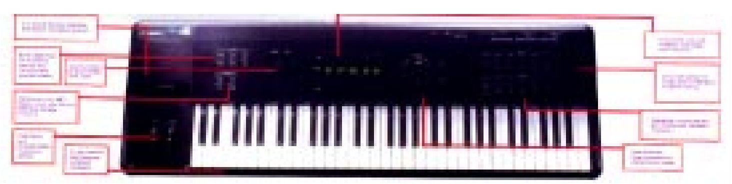
What do these things have in common?

Stradivarius Violin (1742)

Martin Acoustic Guitar (1980)

Fender Stratocasters (1964)



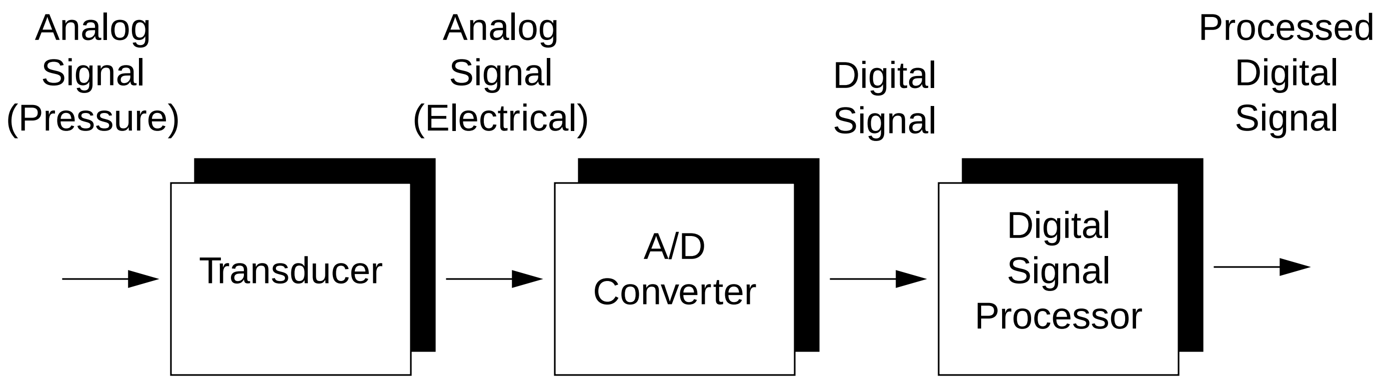
Peavey DMP4 (1994)

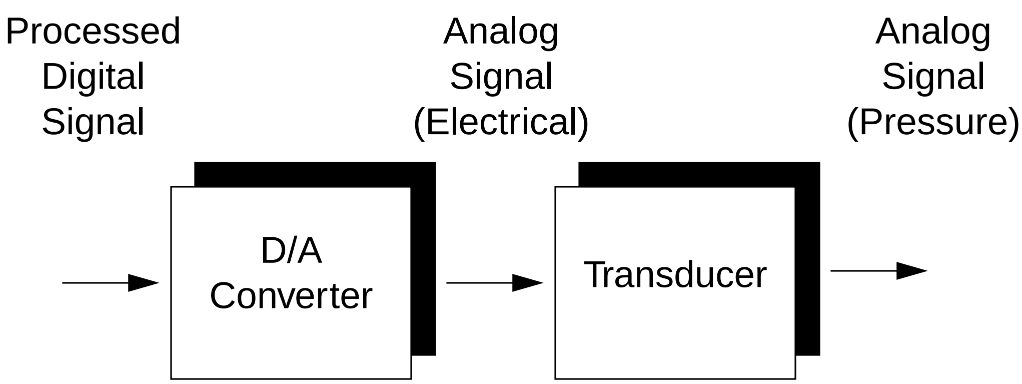
Why Digital Signal Processing?

- Flexibility
- Cost
- Testability

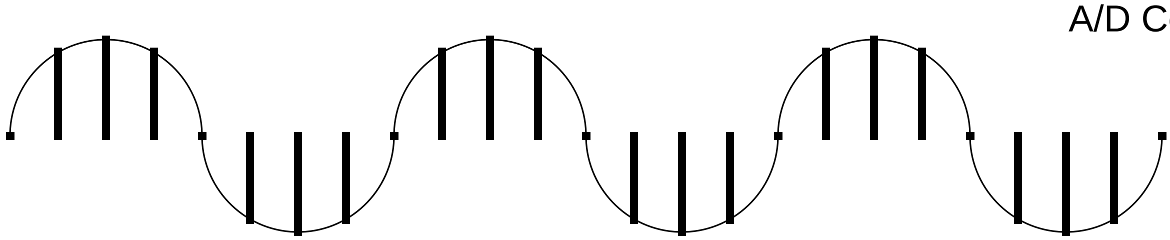
Together, these translate to better **quality** and **reliability**.

The Basic Digital Signal Processing System:

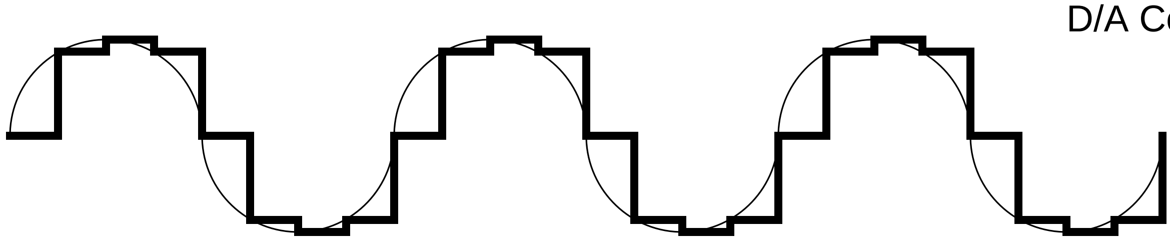




A/D Conversion



D/A Conversion



(Note: linear and quadratic interpolators are possible.)

Four ways to classify a signal:

Continuous: $x(t) = A\cos(2\pi 1000t)$

Discrete: $x(n) = A\cos\left(2\pi 1000\left(\frac{n}{f_s}\right)\right)$

Deterministic: uniquely described by a mathematical expression (everything is known about the signal for all time from the value of all derivatives of the signal at one point in time)

Random: unpredictable

Note: random number generators on computers are usually deterministic

Given a continuous signal:

$$x(t) = A\cos(2\pi ft + \theta),$$

A discrete-time sinusoid may be expressed as:

$$x(n) = A\cos\left(2\pi f\left(\frac{n}{f_s}\right) + \theta\right),$$

which, after regrouping, gives:

$$x(n) = A\cos\left(2\pi\left(\frac{f}{f_s}\right)n + \theta\right),$$

or,

$$x(n) = A\cos(\omega n + \theta),$$

where $\omega = 2\pi\left(\frac{f}{f_s}\right)$, and is called normalized radian frequency.

A discrete-time signal is said to be periodic only if its frequency is a rational number:

$$x(n + N) = x(n) \qquad \forall n,$$

The smallest value of N for which the above equation is true is called the fundamental period.

The highest rate of oscillation in a discrete-time sinusoid is attained when $\omega = \pi$, or equivalently, $\frac{f}{f_s} = \frac{1}{2}$.

Consider:

$$x_1(t) = \cos(2\pi 10t) \text{ and } x_2(t) = \cos(2\pi 50t) .$$

Let $f_s = 40\text{ Hz}$. Then:

$$x_1(n) = \cos\left(2\pi\frac{10}{40}n\right) = \cos\left(\frac{\pi}{2}n\right)$$

and,

$$x_2(n) = \cos\left(2\pi\frac{50}{40}n\right) = \cos\left(5\frac{\pi}{2}n\right) = \cos\left(2\pi n + \frac{\pi n}{2}\right) = \cos\left(\frac{\pi}{2}n\right) .$$

Hence,

$$x_1(n) = x_2(n).$$

This observation gives rise (sort of :) to the sampling theorem:

If the highest frequency contained in an analog signal, $x_a(t)$, is $F_{max} = B$ and the signal is sampled at a rate $F_s > 2F_{max} = 2B$, then $x_a(t)$ can be EXACTLY recovered from its sample values using the interpolation function:

$$g(t) = \frac{\sin(2\pi Bt)}{2\pi Bt} .$$

$x_a(t)$ may be expressed as:

$$x_a(t) = \sum_{n=-\infty}^{\infty} x_a\left(\frac{n}{F_s}\right) g\left(t - \frac{n}{F_s}\right)$$

where $x_a\left(\frac{n}{F_s}\right) = x_a(nT) = x(n)$.