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Problem	Points	Score
1a	10	
1b	10	
1c	10	
2a	10	
2b	10	
2c	10	
2d	10	
3a	10	
3b	10	
3c	10	
Total	100	

Notes:

1. The exam is closed books/closed notes - except for one page of notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

3. Please indicate clearly your answer to the problem.

Problem No. 1: For the linear time-invariant system: $H(s) = \frac{s+2}{(s^2 - 2s - 3)}$

(a) Find the state variable description of the system.

The first thing that is done is to transform the system into a form that can be used to solve using partial fractions.

$$H(s) = \frac{(s+2)}{s^2 - 2s - 3} = \frac{(s+2)}{(s+1)(s-3)} = \frac{A}{s+1} + \frac{B}{s-3}$$

The heaviside theorem is then used to find A and B:

$$A = \left.\frac{(s+2)}{s-3}\right|_{s = -s-1} = \frac{(-s-1+s+2)}{-s-1+s-3} = -\frac{1}{4}$$

$$B = \left.\frac{(s+2)}{s-3}\right|_{s = -s+3} = \frac{(-s+3+s+2)}{-s+3+8+1} = \frac{5}{4}$$

A and B are put into a form that can be used to find X1(s) and X2(s):

$$H(s) = \frac{Y(s)}{X(s)} = -\frac{25}{s+1} + \frac{1.25}{s-3} = -25X1(s) + 1.25X2(s)$$

These are the values for X1(s) and X2(s):

$$X1(s) = \frac{1}{s+1}, -X1 + u \qquad X2(s) = \frac{1}{s-3}, 3X2 + u$$

Using the values for X1(s) and X2(s) the state equations are found, as follows:

$$\dot{x}1 = -X1 + u, \dot{x}2 = 3X2 + u, y = 0.25X1 + 3.75X2 + 2u$$

The state equations are put into matrices as shown:

$$\begin{bmatrix} \dot{x}1 \\ \dot{x}2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} X1 \\ X2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} (u)$$

The output of the system is shown below:

$$y = \begin{bmatrix} 0.25 & 3.75 \end{bmatrix} \begin{bmatrix} X1 \\ X2 \end{bmatrix} + 2u$$

The matrix designated A in the text is shown below:

$$A = \begin{bmatrix} 0 & -1 \\ 0 & 3 \end{bmatrix}$$

b) Compute the state transition matrix, $\Phi(t)$.

The transition matrix is computed by using a formula in the text, which is shown below. First the inverse of the matrix is found in the frequency domain:

$$\Phi(s) = (sI - A)^{-1} = \begin{bmatrix} s+1 & 0 \\ 0 & s-3 \end{bmatrix}^{-1} = \frac{1}{s^2 - 2s - 3}\begin{bmatrix} s+1 & 0 \\ 0 & s-3 \end{bmatrix} = \begin{bmatrix} \frac{1}{s-3} & 0 \\ 0 & \frac{1}{s+1} \end{bmatrix}$$

The above matrix is Laplace inversed to give the state transition matrix:

$$\Phi(t) = \begin{bmatrix} e^{-3t} & 0 \\ 0 & e^{-t} \end{bmatrix}$$

(c) Using the state variable representation, implement this as an RLC circuit.

The state variable equations calculated in part (a) cannot implement a RLC circuit.

Problem No. 2: This problem deals with various aspects of Z-Transforms.

(a) **Derive** the expression for the Z-Transform of $x(n) = na^{-n}u(n)$.

Given equation:

$x(n) = na^{-n}u(n)$

Using the Z-transform table to match functions, this is the function that matches the one above:

$$Z[na^{-n}u(n)] = \frac{1}{e^{-at}z^{-1}} = \frac{1}{1-Kz^{-1}}$$

The samples by which the unit exponential sequence is defined is first established, which is nothing more than a decaying exponential:

$(x(nT) = e^{-at}), a > 0, n \geq 0$

The sequence above is Z-transformed, which is shown below:

$$X(z) = \sum_{n=0}^{\infty} e^{-ant}z^{-n} = \sum_{n=0}^{\infty} (e^{-at}z^{-1})^n$$

Using the equation 1/(1-x) the following result is achieved:

$$(x = e^{-at}z^{-1}), X(z) = \frac{1}{1-e^{-at}z^{-1}}, |z| > e^{-at}$$

With the sampling period T and the parameter alpha fixed, exp(-at) is a constant. Letting:

$K = e^{-at}$

gives the transform pair, which is found in the Z transform table and is the most common form of this equation:

$$X(z) = \Im[K^n] = \frac{1}{1-Kz^{-1}}, |z| > K$$

The equation above can be rewritten in the form shown below:

$$\frac{1}{1-Kz^{-1}} = \frac{z}{z-K}$$

X(z) has a zero at z=0.
X(z) has a pole at z=K.

(b) For the transfer function, $H(z) = \frac{1-(1/2)z^{-1}}{1-(3/2)z^{-1}-z^{-2}}$, find a closed-form expression for $h(n)$ (don't use long division).

The equation is first multiplied by z-squared on the top and bottom of the equation. This will put the equation in the right form to use partial fractions to solve for A and B.

$$\frac{H}{z}(z) = \left(\frac{z^2}{z^2}\right)\frac{1-5z^{-1}}{1-1.5z^{-1}-z^{-2}}$$

The equation is in the right form for partial fractions:

$$\frac{H}{z}(z) = \frac{z^2-0.5z}{-1-1.5z+z^2}$$

The equation is now in the form to find A and B:

$$\frac{H}{z}(z) = \frac{z^2-0.5z}{(z-2)(z+0.5)} = \frac{A}{(z+0.5)} + \frac{B}{(z-2)}$$

Using Heaviside's theorem A is found:

$$A = \frac{z^2-0.5z}{(z-2)} withz = -0.5$$

A=-0.2

Using Heaviside's theorem B is found:

$$B = \frac{z^2-0.5z}{z+0.5} withz = 2$$

B=1.2

The equations with A and B filled in:

$$\frac{H}{z}(z) = \frac{-0.2}{(z+0.5)} + \frac{1.2}{(z-2)}$$

The equations multiplied by z:

$$H(z) = \frac{-0.2z}{(z+0.5)} + \frac{1.2z}{(z-2)}$$

The equation is now in the proper form to transform it from the frequency domain to the time domain:

$$H(z) = \frac{-0.2}{1-2z^{-1}} + \frac{1.2}{1+0.5z^{-1}}$$

The time domain solution:

$$X(nT) = 1.2(2)^n - 0.2(-5)^n, n \geq 0$$

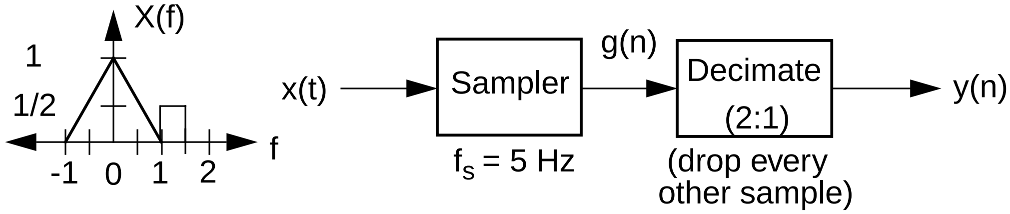
(c) Is the system stable?

No

(d) Is the system causal? Explain.

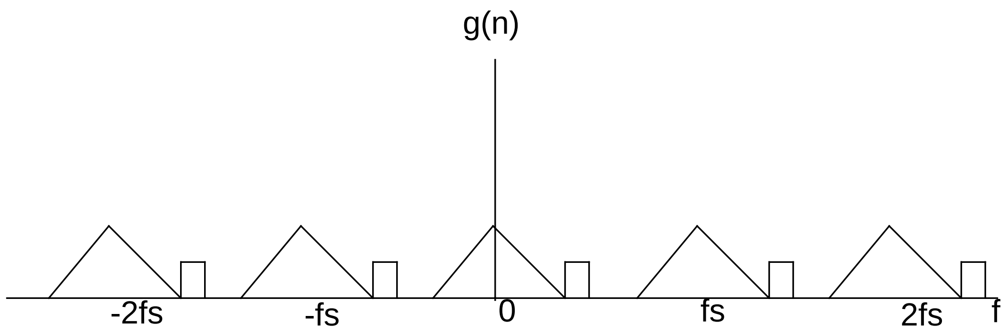
Yes. It is causal because h(n) is 0 for n<0.

Problem No. 3: For the system shown:



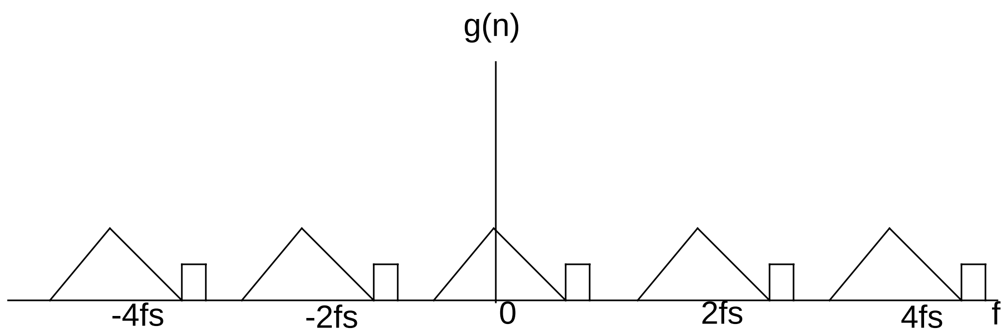
(a) Plot the spectrum of g(n):

The plot g(n) is shown below:



(b) Plot the spectrum of y(n):

y(n) is the sampled signal decimated. All this means is that every second sample is skipped. Therefore the output would be as shown below.



(c) How do you explain the fact that the sample frequency of y(n) is less than the Nyquist rate, yet there is no distortion?

Because every other sample is dropped, the Nyquist rate, which is 2fs, will not cause the system to create distortion. The reason for this is so is that the system is now being sampled at fs=2.5 instead of fs=5.