Name: ______________________________

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total	**100**	

Notes:

1. The exam is closed book / closed notes. You are allowed a copy sheet — only **one** side of **one** standard US-size (8.5” x 11”) sheet of paper — on which you can write relevant information such as equations. You are allowed to bring copy sheets from previous exams.

2. Please show **all** work. Incorrect answers with no supporting explanations or work will be given no partial credit.

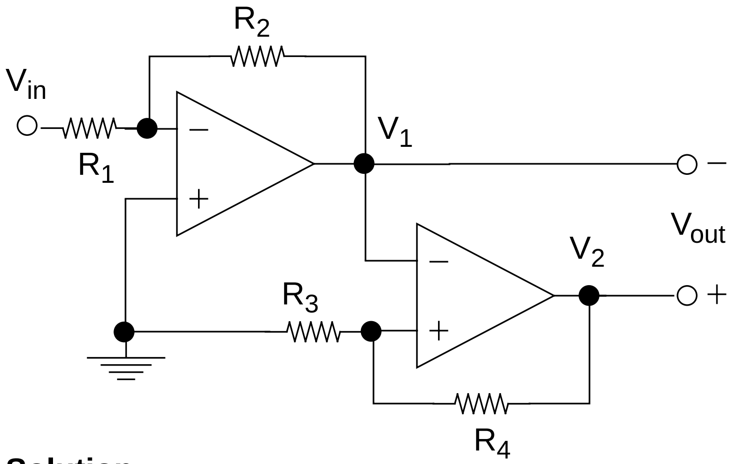
3. If I cannot read or follow your solution, it is wrong; and no partial credit will be given — **PLEASE BE NEAT**!

4. Please indicate clearly your answer to every problem.

5. There is sufficient space after each problem to write your solution. In case you need extra paper please see the instructor.

6. If specified, do **not** use calculators. Show complete work and detailed steps for proper credit.

Problem No. 1:



Find the voltage gain

$$A_v = \frac{V_{out}}{V_{in}}$$

for the adjacent op-amp circuit. Use

$R_1 = 10k\Omega$ $R_2 = 33k\Omega$
$R_3 = 1.2k\Omega$ $R_4 = 10k\Omega$.

Solution:

The first op-amp with R_1 and R_2 is in a standard inverting amplifier configuration, with V_{in} as input and V_1 as the output voltage. Therefore, we have

$$V_1 = -\frac{R_2}{R_1}V_{in}.$$

The second op-amp with R_3 and R_4 is a non-inverting amplifier with V_1 as the input and V_2 as the output. Therefore

$$V_2 = \left(1 + \frac{R_4}{R_3}\right)V_1$$

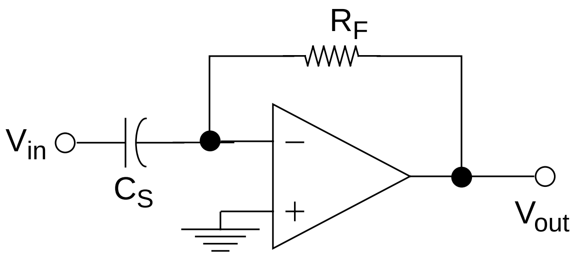
The output voltage is given by

$$V_{out} = V_2 - V_1 = \left(1 + \frac{R_4}{R_3}\right)V_1 - V_1 = \frac{R_4}{R_3}V_1 = -\frac{R_2R_4}{R_1R_3}V_{in}$$

Thus the voltage gain is

$$\frac{V_{out}}{V_{in}} = -\frac{R_2R_4}{R_1R_3} = -27.5$$

Problem No. 2:



Identify the adjacent op-amp circuit (is it a differentiator or an integrator?).

The input signal is given by $V_{in} = 10\sin(4000\Pi t)$ V.

Calculate the output voltage V_{out} if $R_F = 10k\Omega$ and $C_S = 0.01\mu F$.

Solution:

This op-amp circuit is a **differentiator**.

For a differentiator op-amp circuit,

$$\begin{aligned} V_{out} &= -R_F C_S \frac{dV_{in}}{dt} \\ &= -10k\Omega \times 0.01\mu F \times \frac{d}{dt} 10\sin(4000\Pi t) V \\ &= -10^{-4} \times 40000\Pi\cos(4000\Pi t) V \\ &= -4\Pi\cos(4000\Pi t) V \end{aligned}$$

Problem No. 3:

The relationship between the force $f(t)$ on the piston of an engine, and the displacement $x(t)$ of the piston is given by

$$\frac{d^2x}{dt^2} + 100\frac{dx}{dt} + 5x = -10f(t)$$

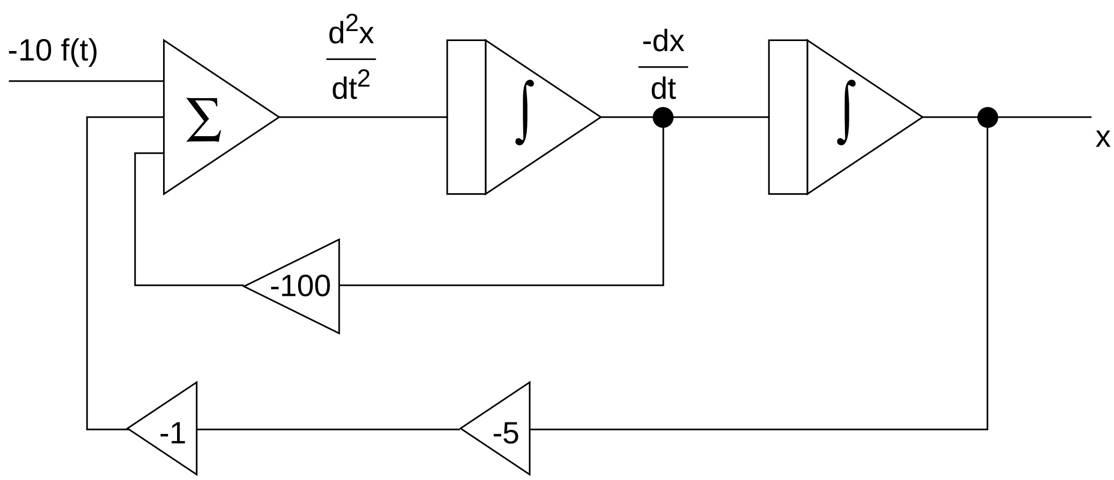
Construct an analog computer simulation (i.e. draw a circuit diagram for the analog computer) that implements the above differential equation.

Solution:

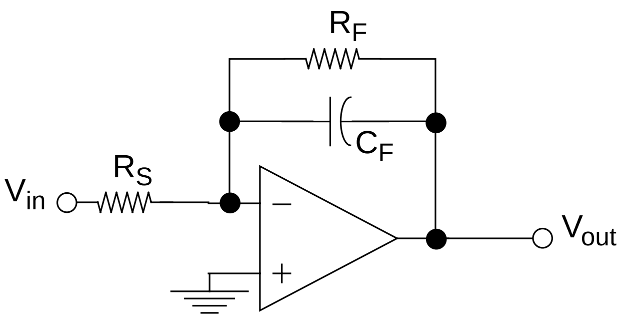
We can re-write the above equation as

$$\frac{d^2x}{dt^2} = -100\frac{dx}{dt} - 5x - 10f(t)$$

Therefore, the analog computer used to solve this equation is given by



Problem No. 4:



Identify the adjacent op-amp filter (low-pass, band-pass or high-pass?).

The cut-off frequency of the filter is $\omega_0 = 100Hz$. Also, $R_S = 10k\Omega$ and $C_F = 0.5\mu F$.

Find R_F and the voltage gain $A_V(j\omega)$.

Solution:

This is a **low-pass filter** circuit.

For a low-pass filter, we have

$$A_V(j\omega) = \frac{-R_F/R_S}{1 + j\omega R_F C_F} \qquad \omega_0 = \frac{1}{R_F C_F}$$

Therefore the feedback resistance is

$$R_F = \frac{1}{\omega_0 C_F} = \frac{1}{100 \times 0.5 \times 10^{-6}} = 2 \times 10^4 = 20k\Omega$$

And the frequency response is given by

$$A_V(j\omega) = \frac{-20k\Omega/10k\Omega}{1 + j\omega/100} = \frac{-2}{1 + 0.01j\omega}$$

Problem No. 5:

Convert the following number from decimal to binary. Then convert it to hexadecimal.

631.3125_{10}

Do not use calculators of any kind. Show in detail all the steps involved.

Solution:

We can split the decimal to binary conversion in its integer and fraction portions.

Integer part:

631	÷	2	=	315	1	↑
315	÷	2	=	157	1	
157	÷	2	=	78	1	
78	÷	2	=	39	0	
39	÷	2	=	19	1	
19	÷	2	=	9	1	
9	÷	2	=	4	1	
4	÷	2	=	2	0	
2	÷	2	=	1	0	
1	÷	2	=	0	1	

Fraction part:

0.3125	x	2	=	0.625	0	↓
0.625	x	2	=	1.25	1	
0.25	x	2	=	0.5	0	
0.5	x	2	=	1.0	1	

Thus the binary equivalent number is

1001110111.0101_2

To convert this into hexadecimal, divide the bits into groups of 4 bits each and replace each group with the corresponding hexadecimal digit. Pad with 0s wherever necessary.

0010	0111	0111	.	0101
2	7	7		5

The corresponding hexadecimal value is 277.5_{16}

Problem No. 6:

a) Perform any *one* of the following binary arithmetic operations (assume unsigned numbers). **Do not use calculators of any kind.**

1. 10101×110
2. $110101 \div 101$

Solution 1:

$$\begin{array}{r} 10101 \\ \times\, 110 \\ \hline 00000 \\ 101010 \\ \underline{1010100} \\ 1111110 \end{array}$$

Solution 2:

$$\begin{array}{r|l} & 1010 \\ \hline 101\ ) & 110101 \\ & \underline{101} \\ & 00110 \\ & \underline{101} \\ & 0011 \end{array}$$

Quotient: 1010
Remainder: 11

b) Assume 6 bit signed binary arithmetic with 2's complement representation of negative numbers. Perform the following subtraction and indicate whether there is an overflow. **Do not use calculators of any kind.**

$$010110 - 111011$$

Solution:

2's complement of 111011 is 000101.

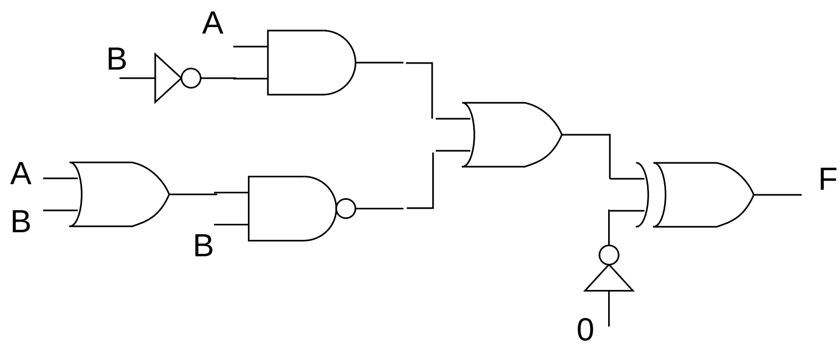
In 2's complement binary arithmetic, we have

$$\begin{array}{r} 010110 \\ -111011 \\ \hline \end{array} \quad \equiv \quad \begin{array}{r} 010110 \\ +\,000101 \\ \hline 011011 \end{array}$$

There is no overflow, as the expected result is also positive (a negative number subtracted from a positive number).

Problem No. 7:

Find the minimum expression for the output of the logic circuit shown below.



Solution:

$$
\begin{aligned}
F &= \bar{0} \oplus (A\overline{B} + \overline{(A+B)B}) \\
&= 1 \oplus (A\overline{B} + \overline{(AB+B)}) \\
&= 1 \oplus (A\overline{B} + \overline{B}) \\
&= 1 \oplus \overline{B} \\
&= B
\end{aligned}
$$

Problem No. 8:

Using only the laws of Boolean algebra, simplify the following expression to its minimum sum-of-products form.

$$(\bar{A} + B)(\bar{A} + C)(C + D)(B + D)$$

Solution:

$$(\bar{A} + B)(\bar{A} + C)(C + D)(B + D)$$

$$= (\bar{A} + BC)(BC + D) \qquad \ldots(X + Y)(X + Z) = X + YZ$$

$$= \bar{A}D + BC \qquad \ldots(X + Y)(X + Z) = X + YZ$$

Problem No. 9:

A game machine at the Digital Casino consists of four slots ABCD. You throw four darts simultaneously towards the slots. If any two of the slots are hit, you win a dollar. Having all slots empty or filled with darts is not allowed. Otherwise, you lose a dollar.

Draw a truth table for a digital circuit that simulates this game, with inputs ABCD and output F indicating a win. Derive a minimum expression for F using Karnaugh maps. Account for the don’t care terms appropriately.

Solution:

The truth table is as follows —

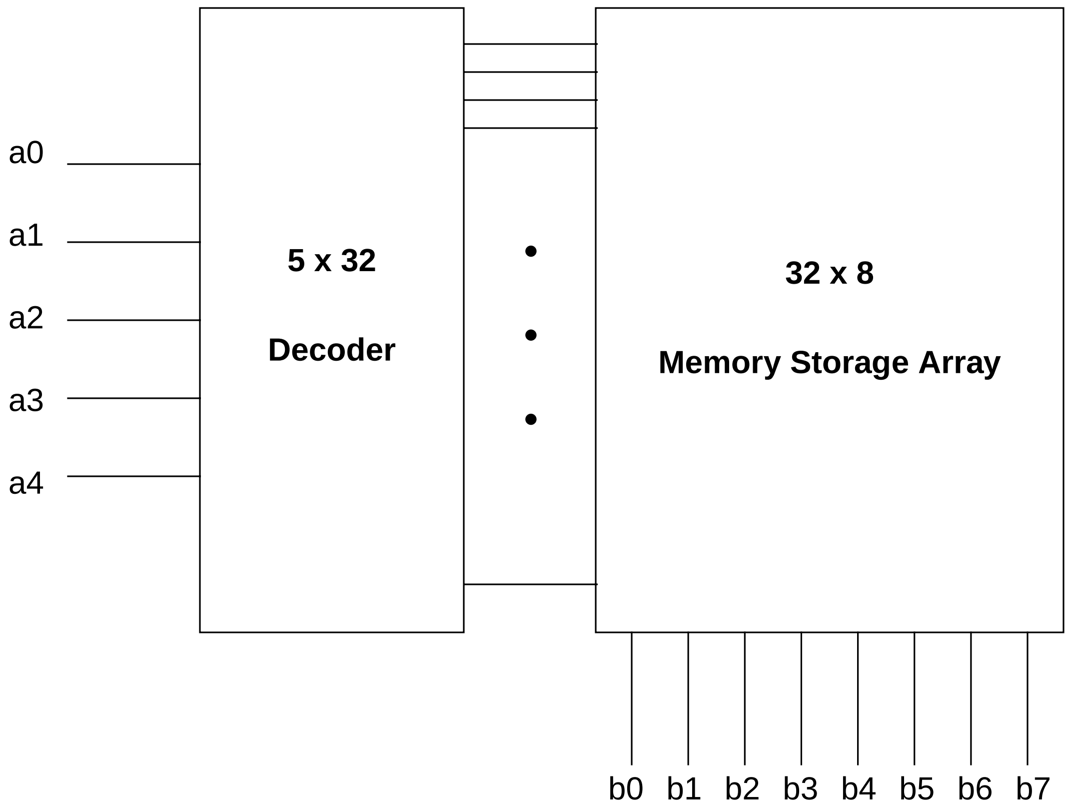
ABCD	F
0000	X
0001	0
0010	0
0011	1
0100	0
0101	1
0110	1
0111	0
1000	0
1001	1
1010	1
1011	0
1100	1
1101	0
1110	0
1111	X

CD \ AB	00	01	11	10
00	X	0	1	0
01	0	1	0	1
11	1	0	X	0
10	0	1	0	1

$$F = \bar{A}\bar{B}CD + \bar{A}B\bar{C}D + \bar{A}BC\bar{D} + A\bar{B}\bar{C}D + A\bar{B}C\bar{D} + AB\bar{C}\bar{D}$$

Problem No. 10:

Draw a block diagram of a 32x8 ROM. What is the number of address bits required? What is the largest value that can be stored in this ROM?



Since there are 32 address lines, the number of address bits required is 5 (since $2^5 = 32$).

The largest value that can be stored in the ROM is the largest 8 bit number, which is 11111111_2 i.e. 255_{10}.