Name:

Problem	Points	Score
1a	10	
1b	10	
1c	10	
1d	10	
2a	10	
2b	10	
2c	10	
3a	10	
3b	10	
3c	10	
Total	100	

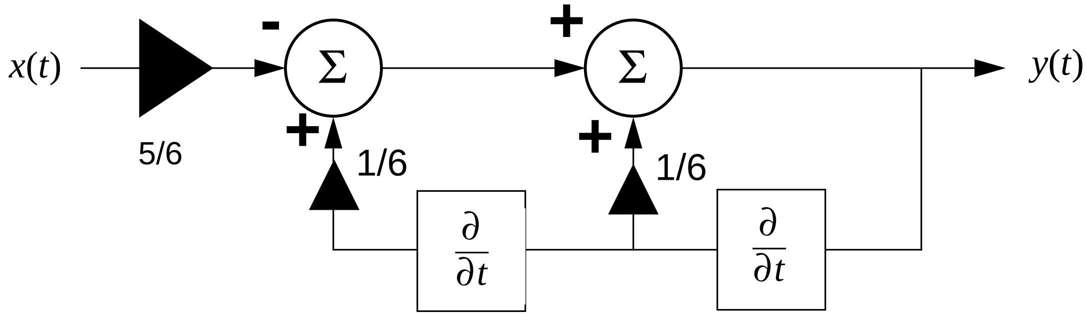
Notes:

1. The exam is closed books/closed notes - except for one page of notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

3. Please indicate clearly your answer to the problem. If I can’t read it (and I am the judge of legibility), it is wrong. If I can’t follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)

Problem No. 1: Block Diagrams



(a) Find the transfer function of this system using Laplace transforms.

$y(t) = -\frac{5}{6}x(t) + \frac{1}{6}\frac{d}{dt}y(t) + \frac{1}{6}\frac{d^2}{dt^2}y(t)$ Letting the initial conditions be zero and taking the Laplace Transform:

$$Y(s) = -\frac{5}{6}X(s) + \frac{1}{6}sY(s) + \frac{1}{6}s^2Y(s)$$

$$Y(s) - \frac{1}{6}sY(s) - \frac{1}{6}s^2Y(s) = -\frac{5}{6}X(s)$$

$Y(s)\left(1 - \frac{1}{6}s - s^2\right) = -\frac{5}{6}X(s)$ with $H(s) = \frac{Y(s)}{X(s)}$ This results in;

$$H(s) = -\frac{\frac{5}{6}}{1 - \frac{1}{6}s - s^2} = \frac{5}{s^2 + s - 6} = \frac{5}{(s+3)(s-2)}$$

This transfer function was found by using the technique of placing an alternate variable $Z_n(s)$ after each block then solving for each. This resulted in the above transfer function.

(b) Find the impulse response.

$H(s) = \frac{5}{((s+3)(s-2))}$ Using Partial Fraction Expansion:

$$\frac{5}{((s+3)(s-2))} = \frac{A}{s+3} + \frac{B}{s-2}$$

$\frac{5}{(s-2)} = A + B\frac{(s+3)}{(s-2)}\bigg|_{s=-3}$ so $A = -1$

$\frac{5}{s+3} = A\frac{(s-2)}{s+3} + B\big|_{s=2}$ so $B = 1$

$H(s) = \frac{1}{s-2} - \frac{1}{s+3}$ this gives; $h(t) = (e^{2t} - e^{-3t})u(t)$

(c) Determine whether the system is stable or unstable. Show ALL work — the correct answer with no supporting work gets no points. Be as detailed as possible.

$$H(s) = \frac{5}{((s+3)(s-2))}$$

By looking at the denominator of the transfer function, the poles are located at s = -3 and s = 2. This process determines that there is a pole in the right half plane making the system unstable.

(d) Sketch the frequency response of the system using Bode plots.

Analyzing the transfer function, H(s), we can calculate the poles and zeros associated with it which are necessary to sketch the frequency response of the system using a Bode plot:

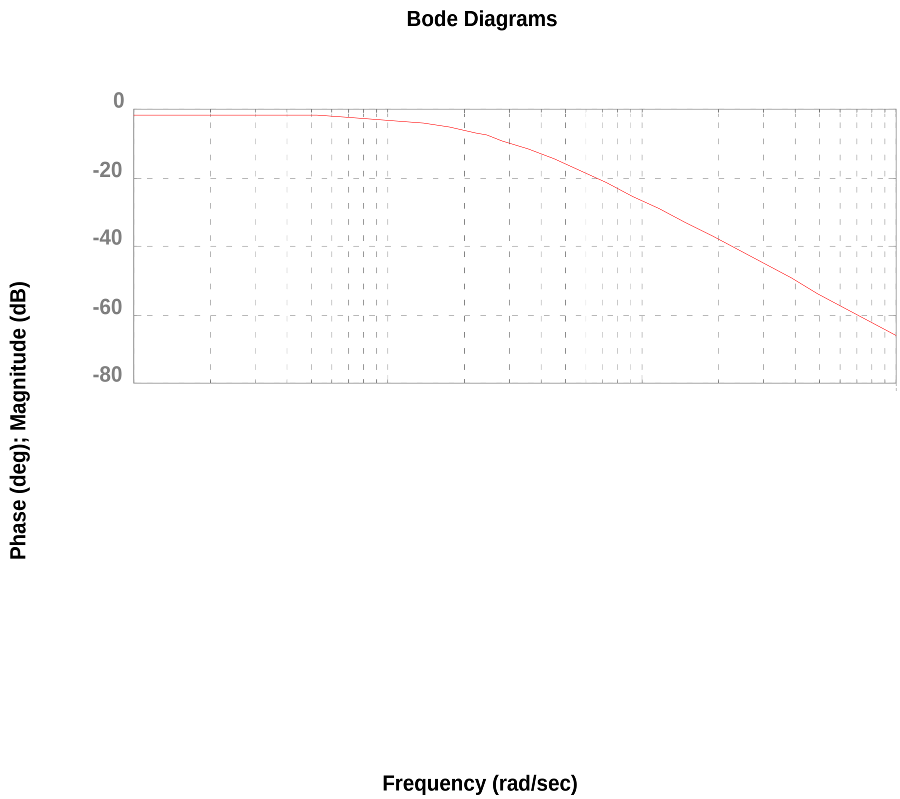
Zeros:

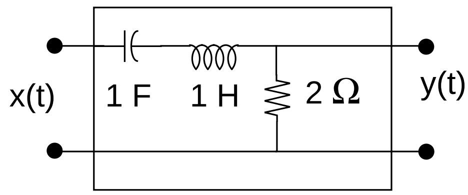
There is a constant, K, value: K = 5/6
Therefore, to find the dc offset in dB: 20 log(5/6) = -1.584 dB

Poles:

There are two simple poles: $\omega\tau = 2, -3$ where $\omega\tau$ is the break frequecy
The negative break frequency is simply mirrored about the vertical axis.
The frequency response for both poles is -20 dB/decade.

The two poles essentially compound to form an aymptote at 2 with a slope of -20 dB/decade, and an asymptote at 3, with a slope of -20 dB/decade. The asymptote at 3 adds to the asymptote from 2 to form an asymptote with a slope of -40 dB/decade beginning at 3. The resulting magnitude of the frequency response is going to be a curve that approximates the asymptotes mentioned above. The largest errors will occur at 2, where there is a -3 dB difference between the asymptote and the actual curve; and at 3, there is a -6 dB difference between the asymptote and the actual curve of the frequency response. A Bode plot was generated using Matlab to clearly demonstrate the frequency response of this system and is included on the next page.


Bode Diagrams
0
-20
-40
-60
-80
Phase (deg); Magnitude (dB)
Frequency (rad/sec)


x(t)
1 F
1 H
2 Ω
y(t)

Problem No. 2: Circuit analysis using Fourier transforms.

For the circuit shown below:

(a) State all the Fourier transform theorems that are invoked when you compute the transfer function of this circuit. You must give specific evidence to support each theorem described (and I must be able to understand your logic!).

Linearity: this is shown by $ax(t) + bh(t) = aX(f) + bH(f)$

Differentation: This is shown by $Z = \frac{V}{I} = \frac{\left(\frac{d}{dt}i(t)\right)}{i(t)} = j\omega L$

Integration: This is shown by $Z = \frac{V}{I} = \frac{\left(\frac{1}{C}\int i(t)dt\right)}{\left(C\frac{dv}{dt}\right)} = \frac{1}{(j\omega C)}$

(b) State and prove the Frequency Translation Theorem.

$$F\{x(t)e^{j\omega_0 t}\} = X(\omega - \omega_0)$$

$$\int_{-\infty}^{\infty} [x(t)e^{j\omega_0 t}]e^{-j\omega t}dt = X(\omega - \omega_0)$$

$$\int_{-\infty}^{\infty} xt)e^{-j(\omega - \omega_0)}dt = X(\omega - \omega_0)$$

Therefore, the righthandsideof the equation is simply the Fourier Transform of x(t) with ω replaced by $(\omega - \omega_0)$.

(c) Find the impulse response of the circuit using Fourier Transforms.

$$H(j\omega) = Y\frac{(j\omega)}{X(j\omega)} = \frac{R}{\left(R + \left(j\omega L + \frac{1}{j\omega C}\right)\right)}$$

$$\frac{R}{\left(\frac{(j\omega RC + (j\omega)^2 LC + 1)}{j\omega C}\right)} = Hj\omega)$$

$$H(j\omega) = \frac{(j\omega RC)}{(j\omega)^2 LC + j\omega RC + 1}$$

Inputing values for the components,

$$H(j\omega) = \frac{(j\omega 2)}{(j\omega)^2 + 2j\omega + 1} = \frac{(j\omega 2)}{(j\omega + 1)^2}$$

Now using Partial Fraction Expansion,

$$H(j\omega) = \left(\frac{A}{j\omega + 1} + \frac{B}{(j\omega + 1)^2}\right)$$

Finding the coefficients:

$$2j\omega = A(j\omega + 1) + B$$

$A = 2$ and $B = -A = -2$

Therefore,

$$H(j\omega) = \left(\frac{2}{j\omega + 1} - \frac{2}{(j\omega + 1)^2}\right)$$

$$h(t)) = F^{-1}\{H(j\omega)\} = F^{-1}\left\{\frac{2}{j\omega + 1}\right\} - F^{-1}\left\{\frac{2}{(j\omega + 1)^2}\right\}$$

$$h(t) = [2e^{-t} - 2te^{-t}]u(t)$$

Problem No. 3: The Dreaded Thought Problem

Signal to Noise (SNR) ratio is defined as the ratio of the power of a signal and the power of the noise in a system, computed on a log scale and measured in dB:

$$SNR|_{dB} = 10\log_{10}\left(\frac{P_{signal}}{P_{noise}}\right)$$

Assume the signal is given by $x(t) = \sin\omega_o t$, and the noise is given by $w(t) = e^{-\alpha|t|}$.

(a) Compute SNR in the time domain.

$$P[x(t)] = \frac{A^2}{2} = \frac{1}{2}$$

$$P[w(t)] = \int_0^1 e^{-2\alpha|t-n|}dt$$

let n=0

$$P[w(t)] = -\frac{1}{(2\alpha)}e^{-2\alpha t}\Big|_0^1 = -\frac{1}{(2\alpha)} + \frac{1}{(2\alpha)}e^{-2\alpha}$$

Therefore;

$$SNR|_{dB} = 10\log\left(\frac{(P[x(t)])}{(P[w(t)])}\right) = \frac{\left(\frac{1}{2}\right)}{\left(\left(-\frac{1}{(2\alpha)}\right) + \left(\frac{1}{2\alpha}\right)e^{-2\alpha}\right)}$$

$$SNR|_{dB} = 10\log\left(\frac{1}{\left(-\frac{1}{\alpha} + \frac{1}{\alpha}e^{-2\alpha}\right)}\right)$$

$$SNR|_{dB} = 10\log\left(\frac{1}{\left(\left(-\frac{1}{\alpha}\right)(1 - e^{-2\alpha})\right)}\right)$$

$$SNR|_{dB} = 10\log\left(\frac{\alpha}{(1-e^{-2\alpha})}\right)$$

(b) Compute SNR in the frequency domain and prove it is equivalent to the time domain calculation.

$$x(t) = \sin\omega_o t$$

$$(s) = \frac{1}{2j}\delta(f-fo) - \frac{1}{2j}\delta(f+fo)$$

$$\int_{-\infty}^{\infty}\left|\frac{1}{2j}\delta(f-fo) - \frac{1}{2j}\delta(f+fo)\right|^2 d$$

Since the area of a delta function is 1

$$P(X(s)) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$$e^{-jm\omega t}\left(\frac{1}{To}\right)\int_0^{To} e^{-\alpha t - jm\omega t}$$

We let $\omega = 2\pi fo$ where $fo = 1$

and $To = 1$ to get the equation in the form of

$$\int_0^1 e^{-\alpha t - jm2\pi t}$$

$$= \int_0^1 e^{t(-\alpha - jm2\pi)}$$

$$= \frac{1}{-\alpha - jm2\pi} e^{t(-\alpha - jm2\pi)}\Big|_0^1$$

$$= \frac{1}{-\alpha - jm2\pi}(1 - e^{-\alpha - jm2\pi})$$

(c) Explain how the SNR varies with ω_0 and α.

As shown below, SNR increases without bound as α increases. Therefore the signal to noise ratio is proportional to alpha but is independent of ω_0.