Name: Damon Reeves

Problem	Points	Score
1a	10	
1b	10	
1c	10	
1d	10	
2a	10	
2b	10	
2c	10	
2d	10	
3a	10	
3b	10	
Total	100	

Notes:

1. The exam is closed books/closed notes - except for one page of notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

3. Please indicate clearly your answer to the problem. If I can't read it (and I am the judge of legibility), it is wrong. If I can't follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)

Problem No. 1: Modeling Concepts

(a) Prove whether the signal $x(t) = te^{-\alpha t}u(t)$ is an energy signal or power signal.

By definition, the Energy in $x(t) = te^{-\alpha t}u(t)$ is $\int_0^{\infty} |te^{-\alpha t}|^2 dt = \int_0^{\infty} t^2 e^{-2\alpha t} dt$

Using the identity located in our Signals and Systems book on page 613:

$$\int_0^{\infty} t^m e^{-\alpha t} = \frac{m!}{\alpha^{n+1}}$$

and integrating the signal we can see that:

$$E = \frac{1}{4\alpha^3}$$

By looking at the output, we can see that the power of the signal is zero, and that the energy is infinite thus making it an energy signal.

(b) Is the signal $x(t) = \sin^2 \omega_0 t$ periodic? If so, what is its period? If not, explain.

This signal is periodic because it satsifies the following conditions:

$$x(t) = x(t + T_0)$$

$$x(t + T_0) = (\sin\omega_0(t + T_0))^2$$

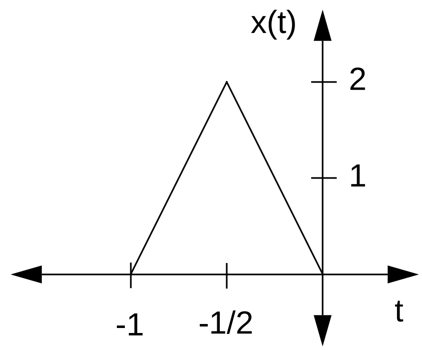
$$= \left(\sin\omega_0\left(t + 2\frac{\pi}{\omega_0}\right)\right)^2$$

$$= (\sin\omega_0 t)^2$$

Because the signal satisfies this condition, we can say it is periodic. The period is found by saying:

$$(\sin\omega_0 t)^2 = \frac{1}{2} - \frac{1}{2}\cos 2\omega_0 t \text{ so } T_0 = \frac{\pi}{\omega_0} = \frac{1}{2f_0}$$

(c) Write the following signal in terms of a weighted sum of a combination of one or all of the following functions: $\delta(t), u(t), r(t)$.



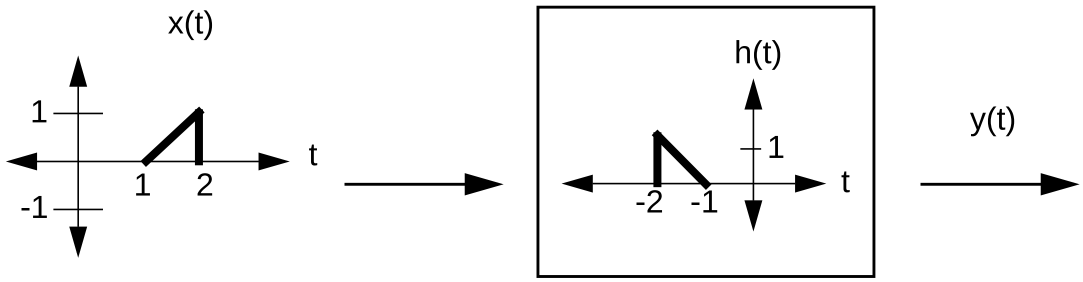
$$x(t) = 4r(t+1) - 8r\left(t + \frac{1}{2}\right) + 4r(t)$$

(d) Compute the energy value of the signal in (c).

$$E = 2\int_0^{0.5} (4t)^2 dt \quad = \quad 2\left(\left(\frac{16}{3}\right)t^3\Big|_0^{0.5}\right) \quad = 1.33 \text{ Joules}$$

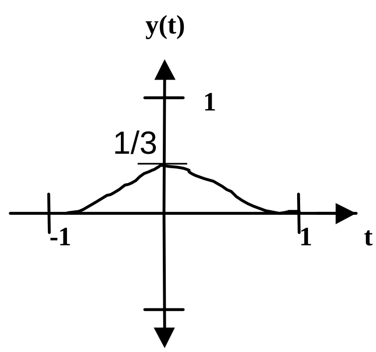
Problem No. 2: Time-Domain Solutions

Consider the signal and system (completely described by its impulse response):



(a) Compute and plot output, $y(t)$, for the system shown above.

The signal starts at time = -1 seconds (-2+1), and ends at time = 1 second(2-1).



The maximum amplitude is equal to the maximum area
shared between x(t) and a convolved h(t). The output is computed by convolution.

$$y(t) = (0, (t < -1), (t > 1))$$

$$x(t) = (t-1), 1 \le t \le 2$$

$$h(t) = (t+2), -2 \le t \le -1$$

$$\int_{-2}^{(t-1)} (t-\lambda-1)d\lambda = -\frac{t^2}{6} + \frac{t}{2} + \frac{1}{3}, -1 \le t \le 0$$

$$-\int_{(t-2)}^{-1} (t-\lambda-1)(\lambda+1)d\lambda = \frac{t^2}{6} - \frac{t}{2} + \frac{1}{3}, 0 \le t \le 1$$

(b) Without using the answer to part (a), explain whether the system is causal.

The system is non-causal because h(t) does have values for time < 0.

(c) Use your answer to part (a) to support your reasoning given in (b).

By looking at the graphs, the output y(t) begins before input x(t). Y(t) begins at -1, while x(t) does not begin until 1. Therefore, the system is noncausal.

(d) Describe the system using as many system modeling concepts as possible (for example, linearity). Justify your answers.

Time invariant - it does not depend on time

Aperiodic -it has no period

Instantaneous - it has no memory elements

Linear - superposition will hold true for this system

Non-causal - lots of reasons, see above

Problem No. 3: Fourier Series

Given the signal $x(t) = 3\sin(1.5\omega_1 t) + 5\cos(1.75\omega_1 t)$,

(a) Using symmetry arguments, explain which Fourier coefficients of the trigonometric

Fourier Series will be zero ({an} and {bn}). Be careful and be precise :)

We can tell by inspection that the signal has no DC value because $a_0 = 0$ and all other coefficients are nonzero. This argument is proved by the mathematical evidence below:

If x(-t) = x(t), then the function is even, and if x(-t) = -x(t), then the function is odd.

$$x(t) = (3\sin(1.5\omega_1 t)) + (5\cos(1.75\omega_1 t))$$
$$x(-t) = (3\sin(-1.5\omega_1 t)) + (5\cos(-1.75\omega_1 t))$$
$$-x(t) = -(3\sin(1.5\omega_1 t)) - (5\cos(1.75\omega_1 t))$$

since x(-t) does not equal x(t), and x(-t) does not equal -x(t), neither a_n nor b_n will be zero.

(b) Compute the Fourier series coefficients.

The signal x(t) is already in standard form for a Trigonometric Fourier Series:

$$x(t) = a_0 + \sum_{n=1}^{\infty} ((a_n \cos n\omega_0 t) + (b_n \sin n\omega_0 t))$$

$$x(t) = (3\sin 1.5\omega_1 t) + (5\cos 1.75\omega_1 t)$$

We can easily see that the above series is periodic with a fundamental frequency of $f_0 = \frac{f_1}{4}$, which can be transformed into $\omega_1 = 4\omega_0$. We can now determine the Fourier coefficients of the series. Knowing the properties of integrals

involving the products of sines and cosines:

$$I_1 = \int_{T_0} (\sin m\omega_0 t) \sin n\omega_0 t dt = 0, m \neq n$$

and

$$I_1 = \frac{T_0}{2}, m = n \neq 0$$

$$I_2 = \int_{T_0} (\cos m\omega_0 t)(\cos n\omega_0 t) dt = 0, m \neq n$$

and

$$I_2 = \frac{T_0}{2}, m = n \neq 0$$

we know that $1.75\omega_1 t = n\omega_0 t$, so the following is true:

$$1.75\omega_1 = n\omega_0$$

$$n = 1.75\frac{\omega_1}{\omega_0}$$

$$n = 1.75\frac{(4\omega_0)}{\omega_0}$$

$$n = 7$$

Therefore, $a_7 = \frac{T_0}{2}$.

Using the same argument for b_n, we see that:

$$1.5\omega_1 = n\omega_0$$

$$n = 1.5\frac{\omega_1}{\omega_0}$$

$$n = 1.5\frac{(4\omega_0)}{\omega_0}$$

$$n = 6$$

Therefore, $b_6 = \frac{T_0}{2}$. So there are only two coefficients for this series, a_7 and b_6. All other coefficients are zero.