Name:

Problem	Points	Score
1a	10	
1b	10	
1c	10	
1d	10	
2a	10	
2b	10	
2c	10	
3a	30	
3b	0	
Total	100	

Notes:

1. The exam is open books/open notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

3. Please indicate clearly your answer to the problem.

4. Several problems on this exam are fairly open-ended. Since the evaluation of your answers is obviously a subjective process, we will use a marketplace strategy in determining the grade. Papers will be rank-ordered in terms of the quality of the solutions, and grades distributed accordingly.

Problem No. 1: Filter Design

(a) Design a first order digital lowpass filter with a cutoff frequency of 2 kHz and a sample frequency of 8 kHz using the bilinear transform design technique and a Butterworth prototype. The final digital filter should have no more than two poles and two zeroes.

First, normalize the sample period to 1: $T = 1$ and $f_c = \frac{2\text{ kHz}}{8\text{ kHz}} = \frac{1}{4}$.

Second, pre-warp: $\Omega = \frac{2}{T}\tan\frac{\omega}{2} = 2\tan\left(\left(\frac{2\pi}{4}\right)\left(\frac{1}{2}\right)\right) = 2\tan\left(\frac{\pi}{4}\right) = 2$

Third, construct $H(s)$: $H(s) = \frac{1}{s - \Omega_c e^{j\pi/2} e^{j\pi/2}} = \frac{1}{s+2}$

Fourth, use the bilinear transformation, $s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$, to map to the z-plane:

$$H(z) = \frac{1}{2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+2} = \frac{1}{2}\,\frac{1+z^{-1}}{(1-z^{-1})+(1+z^{-1})} = \frac{1}{2}(1+z^{-1})$$

This is a pretty bad filter — we really need a minimum of N=2:

$$H(s) = \frac{1}{s - \Omega_c e^{j\pi/2} e^{j\pi/2}}\,\frac{1}{s - \Omega_c e^{j\pi/2} e^{j3\pi/2}}$$

$$= \left(\frac{1}{s+2}\right)\left(\frac{1}{s-2}\right) = \frac{1}{s^2-4}$$

$$H(z) = \frac{1}{2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2 - 4} = \left(-\frac{1}{2}\right)\frac{1+2z^{-1}+z^{-2}}{1+4z^{-1}+z^{-2}}$$

(b) Design a digital lowpass filter with the specifications of part (a) using the impulse invariance technique. Again, the final digital filter should have no more than two poles and two zeroes.

$$H(f) = \begin{cases} 1 & 0 < |f| \le f_c \\ 0 & f_c < |f| < (f_s/2) \end{cases}$$

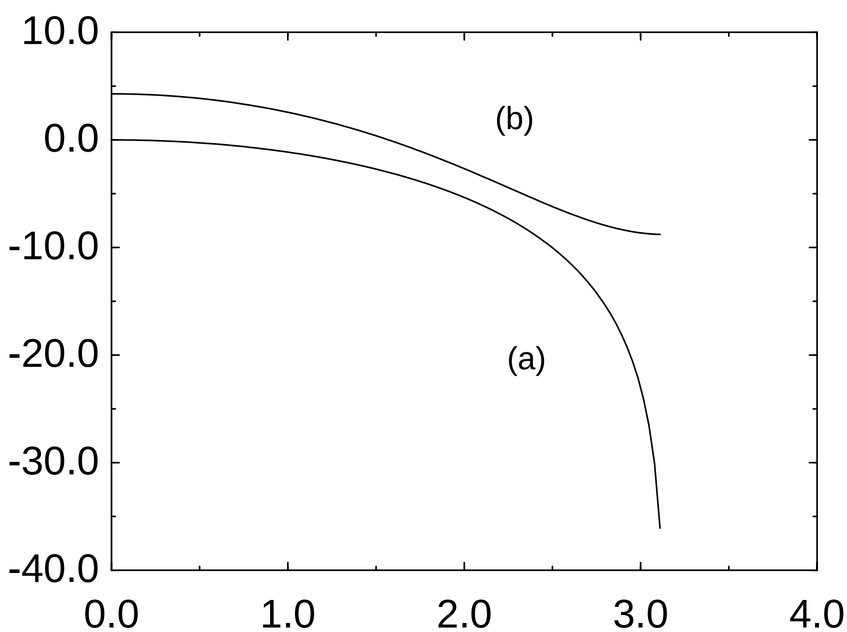
$$h(t) = \frac{\sin(2\pi 2000 t)}{\pi 2000 t}$$

Hence,

$$h(n) = \frac{\sin\frac{\pi}{2}n}{\pi n} = \left\{\ldots, 0, \frac{1}{\pi}, 1, \frac{1}{\pi}, 0, \ldots\right\}$$

$$H(z) = \left(\frac{1}{\pi}\right)z + 1 + \left(\frac{1}{\pi}\right)z^{-1}$$

(c) Compare the magnitude spectra of the two filters.



(d) Use any design technique you want to design a digital bandpass filter with the following specifications:

Center frequency = 2 kHz
Bandwidth = 500 Hz
Passband gain at the center frequency = 2
Sample frequency = 8 kHz
IIR with no more than two poles

Let's use a second order resonator (from the book: page 354):

$$H(z) = \frac{b_0}{1 - 2r\cos\theta z^{-1} + r^2 z^{-2}}$$

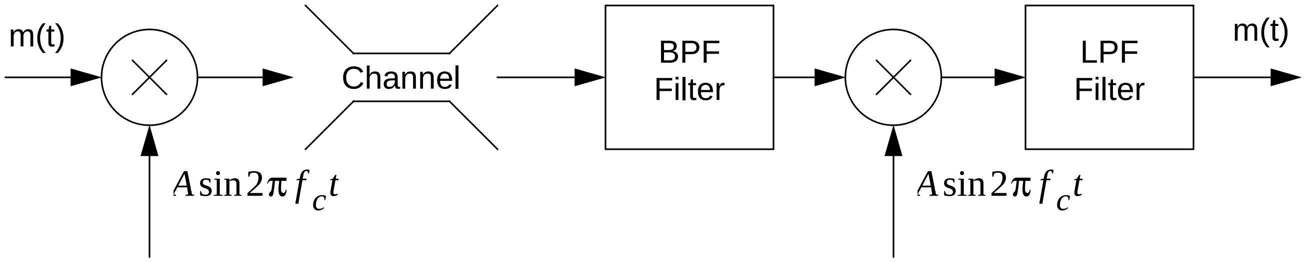
$$\cos\theta = \cos\left(2\pi\frac{2000}{8000}\right) = 0$$

$\Delta\omega \approx 2(1-r)$ implies that $r \approx 0.8$.

$$b_0 = ((1-r)\sqrt{1 + r^2 - 2r\cos 2\theta}) \times 2 = 0.51$$

Problem No. 2: DSP System Design

(a) An analog Amplitude Modulated (AM) radio is shown below:



Assume $m(t)$ is a music signal with a bandwidth of 20 kHz, and that $f_c = 10MHz$.

Implement this system as an end-to-end digital system using A/D and D/A converters, DSPs, and lots of software. Be as specific as possible. Estimate the total cost of the system based on the following models (add any others you feel are relevant):

1 MIP costs $1 and requires 0.01 Watts of power;
1 Mbyte of memory costs $1;
1 bit of accuracy on an A/D or D/A costs $1;
0.1 Watts of power costs
1 line of code costs $1;

Your goal should be to achieve at least a 60 dB SNR with a cost that is competitive with today’s high quality analog radios.

(b) Explain any difficulties and/or weaknesses of your approach. What are the aspects of the system that dominate cost and performance?

(c) Touchtone frequencies from a telephone keypad generate the following frequencies:

		Column			
		1	2	3	4
Row:		1209	1336	1477	1633
1	697	**1**	**2**	**3**	**A**
2	770	**4**	**5**	**6**	**B**
3	852	**7**	**8**	**9**	**C**
4	941	*****	**0**	**#**	**D**

For example, the key marked “1” on a telephone generates sinewaves at 697 Hz and 1209 Hz when pressed.

Describe how you would build a digital touchtone detection system (often called a DTMF detector) that operated on digital data sampled at 8 kHz. Give a specific example for the case of the key marked “1.” Discuss issues such as the ability to detect touchtones during intervals where people are talking, the problem with the touchtone detector falsely detecting touchtones when speech or music is sent over the telephone line your system is monitoring. The system should be capable of recognizing touchtone bursts as short as 40 msec.

Problem No. 3: Summary

(a) Your job next semester will be to offer a short course titled “Fundamentals of Digital Signal Processing” to a group of analog engineers from a company famous for its analog communications and signal processing chips. This will be a one-day course that costs $1000 and assumes students have a strong background in analog signal processing. In the next two pages, give as detailed a description of the course as possible, including an outline of topics. Be sure to justify your choices of topics (and the order of the topics), and to provide sufficient motivation for taking the course. You will be penalized for simply repeating the outline of our current textbook.

(b) Please indicate the grade you think you deserve for the course. Justify your choice. Be honest — think of this as comparable to asking your supervisor for a raise or a higher starting salary.