Name:

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total	100	
Extra Credit	20	

Notes:

1. The exam is open books/open notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

3. If I can't read or follow your solution, it is wrong, and no partial credit will be given — BE NEAT!

4. Please indicate clearly your answer to the problem.

5. Several problems on this exam are fairly open-ended. Since the evaluation of your answers is obviously a subjective process, we will use a marketplace strategy in determining the grade. Papers will be rank-ordered in terms of the quality of the solutions, and grades distributed accordingly.

1. Design the lowest order bandpass filter at a sample frequency of *8 kHz* that has a lower cutoff frequency of *1 kHz*, an upper cutoff frequency of *1.5 kHz*, transition bands of *100 Hz*, a passband gain of *1*, a passband ripple of *1 dB*, and stopband attenuations of *50 dB* (with as much ripple as you want as long as the minimum attenuation is *50 dB*).

2. A causal IIR filter has an impulse response:

$$h(n) = \begin{cases} c_0, & n = 0 \\ c_1 a^{n-1}, & n \geq 1 \end{cases}$$

Derive the difference equation corresponding to this system.

3. Use two completely different methods to check the stability of the following system:

$$H(z) = \frac{2 - 2.05z^{-1}}{1 - 2.05z^{-1} + z^{-2}}$$

4. A sinusoid of frequency *500 Hz* is applied to the following digital filter:

$$H(z) = \frac{b}{1 - az^{-1}}$$

Assume a *10 kHz* sample frequency. Calculate the required values of *a* and *b* such that the gain of this filter is *3 dB* at *500 Hz*.

5. If $x(n)$ is a complex-valued signal, prove the conjugate gradient convolution theorem:

$$F[x(n) \otimes jx(n-\tau)] = \frac{d}{dt}F[x(n-\tau)]$$

6. For the difference equation shown:

$$y(n) = a_1 y(n-1) + a_2 y(n-1) + b_0 x(n) + b_1 x(n-1)$$

Using a signal flow graph, implement this filter as efficiently as possible (using as few multipliers and delay elements as possible).

7. Consider the z-transform pair $x(n) = a^n u(n)$. Applying the derivative operator $\frac{\partial}{\partial a}$ to the pair, derive the z-transform of the sequence $x(n) = na^n u(n)$.

8. Give me a synopsis of JPEG image coding algorithm and tell me what concepts that we covered in the DSP course are included in this algorithm.

9. Given the output sequence $y(n) = \{1, 1/2, 1/4, 1/8\}$, compute the coefficients of a second-order linear predictor. Compare the spectrum implied by this model to that determined by a Fourier transform.

10. In our overview of echo cancellation, which is a special case of an adaptive filter, we introduced a technique to remove echo by minimizing the error between the actual echo and a predicted echo:

$$e(n) = r(n) - \tilde{r}(n)$$

where

$$\tilde{r}(n) = \sum_{k=0}^{N-1} a(n)y(n-k), \quad \text{and} \quad r(n) = \sum_{k=0}^{N-1} h(n)y(n-k).$$

The coefficients $\{a(n)\}$ represent the coefficients of a filter that predicts the echo, while the coefficients $\{h(n)\}$ represent a model of the actual echo. Derive and expression to compute $\{a(n)\}$ such that the error is minimized. (Hint: Follow the strategy used in the derivation of the linear prediction equations.)

(Extra Credit for those who score 50 or more on the previous 10 problems)

What grade do you think you deserve in this course? Explain.

Point to your answers to some of the problems on this final exam as evidence you deserve such a grade. Answers that demonstrate a deep insight into DSP, convince me that you learned something useful in this class, and exhibit quality English grammar and composition, will be awarded a maximum of 20 points of extra credit.