Name:

Problem	Points	Score
1a	10	
1b	10	
1c	10	
1d	10	
2a	10	
2b	10	
2c	10	
3a	10	
3b	10	
3c	10	
Total	100	

Notes:

1. The exam is open books/open notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

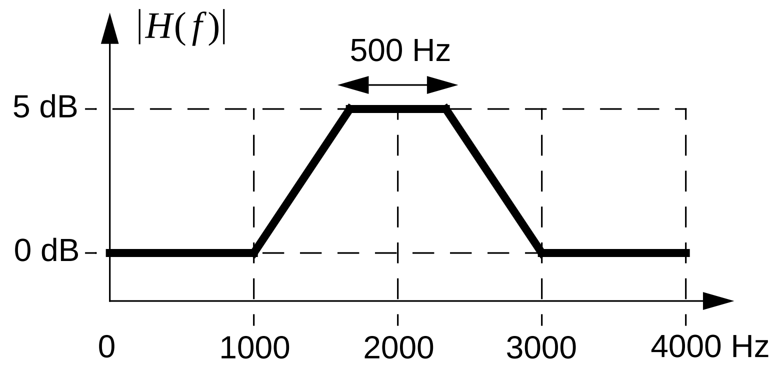
3. If I can’t read or follow your solution, it is wrong, and no partial credit will be given — BE NEAT!

4. Please indicate clearly your answer to the problem.

5. Several problems on this exam are fairly open-ended. Since the evaluation of your answers is obviously a subjective process, we will use a marketplace strategy in determining the grade. Papers will be rank-ordered in terms of the quality of the solutions, and grades distributed accordingly.

Problem No. 1: Mechanics

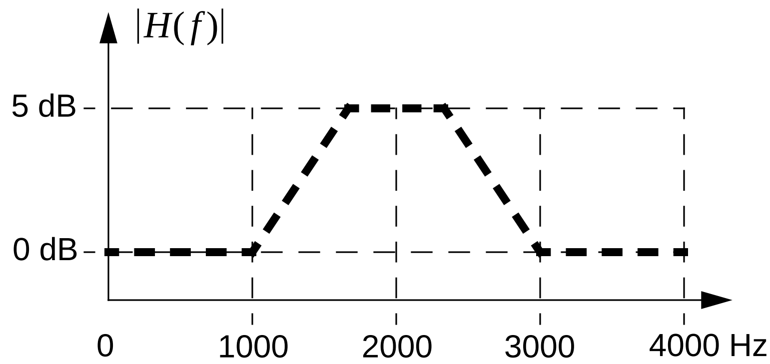
(a) Design a seventh-order linear phase FIR filter (8 coefficients counting $h(0)$) that best approximates the following frequency response:



Answer:

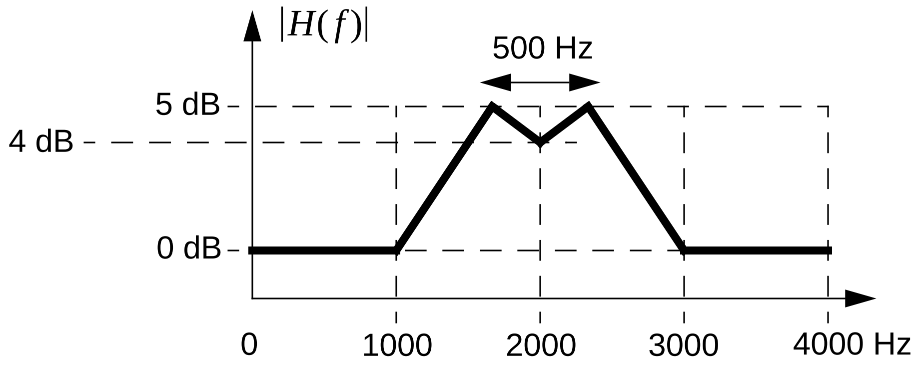
$h(0)$ =	$h(4)$ =
$h(1)$ =	$h(5)$ =
$h(2)$ =	$h(6)$ =
$h(3)$ =	$h(7)$ =

(b) Plot the frequency response of your answer to part (a) and explain why your design deviates from the specification.



(c) The signal $s(t) = 10.0\cos^2(12000\pi t)$ is sampled at 8 kHz using an A/D converter with an anti-aliasing filter that attenuates the analog signal 100 dB at 4 kHz. Compute the output of the system if it is applied to your filter.

(d) The filter specification is changed as shown below. How many more coefficients will you need to do a reasonably good job of matching the frequency response?



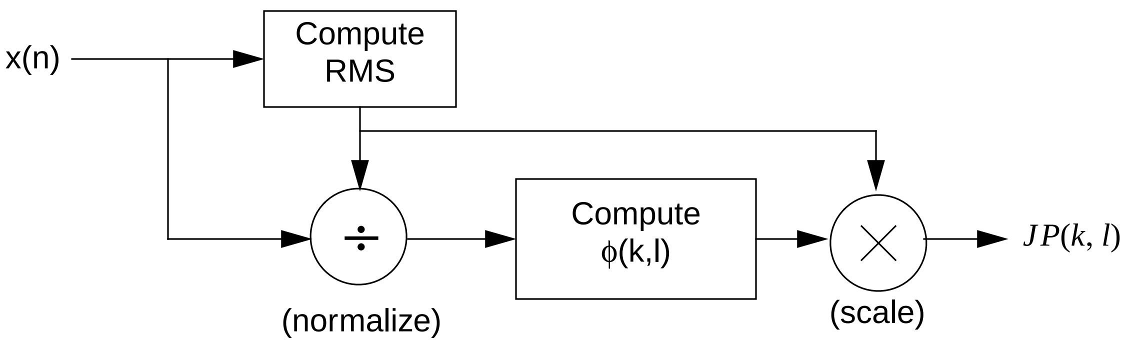
Problem No. 2: Theory

(a) Your DSP professor dies unexpectedly. While you are dancing on his grave at the funeral, you are informed that you are the sole heir to his fortune (turns out he was taking bribes from DSP students on the side) — if you can do one small thing: get a transform named after him.

In his will, he left the following equation:

$$\phi(k, l) = \sum_{n=0}^{N-1} x(n+k)x(n+l)$$

and the following block diagram how to apply this operator:



The National Academy of Science has agreed to name this transform after him, provided you can prove that the principles of linearity and superposition hold.

Do they hold unconditionally (prove this)? If not, under what conditions might they hold?

(b) The two-dimensional discrete Fourier transform is defined as:

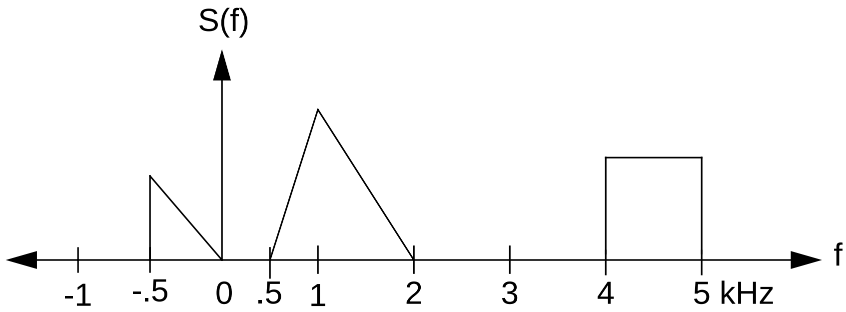
$$X(k, l) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} x(m, n) e^{-j2\pi km/M} e^{-j2\pi ln/N}$$

Prove that if $x(m, n) = x(m)x(n)$, then $X(k, l) = X(k)X(l)$, where $X(k)$ is the one-dimensional discrete Fourier transform.

(c) Prove that the principles of linearity and superposition hold for circular convolution.

Problem No. 3: The Dreaded Thought Problem

Consider a signal $s(t)$ with a power spectral density that looks as follows:



(a) Design a digital system that transmits this signal over the minimum possible bandwidth, and minimizes the computational and memory requirements. Be as detailed as possible — be sure to include the A/D and D/A components of the system. Clearly indicate the amount of bandwidth required to transmit the signal without distortion.

(b) Is the signal in part (a) physically realizable?

(c) We have discussed a number of algorithms and concepts in this course. Most were completely analogous to their analog counterparts. Some, such as convolution, are somewhat easier to understand in the discrete case. Others are less tractable in the discrete case. If there was one property of a transform, or one part of an algorithm you could simplify to make DSP significantly easier, what would it be? Explain.

Your answer will be judged on the impact of the property you select in the real world — and yes, I have a particular one in mind!