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Problem	Points	Score
1a	10	
1b	10	
1c	10	
1d	10	
2a	10	
2b	10	
2c	10	
3a	10	
3b	10	
3c	10	
Total	100	

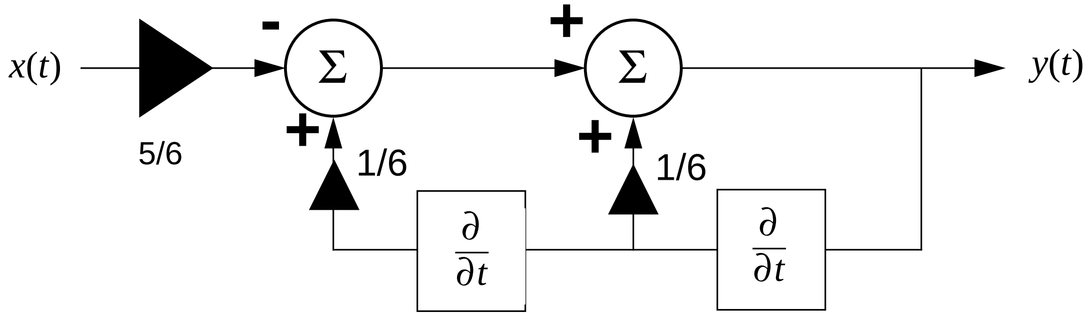
Notes:

1. The exam is closed books/closed notes - except for one page of notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

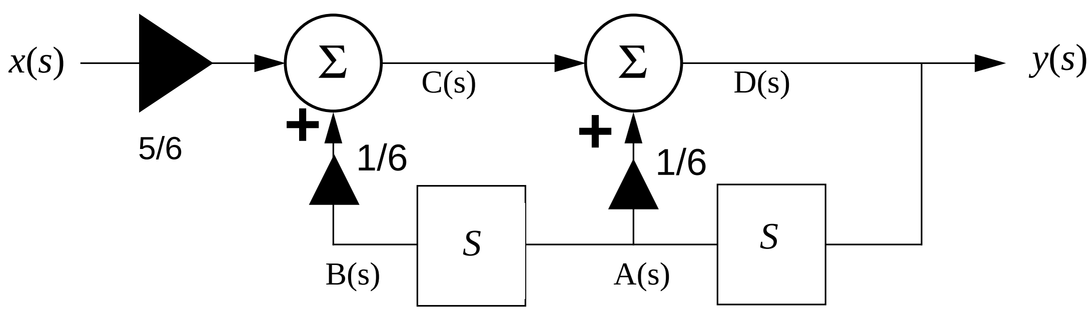
3. Please indicate clearly your answer to the problem. If I can’t read it (and I am the judge of legibility), it is wrong. If I can’t follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)

Problem No. 1: Block Diagrams



(a) Find the transfer function of this system using Laplace transforms.

The first step in solving this problem is to convert the system into the frequency domain.



$$Y(s) = Ay(s)$$
$$A(s) = Y(s)S$$
$$B(s) = S^2Y(s)$$
$$C(s) = \frac{S^2}{6}Ys + \left(-\frac{S}{6}X(s)\right)$$
$$D(s) = A_3(s) + \frac{(A_1(s))}{6} = \frac{S^2}{6}Y(s) + \frac{S}{6}Y(s) + \left(-\frac{S}{6}X(s)\right)$$

Hence:

$$Y(s) = \frac{S^2}{6}Y(s) + \frac{S}{6}Y(s) - \frac{S}{6}X(s)$$

The equation then becomes:

$$\frac{5}{6}\frac{(X(s))}{Y(s)} = \frac{S^2}{6} + \frac{S}{6} - 1$$

In order to obtain H(s), the reciprical of the previous equation had to be taken:

$$\frac{5}{6}\frac{(Y(s))}{X(s)} = \frac{6}{S^2} + \frac{6}{S} - 1$$

Since H(s)=Y(s)/X(s) we conclude the following equation:

$$H(s) = \frac{(Y(s))}{X(s)} = \frac{5}{S^2 + S - 6}$$

(b) Find the impulse response.

$$H(s) = \frac{5}{(S+3)(S-2)} = \frac{(AS+B)}{(S+3)(S-2)}$$

A = -1 and B = 1.

$$H(s) = -\frac{1}{S+3} + \frac{1}{S-2}$$

After applying the Laplaace transform we get the following equation:

$$L\{H(s)\} = H(t) = [(-e^{-3t}) + (e^{2t})]u(t)$$

(c)Determine whether the system is stable or unstable. Show ALL work.
The correct answer with no supporting work gets no points. Be as detailed as possible.

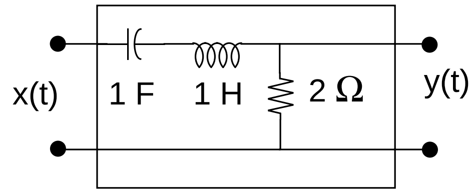
Because the poles are at +2 and -3, the system is unstable. Only negative poles can make a system stable.

(d)Sketch the frequency response (magnitude only) of the system using Bode plots.

The Bode plot does not exist because the system is not stable.

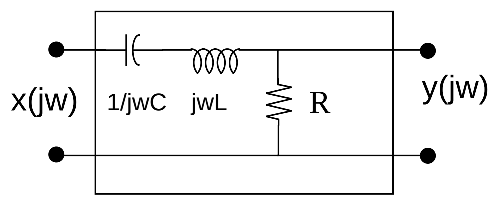
Problem No. 2: Circuit analysis using Fourier transforms.

For the circuit shown below:



(a)State all the Fourier transform theorems that are invoked when you compute the transfer function of this circuit. You must give specific evidence to support each theorem described (and I must be able to understand your logic!).

First observe the system in the frequency domain.



Differentiation: For determining the current flowing through the inductor

$$\frac{\partial}{\partial t}i(t) = jwI(jw)$$

Superposition: For determining Inductance and Capacitance.

$$\frac{1}{j\omega C} + j\omega L = \frac{(1 - LC\omega^2)}{j\omega C}$$

Integration: For determining the voltage across the capacitor

$$\left(-\frac{1}{C}\right)\int_{-\infty}^{t} i(t)dt = I\frac{(jw)}{j(w)} + \frac{1}{2}x(0)\varsigma(jw)$$

(b) State and prove the Frequency Translation Theorem.

$$\Im\{x(t)e^{j\omega wt}\} = X(\omega - \omega_o)$$

$$\int_{-\infty}^{\infty} [x(t)e^{j\omega t}]e^{-j\omega t}dt = X(\omega - \omega_o)$$

$$\int_{-\infty}^{\infty} x(t)e^{-j(\omega - \omega o)}dt = X(\omega - \omega_o)$$

The equation is then multiplied by *jw*C:

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{R}{j\omega L + R + \frac{1}{j\omega C}} = \frac{(j\omega RC)}{((j\omega)^2)LC + j\omega RC + 1}$$

The values given are then entered into the equation:

$$H(j\omega) = \frac{(2j\omega)}{((j\omega)^2) + 2j\omega + 1} = \frac{(2j\omega)}{(j\omega + 1)^2} = \frac{A}{j\omega + 1} + \frac{B}{(j\omega + 1)^2}$$

A=2 and B= -2

$$H(j\omega) = \frac{2}{j\omega + 1} - \frac{2}{(j\omega + 1)^2}$$

Therefore we end up with:

$$h(t) = \Im^{-1}\{H(j\omega)\} = \Im^{-1}\left\{\frac{2}{j\omega + 1}\right\} - \Im^{-1}\left\{\frac{2}{(j\omega + 1)^2}\right\} = (2e^{-t} - 2te^{-t})u(t)$$

Problem No. 3: The Dreaded Thought Problem

Signal to Noise (SNR) ratio is defined as the ratio of the power of a signal and the power of the noise in a system, computed on a log scale and measured in dB:

$$SNR|_{dB} = 10\log_{10}\left(\frac{P_{signal}}{P_{noise}}\right)$$

Assume the signal is given by $x(t) = \sin\omega_o t$, and the noise is given by $w(t) = e^{-\alpha|t|}$.

(a) Compute SNR in the time domain.

$Psignal = PowerofSineWave = \frac{A^2}{2}$ (given in class)

According to the Power equation

$$Pnoise = \frac{1}{T}\int_{-\infty}^{\infty} e^{-\alpha|t-n|}dt = \frac{1}{T}\int_{-\infty}^{\infty} e^{-\alpha|t-n|}dt$$

Since this signal is periodic, we can take the integral over one period (T=1)

$$Pnoise = \int_0^1 e^{-2\alpha|t-n|}dt$$

Letting n=0:

$$P = -\frac{1}{2\alpha}e^{-2\alpha 1} + \frac{1}{2\alpha}e^{-2\alpha 0} = \frac{1}{2\alpha}((e^{-\alpha}) - 1)$$

$$SNR = 10\log\left(\frac{\left(\frac{(A^2)}{2}\right)}{\frac{((e^{-2\alpha})-1)}{\alpha}}\right) = 10\log\frac{\alpha}{((e^{-2\alpha})-1)}$$ Where t=1.

(b) Compute SNR in the frequency domain and prove it is equivalent to the time domain calculation.

Parseval's theorem is stated as follows:

$$\left(\int_{-\infty}^{\infty}((|X(t)|) \wedge 2dt)\right) = \int_{-\infty}^{\infty}(|X(f)| \wedge 2)df$$

Therefore the output by using Parseval's Theorem is as follows:

$$SNR = 10\log\left(\frac{\left(\frac{(A^2)}{2}\right)}{\frac{((e^{-2\alpha})-1)}{\alpha}}\right) = 10\log\frac{\alpha}{((e^{-2\alpha})-1)}$$

(c) Explain how the SNR varies with ω_0 and α.

-As α gets larger, SNR gets larger and complex.
-As α gets smaller, SNR gets smaller.
-ω_0 has no effect on SNR.