Name: ______________________________

Problem	Points	Score
1a	5	
1b	5	
2a	5	
2b	10	
2c	10	
3a	10	
3b	10	
4	10	
5	20	
6a	5	
6b	5	
6c	5	
Total	**100**	

Notes:

1. The exam is closed book / closed notes. Students are allowed a copy sheet — only **one** side of **one** standard US-size (8.5” x 11”) sheet of paper — on which they can write relevant information such as theorems.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

3. If I can’t read or follow your solution, it is wrong, and no partial credit will be given — PLEASE BE NEAT!

4. Please indicate clearly your answer to every problem.

5. There is sufficient space after each problem to write your solution. In case you need extra paper please see the instructor.

6. Calculators of any kind are not allowed.

Problem No. 1:

a) Convert the following number from decimal to octal, and then to binary.

$$265.375_{10}$$

Solution:

To convert to octal form, for the integer part, use divide by 8 method.

$$\begin{array}{rl} 265 \div 8 = 33 & \text{Remainder} = 1 \\ 33 \div 8 = 4 & \text{Remainder} = 1 \\ 4 \div 8 = 0 & \text{Remainder} = 4 \end{array}\Bigg\uparrow$$

For the fraction part use multiply by 8 method.

$$0.375 \times 8 = 3.000 \qquad \text{Integer} = 3$$

Octal representation : 411.3

To convert the octal number to binary, use bit-substitution.

$$\begin{array}{ccccc} 4 & 1 & 1 & . & 3 \\ 100 & 001 & 001 & & 011 \end{array}$$

Binary representation : 100001001.011

b) Convert the following number from binary to hexadecimal and then to decimal.

$$1101000.011_2$$

Solution:

To convert to hexadecimal from binary, use bit-substitution.

$$\underbrace{0111}_{6}\ \underbrace{1000}_{8}\ .\ \underbrace{0110}_{6}$$

Hexadecimal representation : 68.6

To convert to decimal from hexadecimal form, use power series expansion.

$$\begin{aligned} 68.6 &= 6 \times 16^1 + 8 \times 16^0 + 6 \times 16^{-1} \\ &= 6 \times 16 + 8 \times 1 + \frac{6}{16} \\ &= 96 + 8 + 0.375 \\ &= 104.375 \end{aligned}$$

Decimal representation : 104.375

Problem No. 2:

a) Perform the following binary division. Clearly indicate the quotient and the remainder in the space provided.

$$10111001 \div 1101$$

Solution:

$$
\begin{array}{r@{\,}l}
 & 1110 \\
\cline{2-2}
1101 \,\big) & 10111001 \\
 & 1101 \\
\cline{2-2}
 & 10100 \\
 & 1101 \\
\cline{2-2}
 & 1110 \\
 & 1101 \\
\cline{2-2}
 & 00011
\end{array}
$$

Quotient: **1110**

Remainder: **11**

b) Add the following two numbers in binary using a 5-bit 1's complement representation. Indicate if there is an overflow.

$$(-3) + (-11)$$

Solution:

Convert the numbers to 1's complement first.

$$\begin{aligned} 3 &= 00011 & \therefore -3 &= 11100 \\ 11 &= 01011 & \therefore -11 &= 10100 \end{aligned}$$

Now add the two numbers —

$$\begin{array}{lr} & 11100 \\ & +\ 10100 \\ \hline & 1|\ 10000 \\ \text{End-around carry} & +\ 1 \\ \hline & 10001 \end{array}$$

The sum is **10001**. There is **no overflow**, as the sum of two negative numbers is as expected (Check — 1's complement of 10001 is 01110 i.e. the answer is -14).

c) Perform the following subtraction assuming 2's complement representation of the binary numbers. Indicate if there is an overflow.

$$10011 - 11010$$

Solution:

To subtract, convert the second number to 2's complement first, and then add to the first number.

The 2's complement of 11010 is 00110. Now add the two numbers —

$$\begin{array}{r} 10011 \\ +\ 00110 \\ \hline 11001 \end{array}$$

The answer is **11001**. There is **no overflow**, as the answer is the expected one (Check — 10011 is -13, 11010 is -6. Their difference is -7, which is 11010 in 2's complement).

Problem No. 3:

a) Find the complement of F. Do **not** simplify the expression.

$$F = AB + (A' + C)(B + E')[D'E(A' + B) + 1]$$

Solution:

Using the set of rules for complement, this turns out to be a one-step process. We get

$$F' = (A' + B')[AC' + B'E + (D + E' + AB')0]$$

b) Find the dual of F. Do **not** simplify the expression.

$$F = (A + B')(A'D' + E) + (AC' + BD)[(C + E')(B' + D) + 0]$$

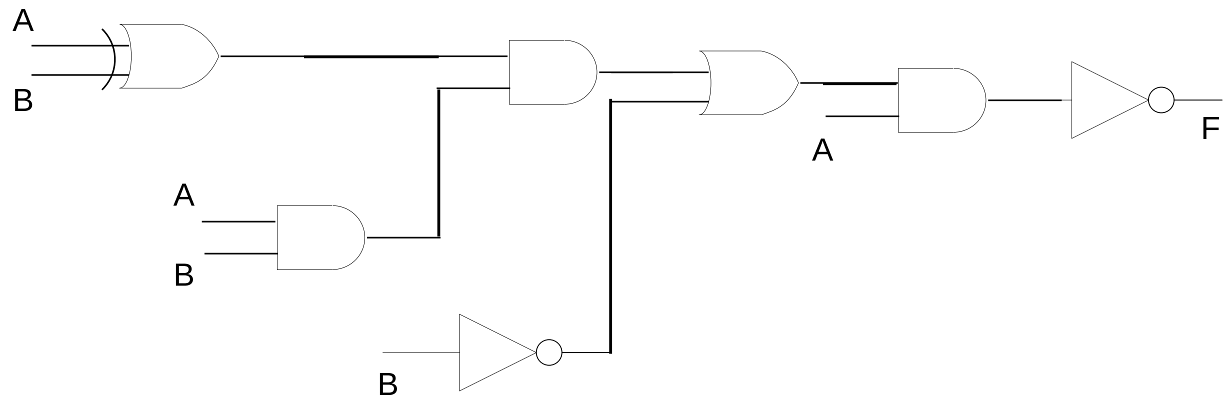
Solution:

Using the set of rules for duals, this turns out to be a one-step process. We get

$$F_D = [AB' + (A' + D')E][(A + C')(B + D) + (CE' + B'D)1]$$

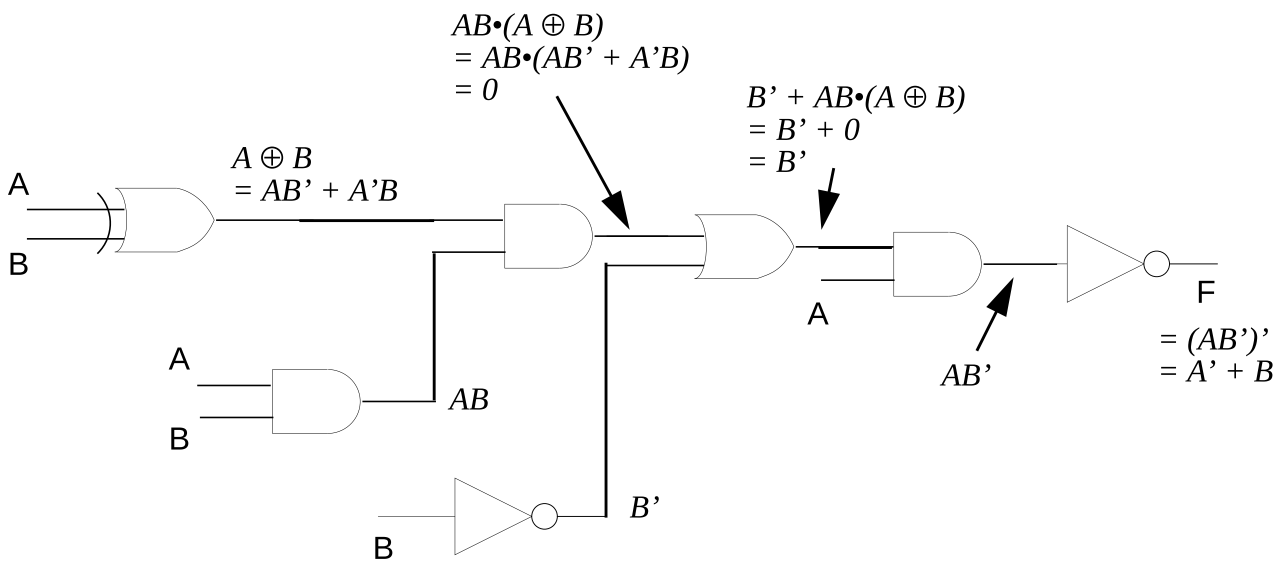
Problem No. 4:

An electronics company wants to cut costs on a circuit being designed in its lab. It has organized a design competition for the students of ECE 3713 to simplify the following circuit and find the minimum expression for F. The winner has to draw a circuit diagram for this minimum form using at most two logic gates. Please send your entry for this competition.



Solution:

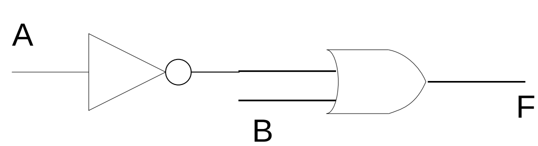
Simplify the circuit at every gate.



Thus

F = A' + B

The resulting circuit looks as shown here —



Problem No. 5:

Assume that you have graduated and now run your own multi-billion electronics company that sells the following circuit —

$$(A \equiv B)(C + D') + (C \equiv D)(A + B')$$

As a smart engineer, you design the following circuit to save costs —

$$(C \oplus D)' + C(A \equiv B)$$

Now you need to check if the two circuits perform the same task. Do so by simplifying both expressions to a minimum form and compare if they are equal.

Solution:

$$\begin{aligned}
\mathrm{FE} &= (A \equiv B)(C + D') + (C \equiv D)(A + B') \\
&= (A'B' + AB)(C + D') + (CD + C'D')(A + B') \\
&= A'B'C + ABC + A'B'D' + ABD' + ACD + AC'D' + B'CD + B'C'D' \\
&= A'B'C + ABC + A'B'D' + ACD + AC'D' + B'CD + B'C'D' \\
&\qquad \ldots ABC + AC'D' + \cancel{ABD'} = ABC + AC'D' \\
&= A'B'C + ABC + A'B'D' + AC'D' + B'CD + B'C'D' \\
&\qquad \ldots ABC + B'CD + \cancel{ACD} = ABC + B'CD \\
&= ABC + A'B'D' + AC'D' + B'CD + B'C'D' \\
&\qquad \ldots A'B'D' + B'CD + \cancel{A'B'C} = A'B'D' + B'CD \\
&= ABC + A'B'D' + AC'D' + B'CD \\
&\qquad \ldots A'B'D' + AC'D' + \cancel{B'C'D'} = A'B'D' + AC'D' \\
&= ABC + A'B'D' + AC'D' + B'CD
\end{aligned}$$

$$\begin{aligned}
\mathrm{SE} &= (C \oplus D)' + C(A \equiv B) \\
&= (CD + C'D') + C(AB + A'B') \\
&= CD + C'D' + ABC + A'B'C
\end{aligned}$$

Thus FE (first expression) and SE (second expression) are not equal, and therefore the circuit you designed is not the same as the old circuit.

Note: This is only one of two possible minimum forms for FE. However, the inequality is valid in either case.

Problem No. 6:

A combinatorial switching network has four inputs A,B,C and D; and two outputs X and Y. The output X goes high if the representation ABCD has no two adjacent 1s, otherwise it is 0. Y equals 1 if there are no two adjacent 0s in the representation ABCD, otherwise it is low.

a) Construct a truth table for this network.

Solution:

The truth table is as follows —

A	B	C	D	X	Y
0	0	0	0	1	0
0	0	0	1	1	0
0	0	1	0	1	0
0	0	1	1	0	0
0	1	0	0	1	0
0	1	0	1	1	1
0	1	1	0	0	1
0	1	1	1	0	1
1	0	0	0	1	0
1	0	0	1	1	0
1	0	1	0	1	1
1	0	1	1	0	1
1	1	0	0	0	0
1	1	0	1	0	1
1	1	1	0	0	1
1	1	1	1	0	1

b) Provide a maxterm representation for X.

Solution:

The maxterm representation for X is

$$X = \prod M(3, 6, 7, 11, 12, 13, 14, 15)$$

c) Provide a minterm representation for Y.

Solution:

The minterm representation for Y is

$$Y = \sum m(5, 6, 7, 10, 11, 13, 14, 15)$$