Name:

Problem	Points	Score
1a	10	
1b	10	
1c	10	
1d	10	
2a	10	
2b	10	
2c	10	
3a	10	
3b	10	
3c	10	
Total	100	

Notes:

1. The exam is open books/open notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

3. Please indicate clearly your answer to the problem.

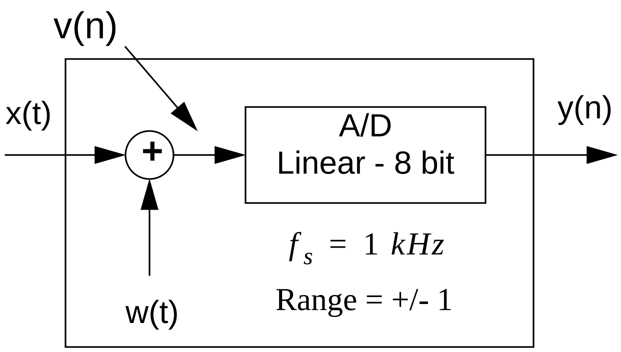
Problem No. 1: Sampling and quantization

(a) What is the single most important new concept introduced in Chapter 6? Explain why.

IMHO: The SNR of a linear quantizer increases 6 dB per bit.

OK, a second choice could be the sampling theorem, but we already saw that before Chapter 6 :-)

(b) Compute the SNR of the system shown below:



SNR = ?

Notes:

$$x(t) = 0.5\sin(2\pi 100t)$$

$$W(f) = 0.0001 \qquad |f| < 0.5\ kHz$$
$$W(f) = 0 \qquad |f| \geq 0.5\ kHz$$

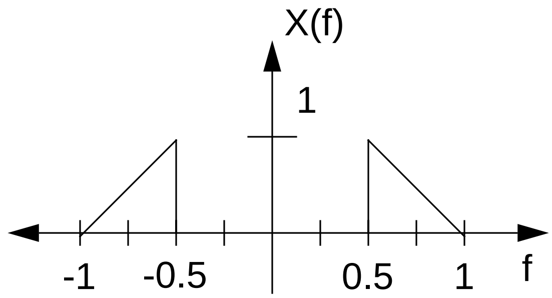
The input to the A/D has an SNR that is given by:

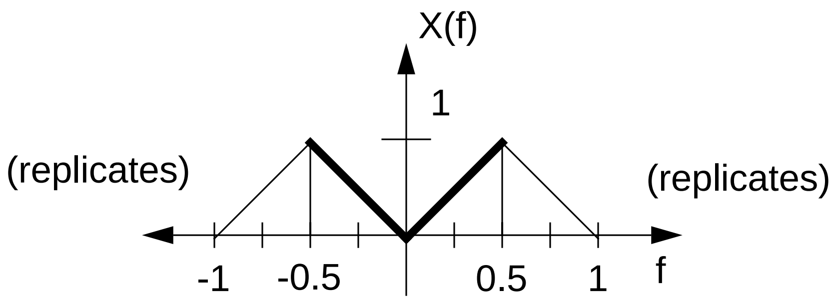
$$SNR_i = \frac{\int_0^{2\pi} (0.5\sin(2\pi 100t))^2}{\int_{-500}^{500} (0.0001)^2 df} = \frac{\frac{0.5^2}{2}}{(0.0001)^2 1000} = 41\text{ dB}$$

The SNR of the quantizer is (6 db/bit x 8 bits + 1.25 dB = 49.25 dB).

Hence, a first-order approximation would be that the overall SNR of the system will be limited by the additive noise, and be 41 dB. In reality, the overall SNR would be lower, because the sinewave and noise would be additionally distorted by the A/D converter. A more accurate approximation, assuming the A/D generates broadband additive noise, would reveal an overall SNR of about 32 dB. (Anything in the range from 30 dB to 50 dB was acceptable).

(c) What is the spectrum of the sampled signal resulting from sampling the signal below at $f_s = 1\ Hz$?



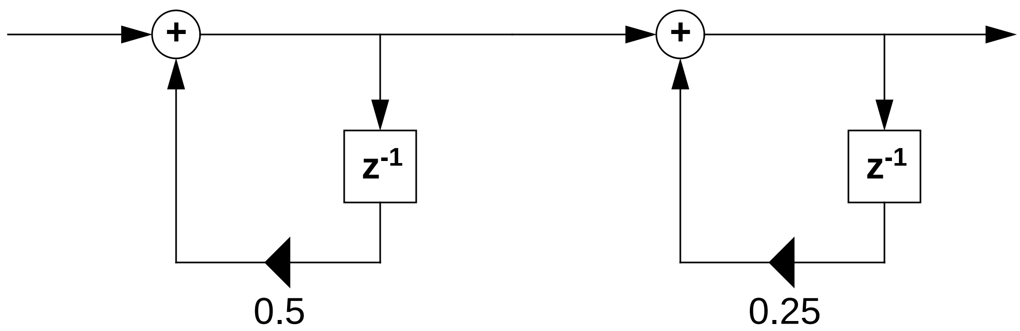


(d) Explain how you would recover the original signal from the sampled signal in (c). Does this violate the sampling theorem?

This, of course, is an example of the bandpass sampling theorem. Simply bandpass filter the sampled signal from 0.5 Hz to 1.0 Hz.

Problem No. 2: Filter Realizations

(a) Implement the filter shown below as a lattice filter.



$$H(z) = \left(\frac{1}{1 - 0.5z^{-1}}\right)\left(\frac{1}{1 - 0.25z^{-1}}\right)$$
$$= \frac{1}{1 - 0.75z^{-1} + 0.125z^{-2}}$$

This implies that (see Equation 7.2.22 on page 478):

$$\alpha_2(1) = -0.75$$
$$\alpha_2(2) = 0.125$$

and (see Equation 7.2.27 on page 479):

$$K_1 = \frac{\alpha_2(1)}{1 + \alpha_2(2)} = -0.67$$
$$K_2 = 0.125$$

(b) Implement the filter shown in (a) as a second order section in a parallel IIR system realization.

See Figure 7.22 on page 495:

$$a_{k1} = -0.75 \qquad b_{k0} = 1$$
$$a_{k2} = 0.125 \qquad b_{k1} = 0$$

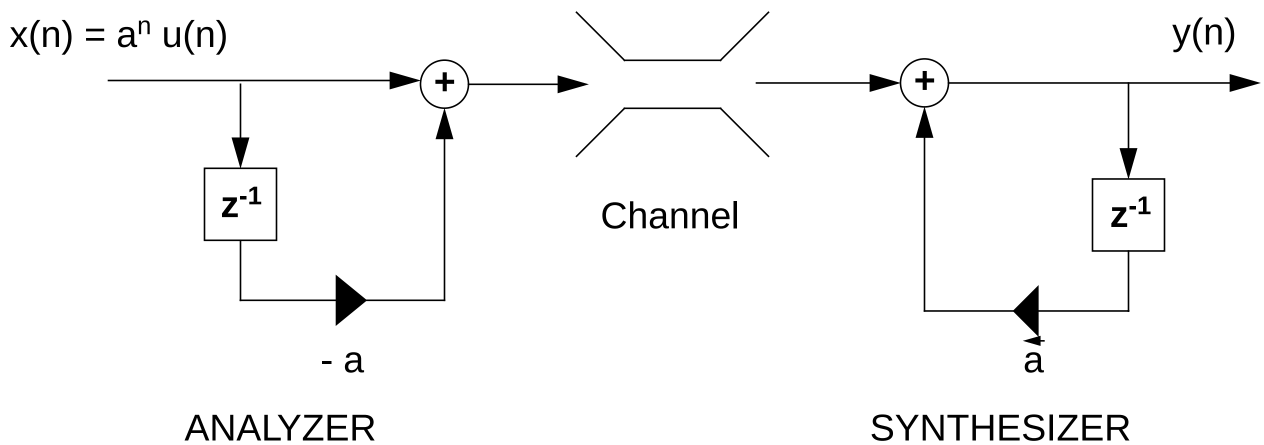
(c) Is the filter stable? Why?

Yes, the magnitudes of the reflection coefficients are less than one.

Problem No. 3: Quantization

When we last left the evil Dr. Anne A. Log, her lab was destroyed, and she was about to join the Darth Vader “I can’t get a break in big budget movies” fan club. Then she remembered that her long lost brother, Phil Terse, had recently completed Mississippi State’s new and improved EE 4773 DSP course, and had taken a job at the famous company DSPs ‘R Us. She called Phil and asked him what she could do to protect her lab in the future.

(a) Phil told her that she obviously needed the world famous Popeil Vegamatic and Channel Deconvolver, which sold for the amazing low price of $9.95. He said it worked as follows ($|a| < 1$):



Compute y(n):

The z-transform of the input is:

$$X(z) = \frac{1}{1 - az^{-1}}$$

From this we can find $Y(z)$ as:

$$Y(z) = X(z)H_{Anal}(z)H_{Synth}(z) = \left(\frac{1}{1 - az^{-1}}\right)(1 - az^{-1})\left(\frac{1}{1 - \bar{a}z^{-1}}\right) = \frac{1}{1 - \bar{a}z^{-1}}$$

Therefore,

$$y(n) = \bar{a}^n u(n)$$

(b) Unfortunately, to reduce the cost of the system, Phil implemented the synthesizer using an inexpensive circuit. The coefficient $\bar{a}$ was linearly quantized over the range [-1,1] using a simple uniform quantization scheme with 4 bits of precision.

For the case $\bar{a} = 0.93$, what is the signal to noise ratio of the overall system for the input signal in (a).

The idea I was trying to get across here was that 0.93 cannot be represented exactly in 4 bits. Since (as mentioned in class) $\bar{a} = Q[a]$, we can make the following assumptions:

$$a = 0.93$$

$$\bar{a} = Q[a] \approx \frac{[Int[(0.93-(-1))\times 2^4]]}{2^4} - 1 = 0.9375$$

(This is a fancy way of expressing a uniform quantizer mathematically. We had actually discussed this specific case in class.)

The SNR is the input signal power divided by the power in the difference between the input signal and the output signal:

$$SNR = \frac{\sum x^2(n)}{\sum (y(n)-x(n))^2} = 10\log_{10}\left(\frac{\sum (a^n)^2}{\sum (\bar{a}^n - a^n)^2}\right)$$

(Any reasonable approximation was accepted) If you approximate this sum by the first term, the SNR is about 41 dB; by two terms it is about 28 dB; it eventually converges to approximately 21 dB.

$$SNR \approx 21 \text{ dB}$$

(c) For the case where both filters are implemented using 8 bits of precision, is the analyzer or the synthesizer more sensitive to quantization error? Why?

The synthesizer is more sensitive because it is an IIR filter with a pole close to the unit circle.