Name:

Problem	Points	Score
1a	10	
1b	10	
1c	10	
1d	10	
2a	10	
2b	10	
2c	10	
2d	10	
3a	10	
3b	10	
Total	100	

Notes:

1. The exam is closed books/closed notes - except for one page of notes.
2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.
3. Please indicate clearly your answer to the problem. If I can't read it (and I am the judge of legibility), it is wrong. If I can't follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)

Problem No. 1: Modeling Concepts

(a) Prove whether the signal $x(t) = te^{-\alpha t}u(t)$ is an energy signal or power signal.

According to the Energy definition $x(t) = te^{-\alpha t}u(t)$ equals $\int_0^\infty |te^{-\alpha t}|^2 dt = \int_0^\infty t^2 e^{-2\alpha t} dt$

Using the identity: $\int_0^\infty t^m e^{-\alpha t} = \frac{m!}{\alpha^{n+1}}$

The integral can be expressed as:

$$E = \frac{1}{4\alpha^3}$$

Therefore, since the energy of the signal is finite, and the power of the signal is zero,
this is an ENERGY SIGNAL.

(b) Is the signal $x(t) = \sin^2 \omega_0 t$ periodic? If so, what is its period? If not, explain.

This signal is periodic because it satsifies the following conditions:

$$x(t) = x(t + T_0)$$

$$x(t + T_0) = (\sin \omega_0 (t + T_0))^2$$

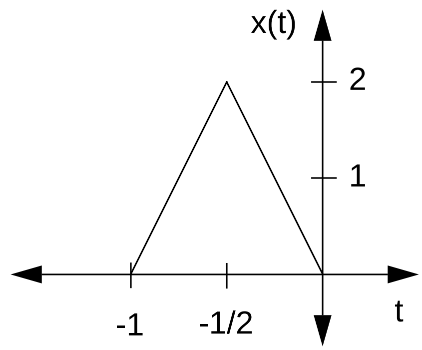
$$= \left(\sin \omega_0 \left(t + 2\frac{\pi}{\omega_0}\right)\right)^2$$

$$= (\sin \omega_0 t)^2$$

Therefore, the condition holds, proving that this signal is periodic. The period can be found by saying:

$$(\sin \omega_0 t)^2 = \frac{1}{2} - \frac{1}{2}\cos 2\omega_0 t \text{ hence } T_0 = \frac{\pi}{\omega_0} = \frac{1}{2f_0}$$

(c) Write the following signal in terms of a weighted sum of a combination of one or all of the following functions: $\delta(t), u(t), r(t)$.



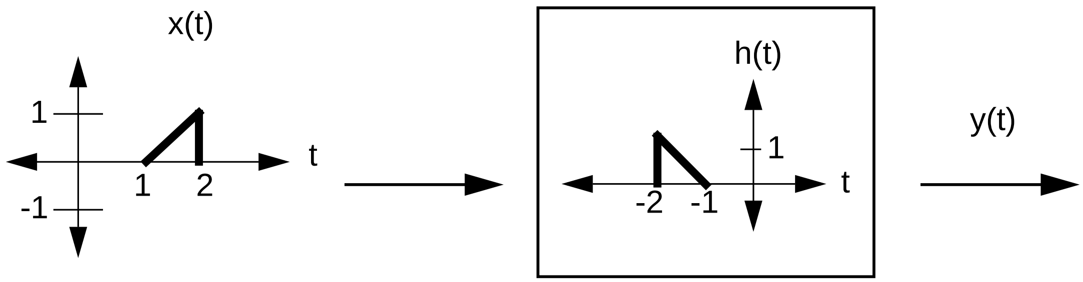
$$x(t) = 4r(t+1) - 8r\left(t + \frac{1}{2}\right) + 4r(t)$$

(d) Compute the energy value of the signal in (c).

$$E = 2\int_0^{0.5} (4t)^2 dt \quad = \quad 2\left(\left(\frac{16}{3}\right)t^3\Big|_0^{0.5}\right) \quad = 1.33 \text{ Joules}$$

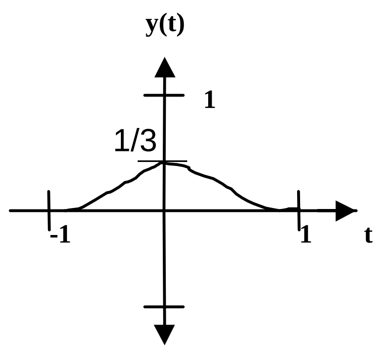
Problem No. 2: Time-Domain Solutions

Consider the signal and system (completely described by its impulse response):



(a) Compute and plot output, $y(t)$, for the system shown above.

The time that the output signal starts is equal to the sum of the lower time limit of x(t) which is 1 and the lower limit of h(t) which is -2. 1+(-2)=-1, so -1 is where the output starts and ends at 1. The maximum amplitude is equal to the maximum area shared between x(t) and a convolved h(t).



The output is computed by the following:

$$y(t) = (0, (t < -1), (t > 1))$$
$$x(t) = (t-1), 1 \le t \le 2$$
$$h(t) = (t+2), -2 \le t \le -1$$
$$\int_{-2}^{(t-1)} (t - \lambda - 1) d\lambda = -\frac{t^2}{6} + \frac{t}{2} + \frac{1}{3}, -1 \le t \le 0$$
$$-\int_{(t-2)}^{-1} (t - \lambda - 1)(\lambda + 1) d\lambda = \frac{t^2}{6} - \frac{t}{2} + \frac{1}{3}, 0 \le t \le 1$$

(b) Without using the answer to part (a), explain whether the system is causal.

The system is non-causal because h(t) is not 0 for negative values of time.

(c) Use your answer to part (a) to support your reasoning given in (b).

According to the graph the input comes behind the output, which indicates that it is a non-causal system.

(d) Describe the system using as many system modeling concepts as possible (for example, linearity). Justify your answers.

Fixed - the signal only shifts and doesn’t change at different points in time

Aperiodic - no period

Instantaneous - no memory elements

Linear - superposition holds

Non-causal - explained above

Continuous - it is defined at every point in time

Problem No. 3: Fourier Series

Given the signal $x(t) = 3\sin(1.5\omega_1 t) + 5\cos(1.75\omega_1 t)$,

(a) Using symmetry arguments, explain which Fourier coefficients of the trigonometric Fourier Series will be zero ({an} and {bn}). Be careful and be precise :)

By inspection, the signal has no constant, or DC value which indicates that this signal has an average value of zero. Therefore, $a_0 = 0$, and all other coefficients are nonzero. This argument is furthered by the mathematical evidence below:

If x(-t) = x(t), then the function is even, and if x(-t) = -x(t), then the function is odd.

$$x(t) = (3\sin(1.5\omega_1 t)) + (5\cos(1.75\omega_1 t))$$
$$x(-t) = (3\sin(-1.5\omega_1 t)) + (5\cos(-1.75\omega_1 t))$$
$$-x(t) = -(3\sin(1.5\omega_1 t)) - (5\cos(1.75\omega_1 t))$$

since x(-t) does not equal x(t), and x(-t) does not equal -x(t), neither a_n nor b_n will be zero.

(b) Compute the Fourier series coefficients.

The signal x(t) is already in standard form for a Trigonometric Fourier Series:

$$x(t) = a_0 + \sum_{n=1}^{\infty} ((a_n \cos n\omega_0 t) + (b_n \sin n\omega_0 t))$$

$$x(t) = (3 \sin 1.5\omega_1 t) + (5 \cos 1.75\omega_1 t)$$

It can easily be seen that the above series is periodic with a fundamental frequency,
$f_0 = \frac{f_1}{4}$, which can be transformed into $\omega_1 = 4\omega_0$. Using this relationship, we can now determine the Fourier coefficients of the series. Knowing the properties of integrals involving the products of sines and cosines:

$$I_1 = \int_{T_0} (\sin m\omega_0 t) \sin n\omega_0 t dt = 0, m \neq n$$

$$I_1 = \frac{T_0}{2}, m = n \neq 0$$

$$I_2 = \int_{T_0} (\cos m\omega_0 t)(\cos n\omega_0 t) dt = 0, m \neq n$$

$$I_2 = \frac{T_0}{2}, m = n \neq 0$$

And since $1.75\omega_1 t = n\omega_0 t$, the following holds true:

$$1.75\omega_1 = n\omega_0$$

$$n = 1.75\frac{\omega_1}{\omega_0}$$

$$n = 1.75\frac{(4\omega_0)}{\omega_0}$$

$$n = 7$$

Therefore, $a_7 = \frac{T_0}{2}$.

Using the same argument for b_n, we see that:

$$1.5\omega_1 = n\omega_0$$

$$n = 1.5\frac{\omega_1}{\omega_0}$$

$$n = 1.5\frac{(4\omega_0)}{\omega_0}$$

$$n = 6$$

Therefore, $b_6 = \frac{T_0}{2}$. So there are only two coefficients for this series, a_7 and b_6. All other coefficients are zero.