Name:

Problem	Points	Score
1a	10	
1b	10	
1c	10	
1d	10	
2a	10	
2b	10	
2c	10	
2d	10	
3a	10	
3b	10	
Total	100	

Notes:

1. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

2. Please indicate clearly your answer to the problem.

3. No z-transforms allowed! No credit will be given for problems solved using the z-transform!

Problem No. 1: Basic properties of continuous and discrete-time signals

(a) Students at MSU are graded on a 5 point scale: A,B,C,D,and F. The Registrar, a former graduate of EE 4773, decides to put this class to good use by developing a program to model the fluctuation of the average GPA at the university. The model the Registrar uses predicts the average GPA for the current year as a function of the number of students enrolled at the beginning of the first semester, the unemployment rate on Jan. 1 of the current year, and the size of the US population for the year corresponding to the current year minus 18 years.

Write an equation that represents a model of this signal.

$$y(n) = \alpha x_1(n) + \beta x_2(n) + \zeta x_3(n-18)$$

Is the signal described above (circle all that apply):

one dimensional	multidimensional
single channel	multichannel
continuous	discrete-time
analog in amplitude	digital in amplitude

(b) What is the Nyquist rate for the signal:

$$x(n) = (\sin(2\pi 1000t + 7.5\pi))^2$$

$$\begin{aligned} x(n) &= (1/2)(1 - \cos(2(2\pi 1000t + 7.5\pi))) \\ &= (1/2)(1 - \cos(2\pi 2000t + 15\pi)) \end{aligned}$$

(The signal is squared, which means its bandwidth doubles.)

The Nyquist sample frequency in Hz is: 4000 Hz

(c) Given the signal $x(t) = \begin{cases} (-1/2)^{1000t} & |t| \le 0.0015 \text{ secs} \\ 0 & elsewhere \end{cases}$, compute the value of $x(t)$ at $t = 0.002 \text{ secs}$ by sampling $x(t)$ at $f_s = 1000 \text{ Hz}$, upsampling the signal to a new sample frequency of 2000Hz, and evaluating this new discrete signal at $n = 4$ (which corresponds to $t = 0.002 \text{ secs}$).

$$x(n) = x(t)|_{t = n/f_s} = \{\ldots, 0, -1/2, \mathbf{1}, -1/2, 0, \ldots\}$$

from the sampling theorem:

$$x_a(t) = (-2)\frac{\sin\left(2\pi(500)\left(t + \frac{1}{1000}\right)\right)}{2\pi(500)\left(t + \frac{1}{1000}\right)}$$

$$+ (1)\frac{\sin(2\pi(500)(t))}{2\pi(500)t}$$

$$+ (-2)\frac{\sin\left(2\pi(500)\left(t - \frac{1}{1000}\right)\right)}{2\pi(500)\left(t - \frac{1}{1000}\right)}$$

$$x_{up}(t = 0.002) = 0 + 0 + 0 = 0$$

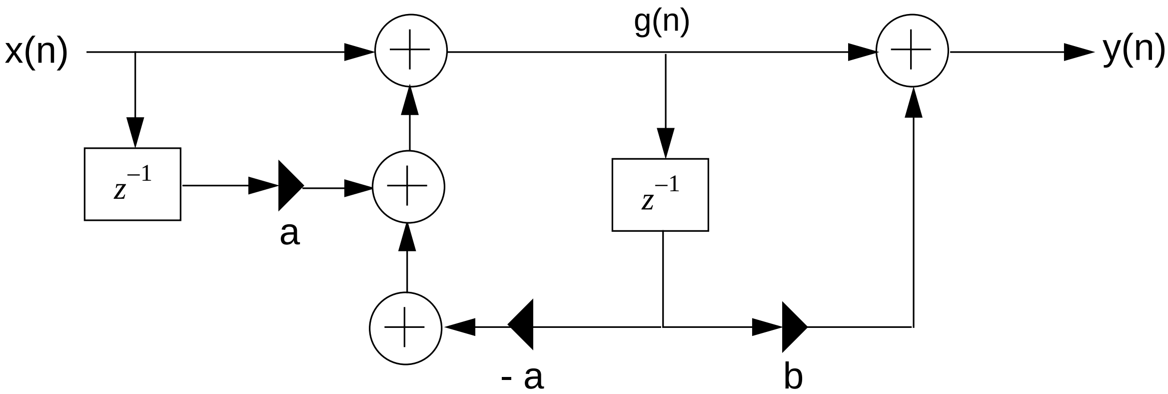
$x_{new}(4) =$ 0

(d) Why is the answer to (c) not equal to $x(t)$ evaluated at $t = 0.002 \text{ secs}$?

In general, since $x(t)$ is time-limited, it is not bandlimited. Hence, the sampled signal is only an approximation to $x(t)$ due to aliasing. However, in this case, since we are sampling the signal at an orthogonal frequency (an integer multiple of the original sample frequency), the values are the same.

Problem No. 2: Discrete-Time Systems

For the system shown below::



(a) Compute the impulse response.

x(0) = 1	g(0) = 1	y(0) = 1
x(1) = 0	g(1) = - a g(0) + a x(0) = 0	y(1) = 0 + b y(0) = b
x(2) = 0	g(2) = 0	y(2) = 0
x(3) = 0	g(3) = 0	y(3) = 0

h(n) = {1, b, 0, 0, ...}

(b) For the signal x(n) = {2,0,0,-2}, compute the output y(n).

$$x(n) = 2\delta(n) - 2\delta(n-3)$$

hence,

$$y(n) = 2h(n) - 2h(n-3)$$

$$y(n) = \{2, 2b, 0, -2, -2b\}$$

(c) For y(n) of part (b), compute $r_{yy}(0)$.

$$r_{yy}(0) = \sum_{n=0}^{4} y^2(n) = \sum_{k=-\infty}^{\infty} r_{hh}(k) r_{xx}(k)$$

$$r_{yy}(0) = 2^2 + (2b)^2 + 0 + (-2)^2 + (-2b)^2$$
$$= 8 + 8b^2$$

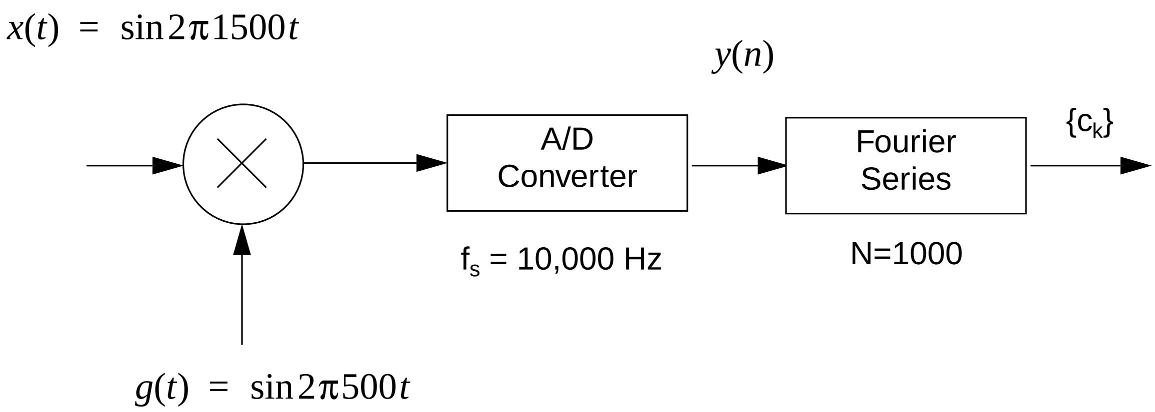
(d) Compute the value of $H(\omega)$ for $\omega = 0$.

This is the DC value of the impulse response.

$$|H(\omega)|\Big|_{\omega=0} = \left|\sum_{n=0}^{1} x(n) e^{-j\omega n}\right|\Bigg|_{\omega=0} = \left|1 + be^{-j\omega}\right|\Big|_{\omega=0} = 1 + b$$

Problem No. 3: Fourier Transforms and Fourier Series

For the system shown below, compute the following:



(a) Compute the c_k for k=10 and k=20.

This is nothing more than the modulation property at work:

$$Y(\omega) = \frac{1}{2}X(\omega - 2\pi 500) + \frac{1}{2}X(\omega + 2\pi 500)$$

The window duration, 1000 samples, is an integral number of periods of the modulated signal, so that the Fourier series coefficients and the Fourier transform are essentially identical at the orthogonal frequencies.

Hence,

$$Y(\omega) = \frac{1}{4}j\delta(\omega - 2\pi 1000) - \frac{1}{4}j\delta(\omega + 2\pi 1000)$$
$$+ \frac{1}{4}j\delta(\omega - 2\pi 2000) - \frac{1}{4}j\delta(\omega + 2\pi 2000)$$

But, since c_{10} corresponds to $f = \left(\frac{10000}{1000}\right)10 = 100\text{Hz}$, and c_{20} corresponds to 200Hz, $c_{10} = c_{20} = 0$.

(b) Recall the definition of the short-term Fourier transform as:

$$Y(\omega) = \sum_{n=-N}^{N} y(n)e^{-j\omega n}$$

Compute $Y(\omega)$ for $\omega = 2\pi 1000$.

From part (a),

$$Y(\omega = 2\pi 1000) = \frac{1}{4}j$$