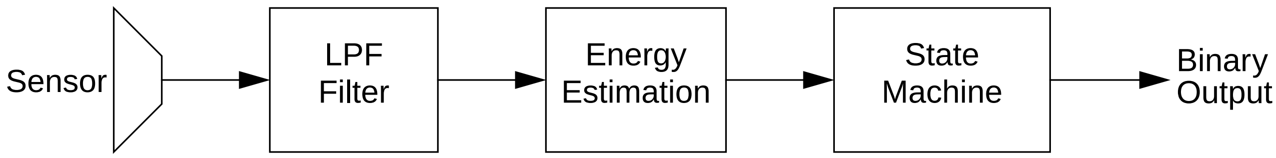
Name:

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total	100	

Notes:

1. The exam is open books/open notes.
2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.
3. If I can't read or follow your solution, it is wrong, and no partial credit will be given — BE NEAT!
4. Please indicate clearly your answer to the problem.
5. Several problems on this exam are fairly open-ended. Since the evaluation of your answers is obviously a subjective process, we will use a marketplace strategy in determining the grade. Papers will be rank-ordered in terms of the quality of the solutions, and grades distributed accordingly.

1. A simple motion detector circuit consists of the following components:



The sensor outputs a nominal signal (non-zero voltage level that is random in nature) when there is no motion in its field of view. When motion occurs, there is a corresponding increase in the output voltage level.

Implement this system as an end-to-end digital system using A/D and D/A converters, processors, and lots of software. Be as specific as possible. Estimate the total cost of the system based on the following models (add any others you feel are relevant):

1 MIP costs $1 and requires 0.01 Watts of power;
1 Mbyte of memory costs $1;
1 bit of accuracy on an A/D or D/A costs $1;
0.1 Watts of power costs $10;
1 line of code costs $1.

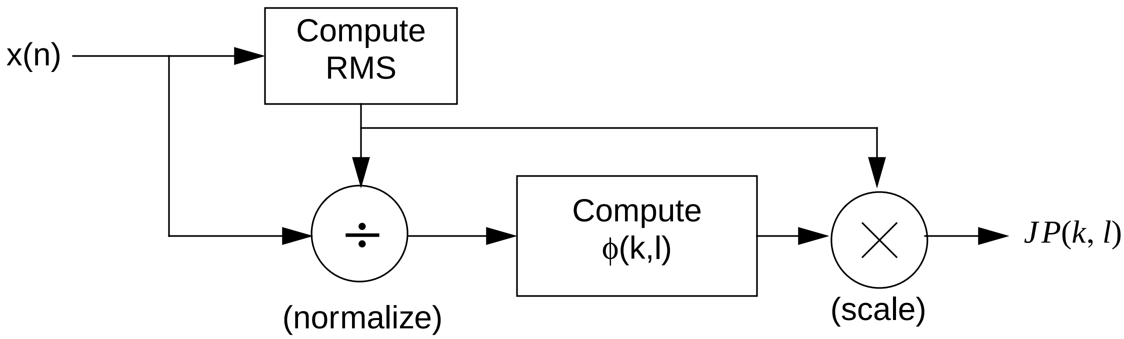
Explain what is the most costly part of the system; justify your explanation. (Continue on the back of this page if you need more space.)

2. Your DSP professor dies unexpectedly. While you are dancing on his grave at the funeral, you are informed that you are the sole heir to his fortune (turns out he was taking bribes from DSP students on the side) — if you can do one small thing: get a transform named after him.

   In his will, he left the following equation:

$$\phi(k, l) = \sum_{n=0}^{N-1} x(n+k)x(n+l)$$

   and the following block diagram how to apply this operator:



   The National Academy of Science has agreed to name this transform after him, provided you can prove that the principles of linearity and superposition hold. Do they hold unconditionally (prove this)? If not, under what conditions might they hold?

3. Consider a two-dimensional (2D) discrete Fourier transform of the image shown to the right. The 2D Fourier transform is essentially a series of one-dimensional transforms taken in every direction (draw a horizontal line, take a 1D transform, rotate the line by one degree about the origin, take another transform, etc.). Sketch the frequency response of the image shown and justify your sketch.

4. For the signal $y(n) = x(n) - ax(n-1)$, derive an expression for a that minimizes the energy in $y(n)$: $E_y = \sum_{n=-\infty}^{\infty} y^2(n)$.

5. Design a bandpass filter (assume an 8 kHz sample frequency) with a lower cutoff frequency of 500 Hz, an upper cutoff frequency of 2000 Hz, and a passband gain of 1.0. The filter must have no more than two poles and two zeroes (with real coefficients). To get more than 5 points for this problem, you must plot the actual frequency response of whatever transfer function you obtain, and demonstrate whether it meets your design criteria.

6. Compute impulse response of the system described by the difference equation:

$$y(n) = ax(n) + bx(n+1) + cy(n-1)$$

Under what conditions is this system stable?

7. The system shown below is time-invariant. The input shown below is applied to the system, and the output is measured as shown. Determine the system impulse response, and whether the system is linear. You MUST justify your answers or no points will be awarded.

$$x(n) = \{1, 2\} \longrightarrow \boxed{H[\ ]} \longrightarrow y(n) = \{0, 2, 3\}$$

8. The input to a linear time-invariant system is $x(n) = \{0,-1,0,1,0,2,0,-2,0,0,\ldots\}$ (note that $x(n)$ starts at time 0). The impulse response of the system is given by the following z-transform: $H(z) = 1 + az$. Compute the output sequence.

9. Consider the signal $x(n) = a^n u(n)$. Compare and contrast the differences in the results when you compute the spectrum using: (a) the Fourier transform; (b) the discrete Fourier transform; (c) the discrete Fourier series. Under what conditions are these representations equivalent for the given signal?

10. What grade do you deserve for this course? (No credit for answering this part.) Justify this by demonstrating what you have learned in this course. Explain in great detail three concepts introduced in this course that you have not previously seen in other courses. In these explanations, identify material that demonstrates you deserve the grade indicated. For example, a student requesting an A should be able to demonstrate insight and understanding beyond that of the average graduate student.