Some Elementary Discrete-Time Signals:

(1) unit sample signal:

$$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

(2) unit step signal:

$$u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

(3) unit ramp signal:

$$u_r(n) = \begin{cases} n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

(4) exponential signal:

$$x(n) = a^n, \qquad \forall n$$

(5) complex exponential signal ($a = re^{j\theta}$):

$$\begin{aligned} x(n) &= (re^{j\theta})^n \\ &= r^n e^{j\theta n} \\ &= r^n(\cos\theta n + j\sin\theta n) \end{aligned}$$

Classification of Discrete-Time Signals:

Energy:

$$E \equiv \sum_{n=-\infty}^{\infty} |x(n)|^2$$

Average Power:

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

Finite Energy:

$$E_N = \sum_{n=-N}^{N} |x(n)|^2$$

$$E = \lim_{N \to \infty} E_N$$

$$P = \lim_{N \to \infty} \frac{1}{2N+1} E_N$$

Comments:

(1) If a signal's energy is finite, $P = 0$.

(2) If a signal's energy is infinite, its power may or may not be zero.

(3) RMS value is the square root of the power.

Examples:

(1) The average power of the unit step sequence is $\frac{1}{2}$.

(2) The average power of a sinewave is $\frac{A^2}{2}$.

Periodic signals:

$x(n)$ is periodic with period N if and only if:

$$x(n+N) = x(n) \qquad \forall n$$

Symmetric:

$x(n)$ is symmetric (even) if:

$$x(-n) = x(n)$$

$x(n)$ is antisymmetric (odd) if:

$$x(-n) = -x(n)$$

Translation:

$$y(n) = x(n-k)$$

Arithmetic:

Scaling:

$$y(n) = a\,x(n)$$

Addition (Linear Combination or Weighted Sum):

$$y(n) = a\,x_1(n) + b\,x_2(n)$$

Multiplication:

$$y(n) = x_1(n)x_2(n)$$

Time-Scaling:

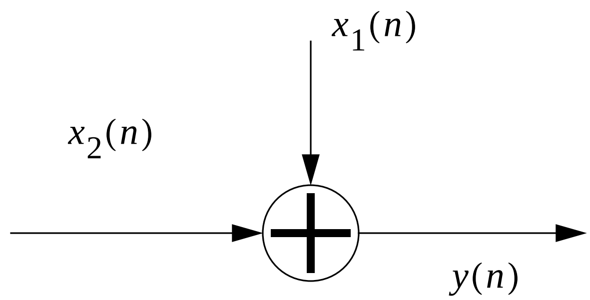
$$y(n) = x(an)$$

Accumulation (Integration):

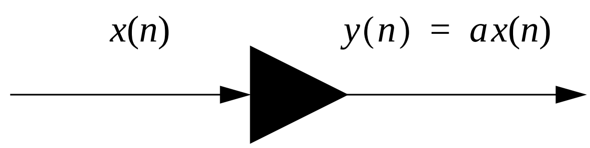
$$y(n) = \sum_{n=-\infty}^{n} x(n)$$

Signal Flow Graphs:

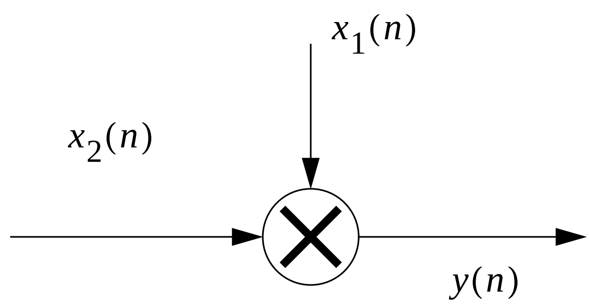
Addition:



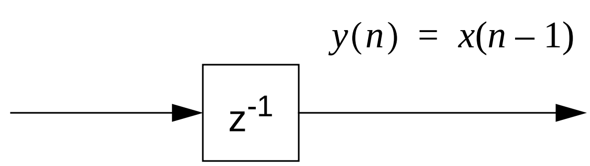
Constant Multiplication (Gain):



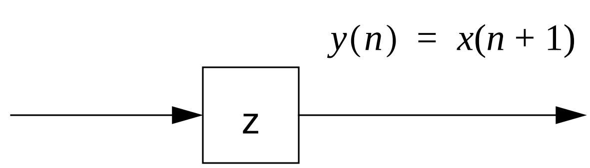
Multiplication:



Unit Delay:



Unit Advance:



Classification of Systems:

Static (or Dynamic): output at time n depends on the input sample at time n and future samples, but not past samples

Time-Invariant (or Time-Variant):

$$x(n-k) \quad \overset{H}{\rightarrow} \quad y(n-k)$$

Linear (or Nonlinear):

$$H[a_1 x_1(n) + a_2 x_2(n)] = a_1 H[x_1(n)] + a_2 H[x_2(n)]$$

Causal:

$$y(n) = F[x(n), x(n-1), x(n-2), \ldots]$$

For Linear and Time-Invariant Systems:

$$y(n) = H_2[H_1[x(n)]] = H_1[H_2[x(n)]]$$