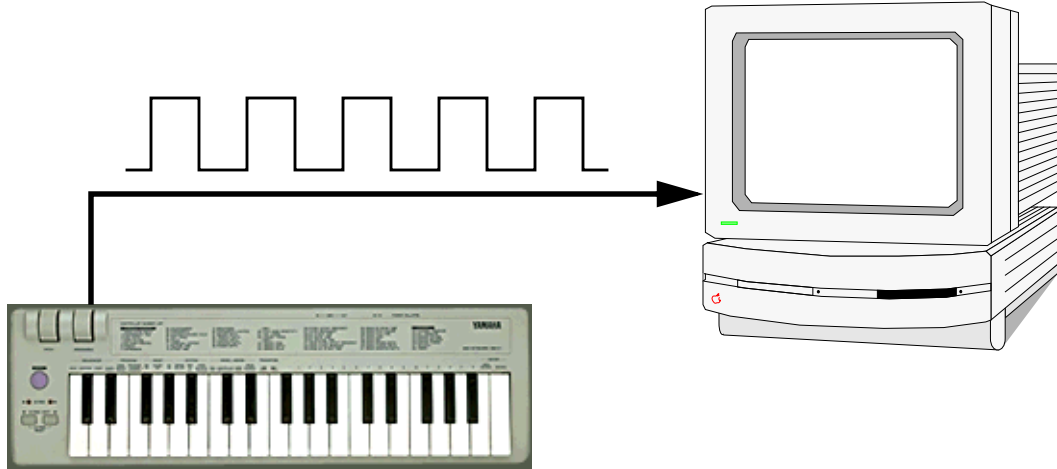
## ABSTRACT

Current music software relies on external input from MIDI capable devices. Because traditional musical instruments are inherently analog, the interaction of musicians and computers is rare. The purpose of this project is to develop a software package for music education utilizing an acoustical instrument interface so that players of all instruments can begin to utilize the computing power of today's world. Musicians who play tones into a microphone will see those tones analyzed in the areas of relative and absolute pitch.



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## A Computer's Musical Experience: MIDI



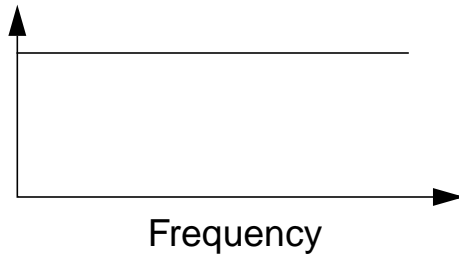
## A Human's Musical Experience: Analog



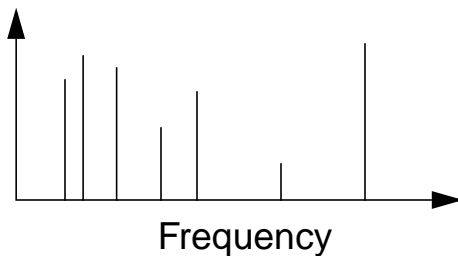
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## Characteristics of a Musical Signal

### Not Music

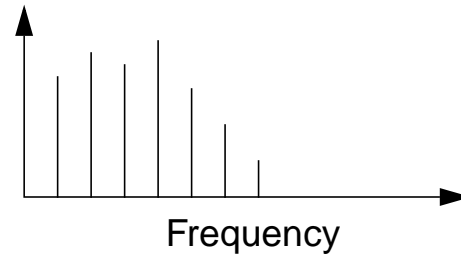


- Signal Energy is Evenly Distributed Across All Ranges of the Frequency Spectrum.



- Signal Energy Is Concentrated at Random Frequencies of the Spectrum.

### Music

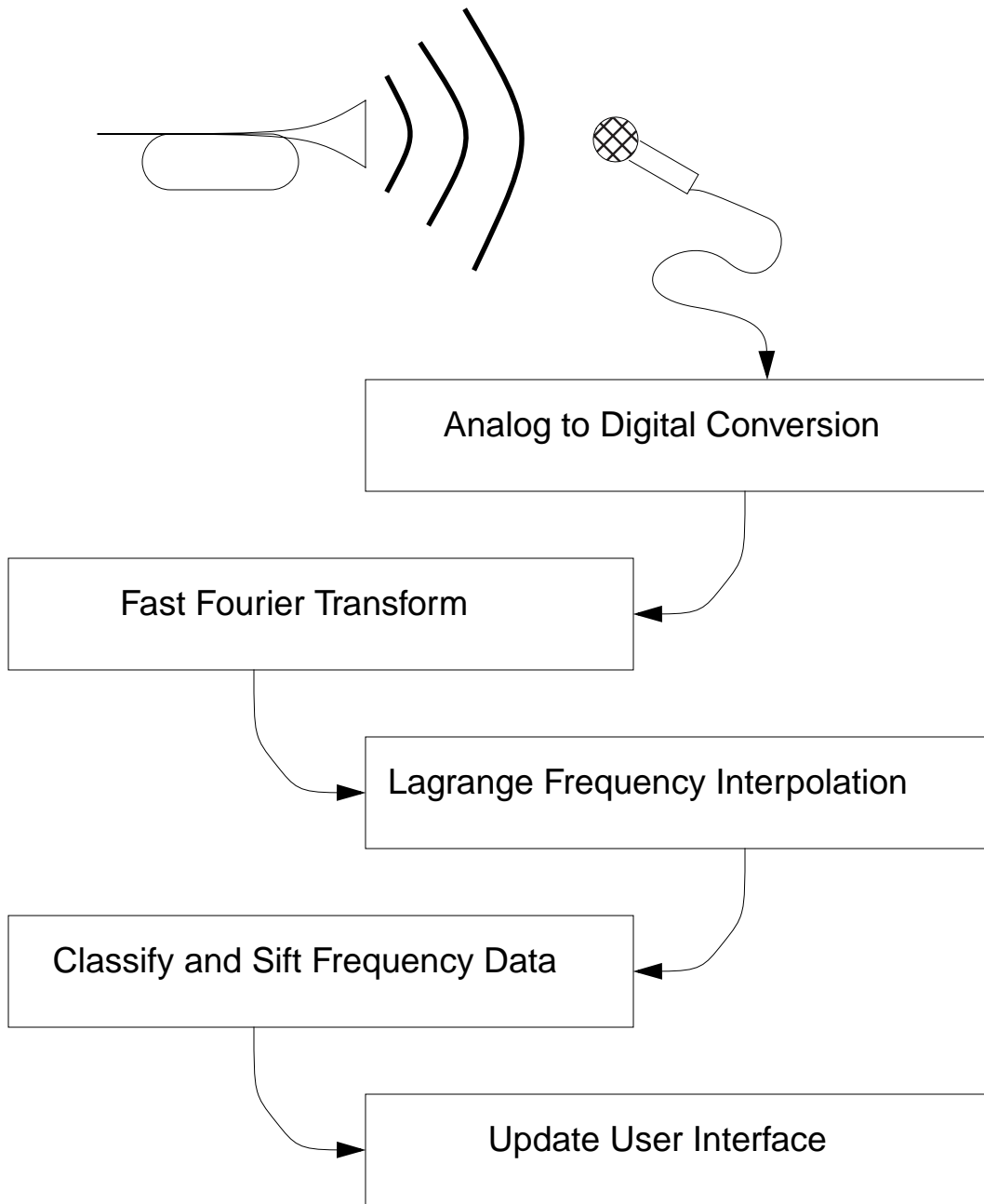


- Signal Energy Is Concentrated in Regular Discrete Portions of the Frequency Spectrum.
- Pattern of Frequency Spectrum Varies Among Different Instruments, But Signal Energy Is Always Concentrated at Integer Multiples of the Lowest (or Fundamental) Frequency,  $f_0$ , of a Note.

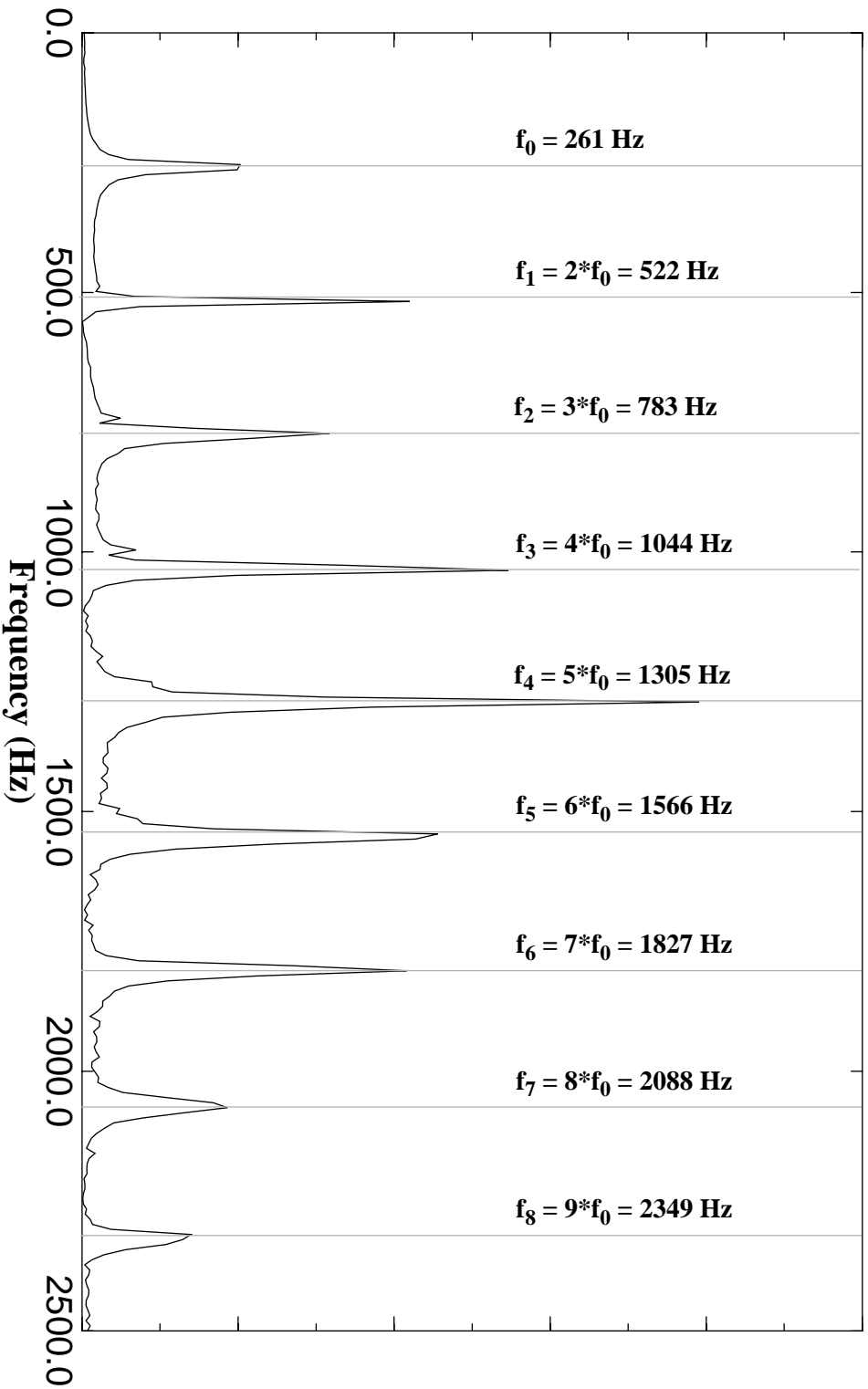


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# Digital Analysis of Musical Sounds



# C4 Played on Trumpet



## The Even-Tempered Scale and Its Derivation

The frequency,  $f_{\text{octave}}$ , of a note,  $N_{\text{octave}}$ , played one octave above a note,  $N_{\text{fundamental}}$ , is twice as high as the frequency of the fundamental,  $f_{\text{fundamental}}$ . That is,

$$f_{\text{octave}} = 2 \cdot f_{\text{fundamental}} \quad (1)$$

Overtones occur at frequencies that are integer multiples of a note's fundamental frequency. Because of this, the same note name does not represent a unique frequency for all keys.

Even-Tempered Tuning is a system where each half-step can be approximated equally well in any scale without retuning the instrument. It is the standard system for tuning musical instruments in the Western world.

The Even-Tempered Scale is designed so that the frequency of each semitone is a factor  $K$  larger than the previous semitone. That is,

$$f_1 = K \cdot f_0 \quad (2)$$

There are 12 semi-tones in one octave. According to equation (1), the thirteenth note, the octave, must be twice the frequency of the first note. Dividing the octave into equal semi-tones, it can be shown that

$$K = \sqrt[12]{2} \approx 1.05946 \quad (3)$$



## Restrictions of FFT: Frequency Resolution

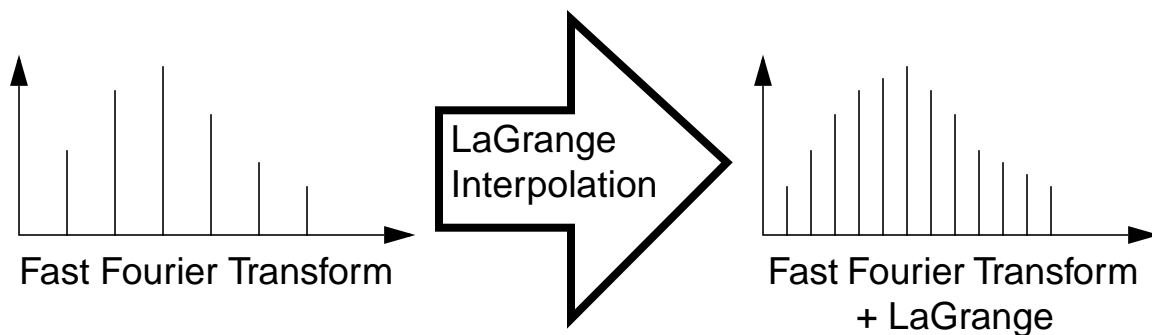
The frequency resolution,  $\Delta f$ , of an N point FFT with sample frequency,  $f_s$ , is defined as

$$\Delta f = \frac{f_s}{N} = \frac{10000}{1024} = 9.765625 \text{ Hz} \quad (1)$$

Recalling the distribution of notes using Even-Tempered Tuning, we realize that for low scale tones, several semi-tones can occur in a 10 Hz window. This is an unacceptable resolution for low frequency pitches. We can improve the resolution of the FFT by interpolating between points.

Given m data points, the LaGrange Interpolation Technique calculates a unique polynomial of order (m-1) that passes through all m points.

Using a 5-point LaGrange Interpolation, the resolution of the data in the frequency domain can be roughly doubled.

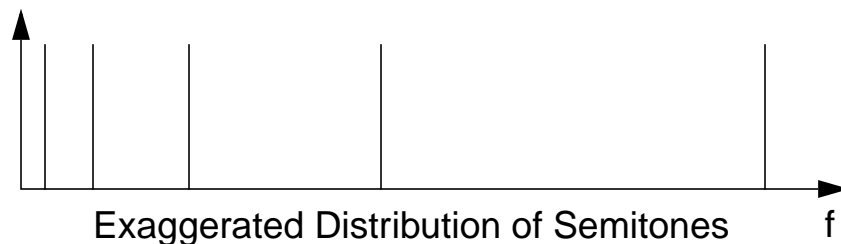




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## Tuning a Note's Overtones to Increase Accuracy

Notes in Even-Tempered Tuning are not evenly spaced throughout the frequency domain. Each semitone is  $K$  times larger than the previous one. Therefore the number of Hertz between semitones increases as the pitch of the note increases.



So the discrepancy between the actual frequency of a peak and the interpolated frequency of that peak becomes less significant as the note's frequency increases.

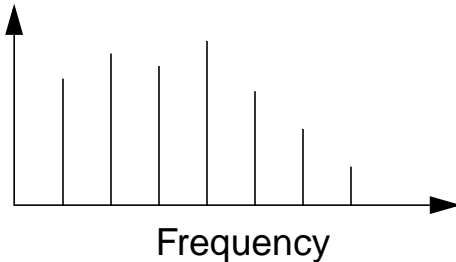
Because a note produces overtones at integer multiples of the fundamental frequency, a direct relationship exists between the frequency of the overtone and the frequency of the fundamental.

Interpolating a note's overtones using LaGrange, we can be within a constant error that becomes less significant for a higher overtone. We can also correct for the intrinsic error introduced by Even-Tempered Tuning.

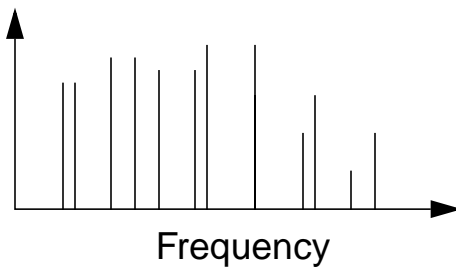


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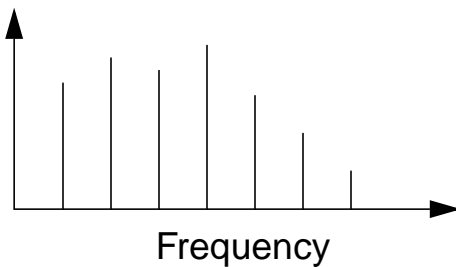
## Project Database



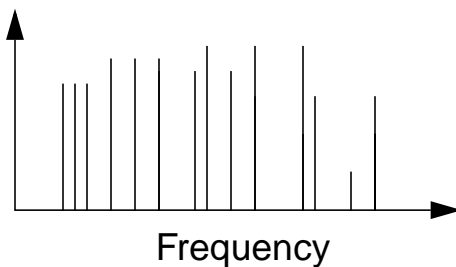
- **Phase I:** Single note  
Played for entire file.  
Each note played for three cases: flat, sharp, and in tune. Built for Tuning.



- **Phase II:** Two notes  
played together. Each interval within an octave recorded. Used for Interval Detection.



- **Phase III:** Three notes  
played sequentially in time. Used for Tuning and detecting note changes.



- **Phase IV:** Three notes  
played at once in a triad.  
Three different qualities of triads for each instrument.



## Results

- Running our software, musicians can play tones into a microphone and see those tones analyzed in the areas of relative and absolute pitch.
- **Using our software to evaluate the Project Database:**
  - The software correctly identifies the fundamental in all files of Phase I.
  - The software tunes the Phase I Database to a great deal of accuracy compared to a commercial tuner.
  - The interaction of multiple notes makes classification of fundamentals more difficult than in Phase I. However, the software classifies the fundamentals properly in most cases.
  - The fundamental was correctly identified in most of Phase III.
- **Using our software to evaluate real-time data:**
  - The software performs in real time using Network Audio as an Analog to Digital Converter.
  - The software performs in real time as well as it does on the Project Database except for data that is very loud or that contains many notes.



## Summary and Areas For Future Research

- We successfully built a tone analyzer that works in real time with Network Audio as an Analog to Digital Converter.
- This research could be used as a basis for music notation or music education software designed to be used with instruments that are not MIDI capable.
- The software analyzes the musical data in the areas of relative and absolute pitch.
- Adjustment of the noise floor filter could improve the performance of the software.
- Adjustment of the “tolerance” for classifying overtones could improve the performance of the software. This could eliminate some unwanted high-frequency spikes by classifying them as overtones instead of as notes.



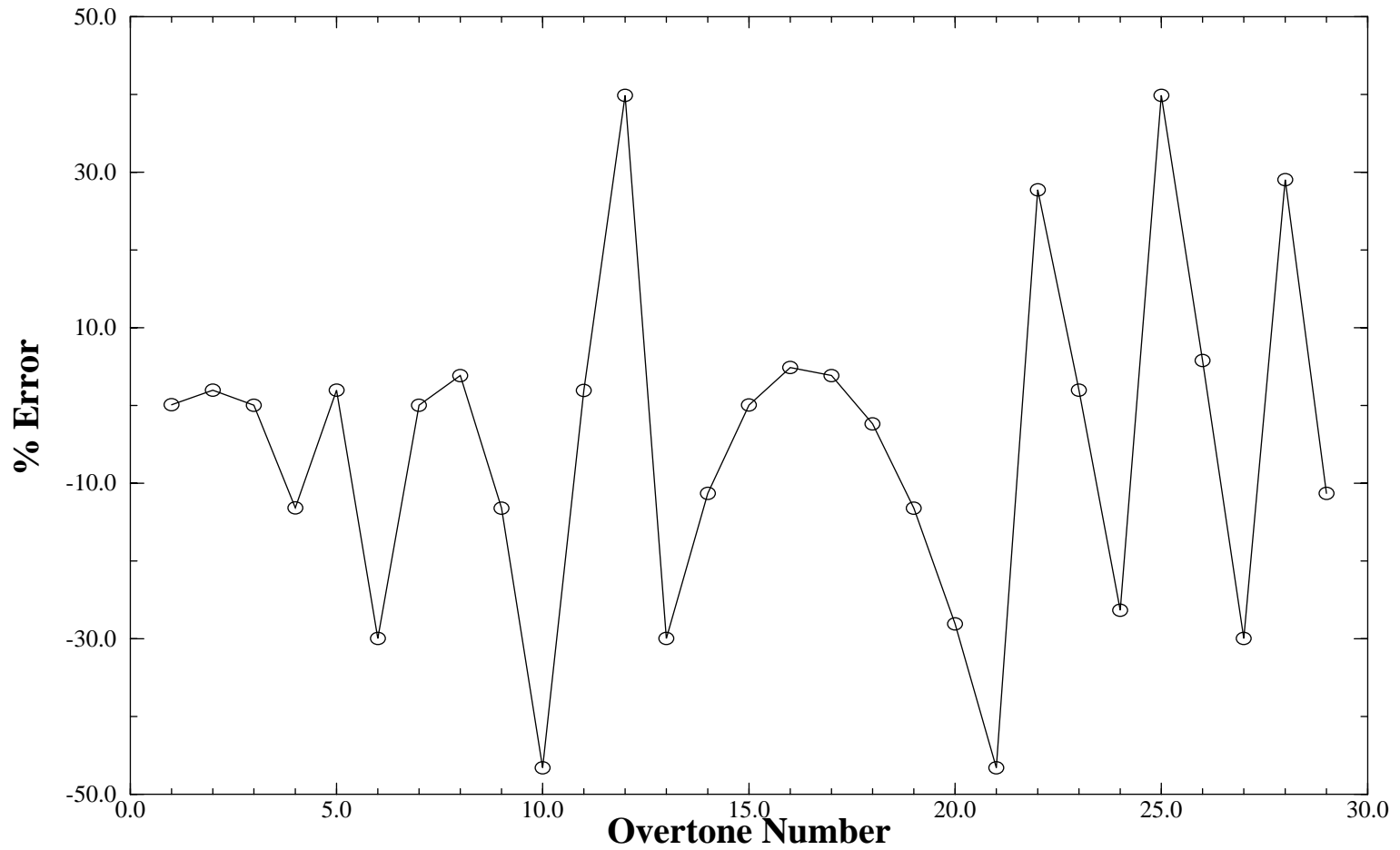
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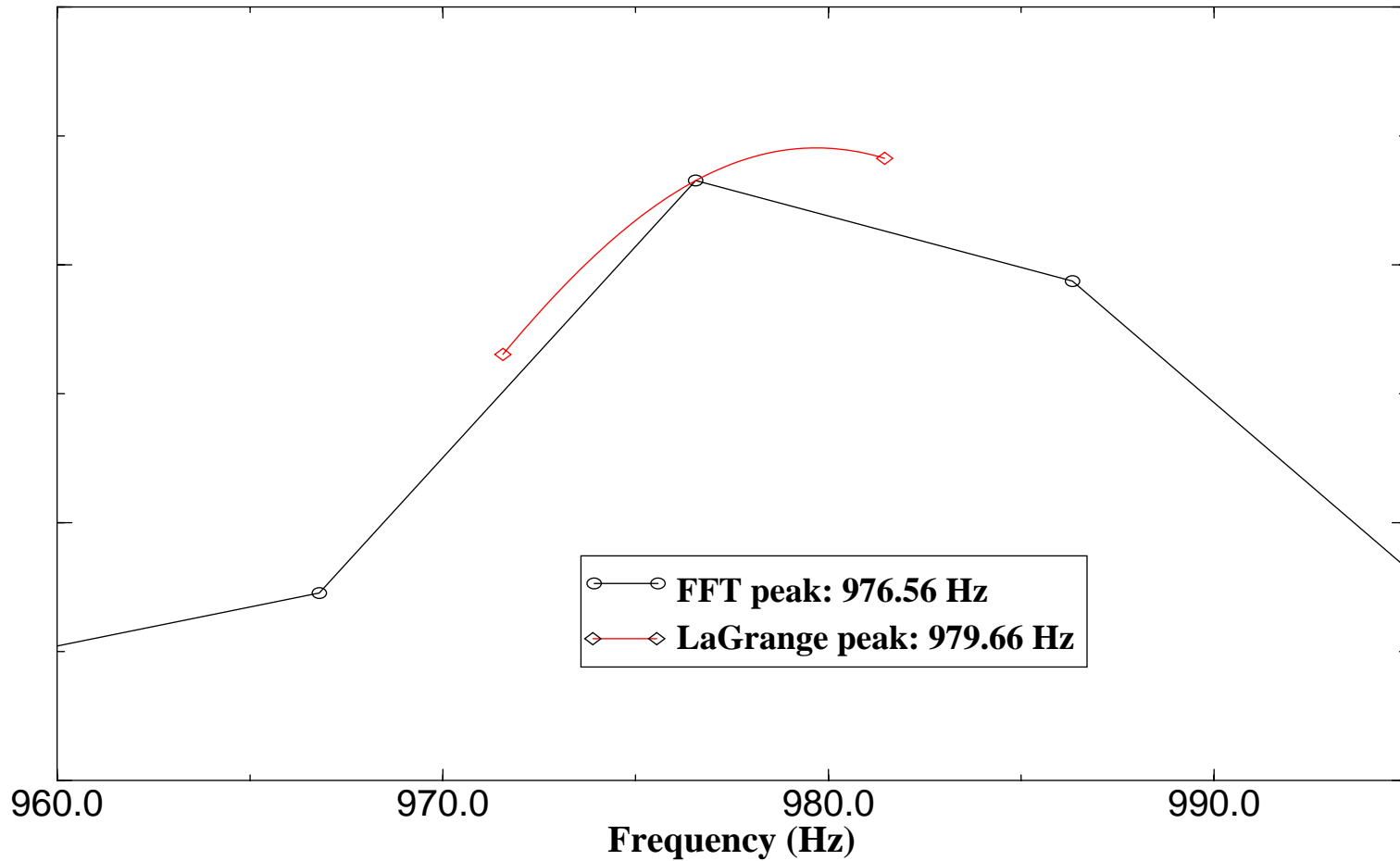
# Overtone Error Using Even-Tempered Scale



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# 981 Hz Peak

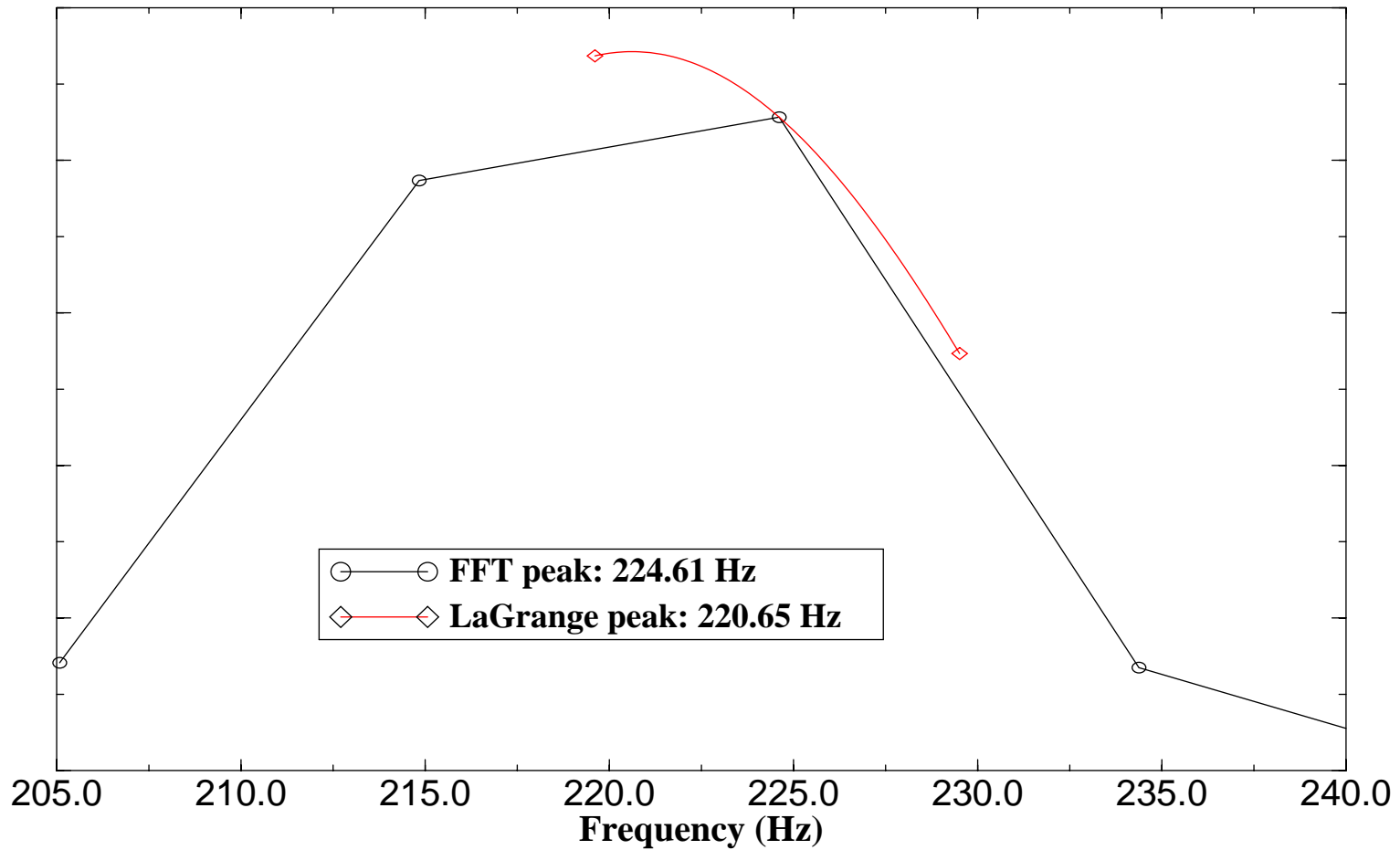
## Increased Accuracy Using LaGrange Interpolation



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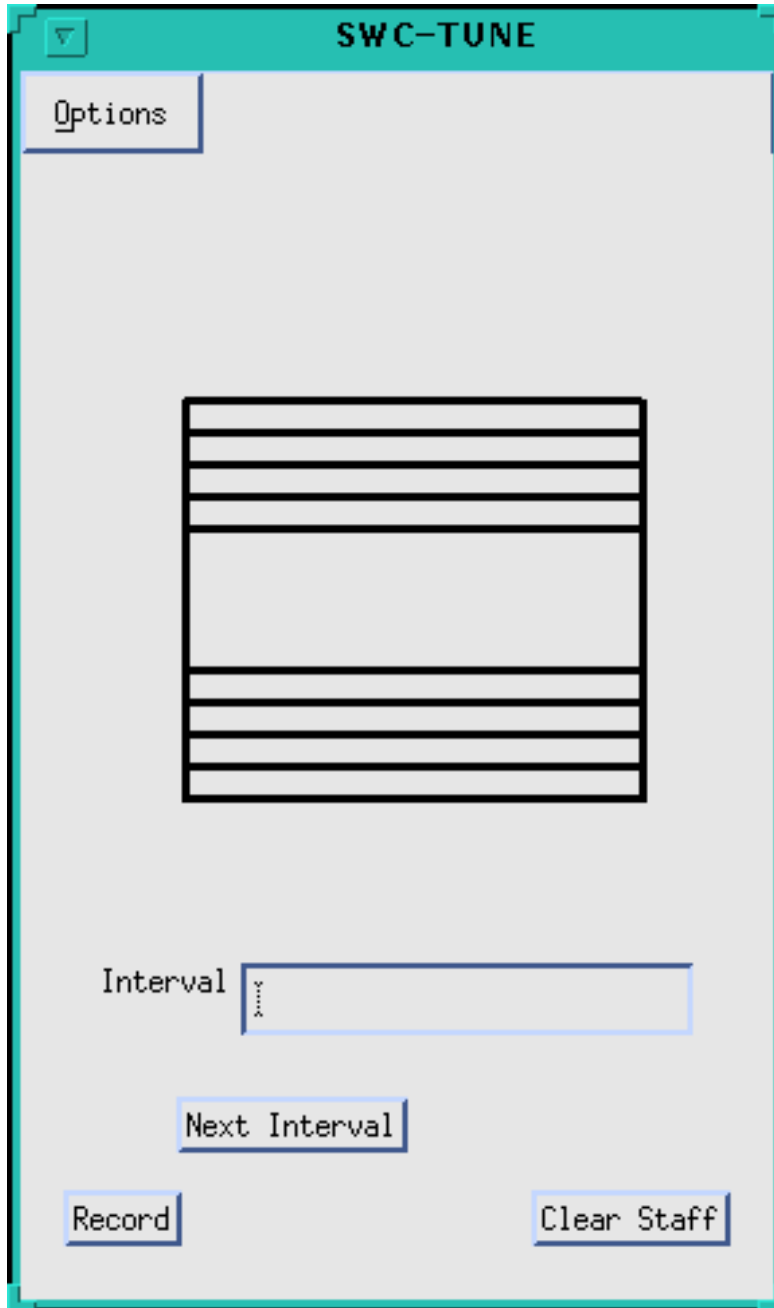
# A3 Played on Bass Guitar

## Increased Accuracy Using LaGrange Interpolation

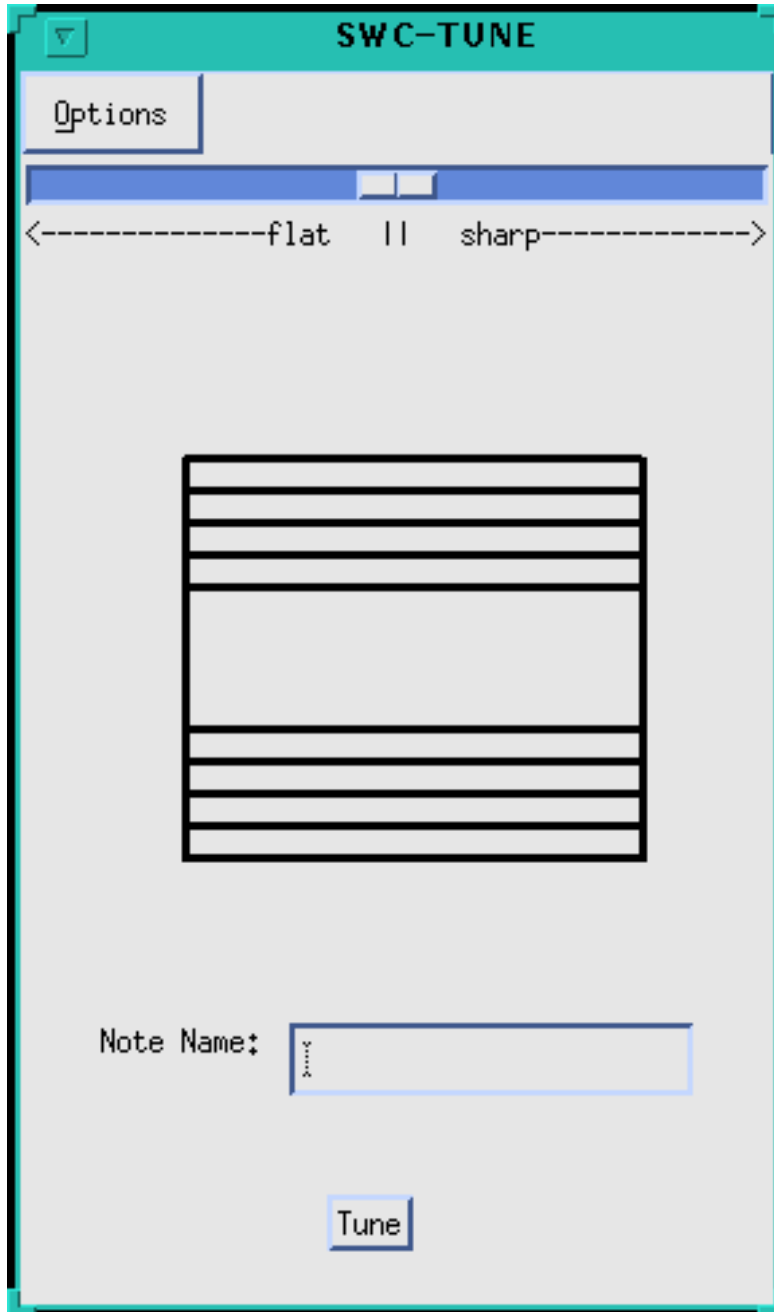




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# An Algorithm to Determine the Scenic Quality of Images

by

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Mississippi State University

## ABSTRACT

The United States Forest Service wishes to determine the scenic quality of images to maintain the beauty of the forest inspite of cutting and also for recreation purpose. This project determines the scenic quality of image on a scale from "0" to "1". We have the database consisting of 680 unique images given by the forestry department. The database has four pictures of the same image taken during all the seasons of the year. This will also help to study the effect of seasons on the scenic quality of the image.

There are subjective scenic beauty ratings available for each of the images in the database. Some of the parameters which effect the scenic quality of the image are the color, number of vertical lines, texture of the image and entropy. For this project we are dealing with only color and number of vertical lines. Histogram is developed to compute the mean of each of the color in the image and edge detection is done to calculate the number of vertical lines.



## Overview

- ❑ Scenic Beauty Estimation
- ❑ Decision making in forest planning
  - ☞ To preserve recreation
  - ☞ To preserve aesthetic resources

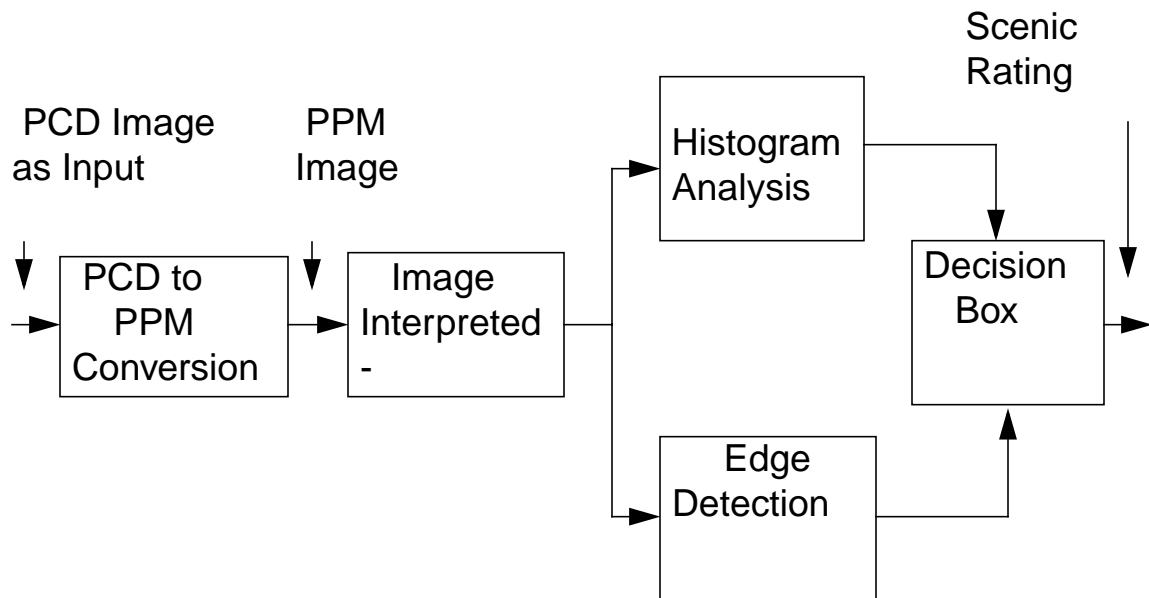


## What to implement

- ❑ Parameters to be determined in scenic beauty determination
- ☞ Color: Histogram
- ☞ Vertical Lines: Edge Detection
- ☞ To compare the derived scenic rating with the actual value



## Block Diagram of the System



## Explanation of the Algorithm

- Calculating the mean of each of the color
- Converting the color image to gray-scale image
- Performing the edge detection
- Calculating the number of vertical lines
- Evaluating the scenic beauty



## Mathematical Equations

### ☞ Color to Gray Conversion

$$Y = 0.299 R + 0.587 G + 0.114 B$$

### ☞ Mask used in Edge Detection

$$G_x = (I_7 + 2I_8 + I_9) - (I_1 + 2I_2 + I_3)$$

$$G_y = (I_3 + 2I_6 + I_9) - (I_1 + 2I_4 + I_7)$$

-1	-2	-1
0	0	0
1	2	1

-1	0	1
-2	0	2
-1	0	1





## Evaluation

- ➔ Computing the scenic beauty based on mean and vertical lines
- ➔ Evaluating the program on the existing database
- ➔ Comparing the derived scenic beauty and the actual beauty

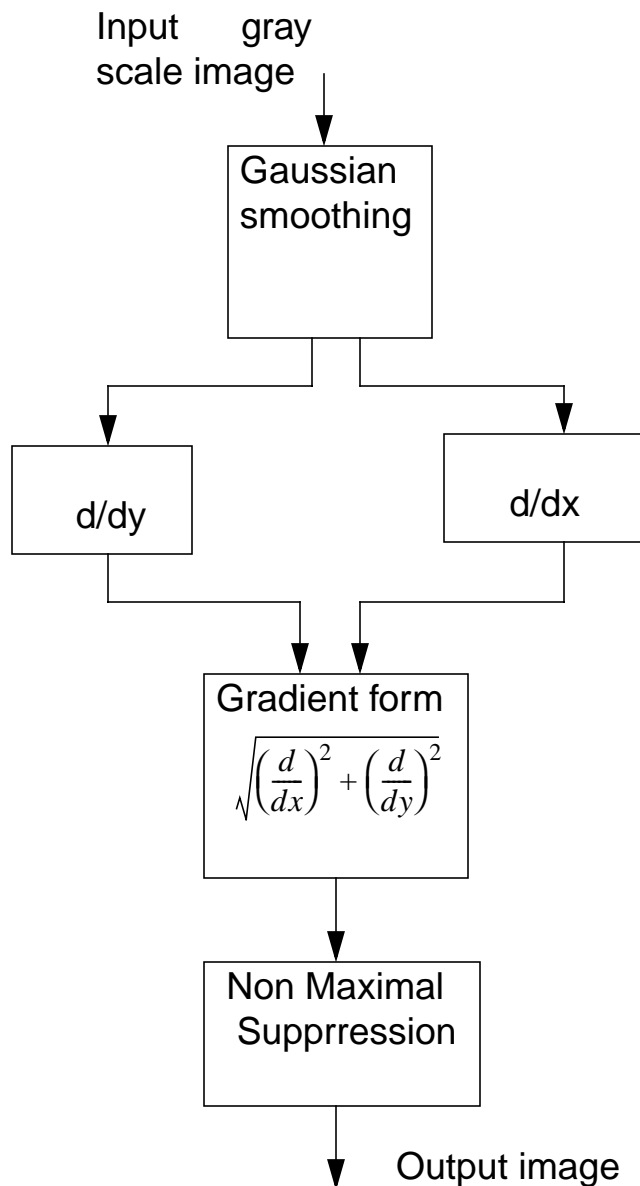
## Database

- ➔ 679 Unique Images with subjective ratings
- ➔ Standardized ratings available for each image



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## Edge detection Algorithm



# Results

Original Image



Gray Image



## Results

- ❑ SBE depends on the mean of each of the color, number of vertical lines
- ❑ The length of the lines is a direct indication of the long trees and bushes

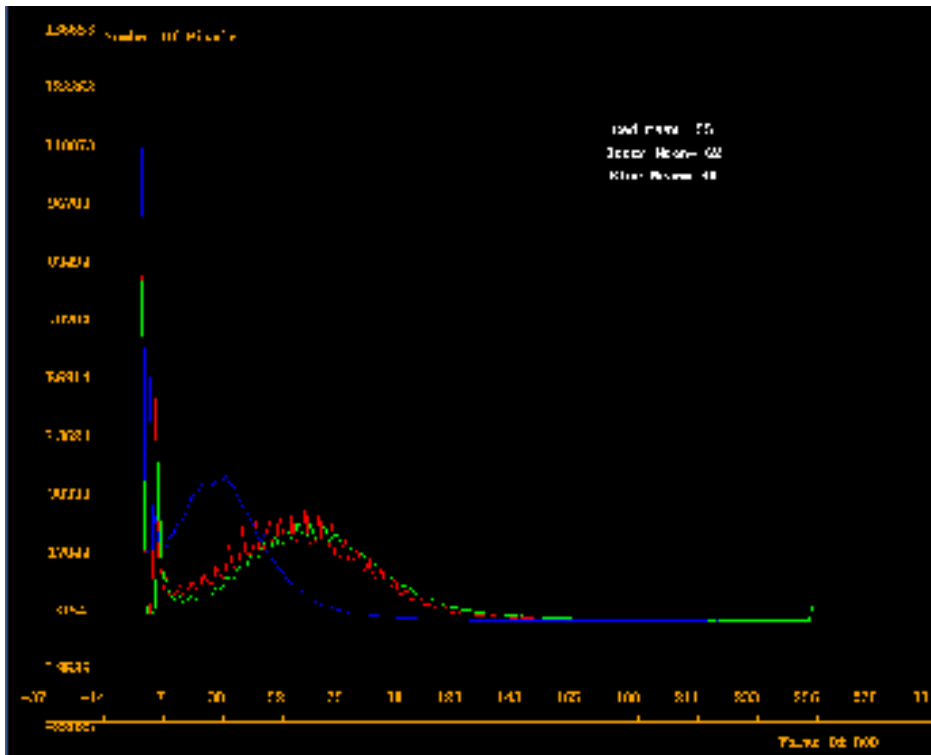
**Table 1: table showing the derived means and the SBE rating**

Redmean	Greenmean	Bluemean	%longlin	%shortli	der_sbe	act_sbe
66	72	36	1.97	78.00	0.40	4.37
46	62	36	2.10	76.10	0.52	4.80
64	78	45	2.20	73.60	0.50	5.16
55	62	30	2.02	77.10	0.42	4.68
46	62	36	2.27	75.60	0.47	4.80

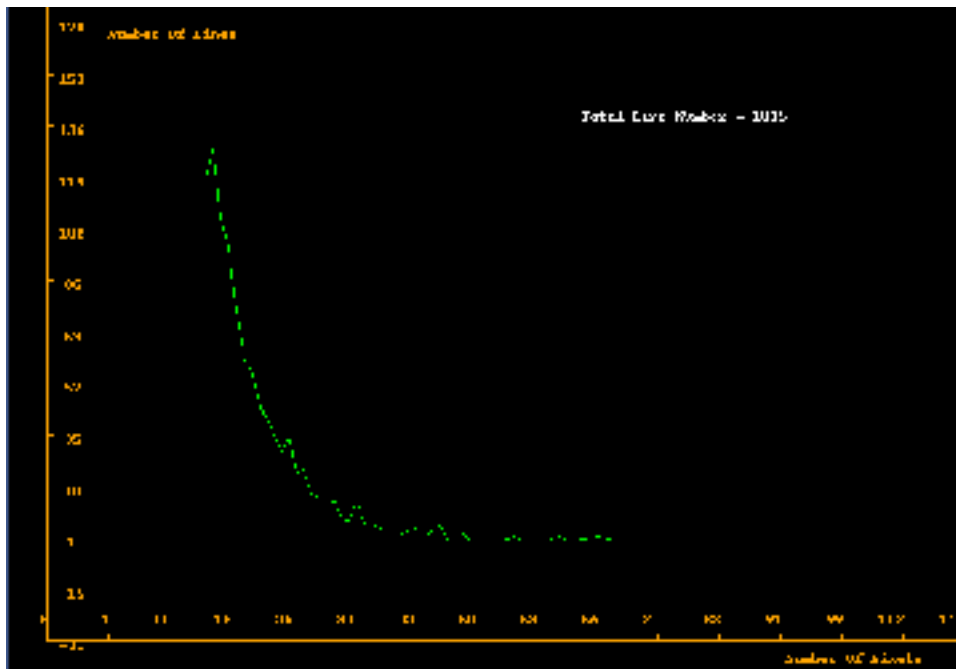


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### Graph showing the number and value of each of the colors



### Graph showing the number and size of the vertical lines



## Summary

- ❑ Dependency of color on the scenic beauty
- ❑ Dependency of vertical lines on the scenic beauty
- ❑ Well organized database

## Future Research

- ❑ Using neural network to evaluate the scenic beauty rating
- ❑ Develop algorithm to check the effect of texture, landform position on the scenic beauty
- ❑ To study the future growth of the trees with the present data



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