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Problem	Points	Score
1a	10	
1b	10	
1c	10	
2a	10	
2b	10	
2c	10	
2d	10	
3a	10	
3b	10	
3c	10	
Total	100	

Notes:

1. The exam is closed books/closed notes - except for one page of notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

3. Please indicate clearly your answer to the problem.

Problem No. 1: For the linear time-invariant system: $H(s) = \dfrac{s+2}{(s^2-2s-3)}$

(a) Find the state variable description of the system.

$$H(s) = \frac{(s+2)}{s^2-2s-3} = \frac{(s+2)}{(s+1)(s-3)} = \frac{A}{s+1} + \frac{B}{s-3}$$

$$A = \left.\frac{(s+2)}{s-3}\right|_{s=-s-1} = \frac{(-s-1+s+2)}{-s-1+s-3} = -\frac{1}{4}$$

$$B = \left.\frac{(s+2)}{s-3}\right|_{s=-s+3} = \frac{(-s+3+s+2)}{-s+3+8+1} = \frac{5}{4}$$

$$H(s) = \frac{Y(s)}{X(s)} = -\frac{25}{s+1} + \frac{1.25}{s-3} = -25X1(s) + 1.25X2(s)$$

$$X1(s) = \frac{1}{s+1}, -X1 + u\ X2(s) = \frac{1}{s-3}, 3X2 + u$$

$$\dot{x}1 = -X1 + u, \dot{x}2 = 3X2 + u, y = 0.25X1 + 3.75X2 + 2u$$

$$\begin{bmatrix}\dot{x}1\\ \dot{x}2\end{bmatrix} = \begin{bmatrix}-1 & 0\\ 0 & 3\end{bmatrix}\begin{bmatrix}X1\\ X2\end{bmatrix} + \begin{bmatrix}1\\ 1\end{bmatrix}(u)\ y = \begin{bmatrix}0.25 & 3.75\end{bmatrix}\begin{bmatrix}X1\\ X2\end{bmatrix} + 2u$$

$$A = \begin{bmatrix}0 & -1\\ 0 & 3\end{bmatrix}$$

b) Compute the state transition matrix, $\Phi(t)$.

$$\Phi(s) = (sI - A)^{-1} = \begin{bmatrix} s+1 & 0 \\ 0 & s-3 \end{bmatrix}^{-1} = \frac{1}{s^2 - 2s - 3}\begin{bmatrix} s+1 & 0 \\ 0 & s-3 \end{bmatrix} = \begin{bmatrix} \frac{1}{s-3} & 0 \\ 0 & \frac{1}{s+1} \end{bmatrix}$$

$$\Phi(t) = \begin{bmatrix} e^{-3t} & 0 \\ 0 & e^{-t} \end{bmatrix}$$

(c) Using the state variable representation, implement this as an RLC circuit.

It is not possible to do this, because of instabilities in the system.

Problem No. 2: This problem deals with various aspects of Z-Transforms.

(a) **Derive** the expression for the Z-Transform of $x(n) = na^{-n}u(n)$.

$$x(n) = na^{-n}u(n)$$

$$Z[na^{-n}u(n)] = \frac{1}{e^{-at}z^{-1}} = \frac{1}{1 - Kz^{-1}}$$

$$(x(nT) = e^{-at}), a > 0, n \geq 0$$

$$X(z) = \sum_{n=0}^{\infty} e^{-ant}z^{-n} = \sum_{n=0}^{\infty} (e^{-at}z^{-1})^n$$

$$(x = e^{-at}z^{-1}), X(z) = \frac{1}{1 - e^{-at}z^{-1}}, |z| > e^{-at} \; K = e^{-at}$$

$$X(z) = \Im[K^n] = \frac{1}{1 - Kz^{-1}}, |z| > K$$

$$\frac{1}{1 - Kz^{-1}} = \frac{z}{z - K}$$

X(z) has a zero at z=0.
X(z) has a pole at z=K.

(b) For the transfer function, $H(z) = \dfrac{1 - (1/2)z^{-1}}{1 - (3/2)z^{-1} - z^{-2}}$, find a closed-form expression for $h(n)$ (don't use long division).

$$\frac{H(z)}{z} = \left(\frac{z^2}{z^2}\right)\frac{1 - .5z^{-1}}{1 - 1.5z^{-1} - z^{-2}}$$

$$\frac{H(z)}{z} = \left(\frac{z^2}{z^2}\right)\frac{1 - .5z^{-1}}{1 - 1.5z^{-1} - z^{-2}}$$

$$\frac{H(z)}{z} = \frac{z^2 - 0.5z}{-1 - 1.5z + z^2}$$

$$\frac{H(z)}{z} = \frac{z^2 - 0.5z}{(z-2)(z+0.5)} = \frac{A}{(z+0.5)} + \frac{B}{(z-2)}$$

$$A = \frac{z^2 - 0.5z}{(z-2)} \text{ with } z = -0.5$$

A=-.2

$$B = \frac{z^2 - 0.5z}{(z+0.5)} \text{ with } z$$

B=1.2

$$\frac{H(z)}{z} = \frac{-0.2}{(z+0.5)} + \frac{1.2}{(z-2)}$$

$$H(z) = \frac{-0.2z}{(z+0.5)} + \frac{1.2z}{(z-2)}$$

$$H(z) = \frac{-0.2}{1-2z^{-1}} + \frac{1.2}{1+0.5z^{-1}}$$

$$x(n) = 1.2(2)^n - 0.2(-0.5)^n, \; n \ge 0$$

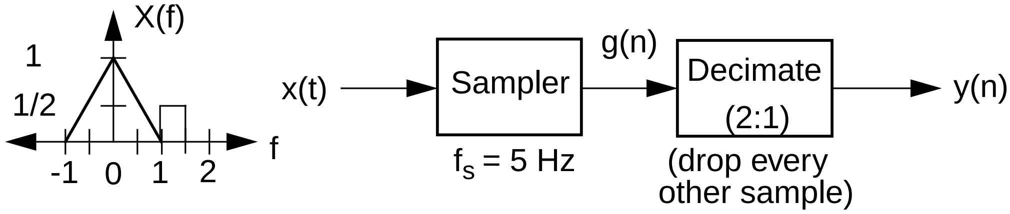
(c) Is the system stable?

No.

(d) Is the system causal? Explain.

Yes. It is causal because the system starts at time t=0.

Problem No. 3: For the system shown:



(a) Plot the spectrum of g(n):

(b) Plot the spectrum of y(n):

(c) How do you explain the fact that the sample frequency of y(n) is less than the Nyquist rate, yet there is no distortion?

If every other sample is dropped, the Nyquist rate will not cause distortion. This is because we would be sampling at fs=2.5 instead of fs=5. Dropping those samples would cause the Nyquist rate would be 2fs.