Name:

Problem	Points	Score
1a	10	
1b	10	
1c	10	
1d	10	
2a	10	
2b	10	
2c	10	
3a	10	
3b	10	
3c	10	
Total	100	

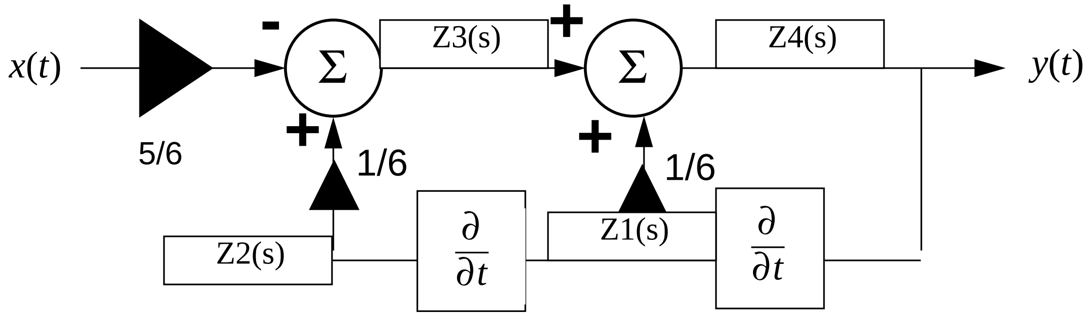
Notes:

1. The exam is closed books/closed notes - except for one page of notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

3. Please indicate clearly your answer to the problem. If I can't read it (and I am the judge of legibility), it is wrong. If I can't follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)

Problem No. 1: Block Diagrams



(a) Find the transfer function of this system using Laplace transforms.

$$Z1(s) = Y(s)s$$
$$Z2(s) = Y(s)s^2$$
$$Z3(s) = -\frac{5}{6}X(s) + \frac{1}{6}Y(s)s^2$$
$$Y(s) = Zy(s)$$
$$Z4(s) = Z3(s) + \frac{1}{6}Z1(s)$$

Now by inspection we find that

$$y(t) = -\frac{5}{6}x(t) + \frac{1}{6}\frac{d}{dt}y(t) + \frac{1}{6}\frac{d^2}{dt^2}y(t)$$

Now, using the Fourier transform theorem of differentiation which states $\frac{d}{dt} = s$ we get:

$$Y(s) = -\frac{5}{6}X(s) + \frac{1}{6}sY(s) + \frac{1}{6}s^2Y(s)$$
$$Y(s) - \frac{1}{6}sY(s) - \frac{1}{6}s^2Y(s) = -\frac{5}{6}X(s)$$

$$Y(s)\left(1 - \frac{1}{6}s - s^2\right) = -\frac{5}{6}X(s)$$ with $H(s) = \frac{Y(s)}{X(s)}$ This results in;

$$H(s) = -\frac{\frac{5}{6}}{1 - \frac{1}{6}s - s^2} = \frac{5}{s^2 + s - 6} = \frac{5}{(s+3)(s-2)}$$

This transfer function was found by using the technique of placing an alternate variable $Z_n(s)$ after each block then solving for each. This resulted in the above transfer function.

$$H(s) = \frac{1}{s-2} - \frac{1}{s+3} \quad \text{this gives;} \quad h(t) = (e^{2t} - e^{-3t})u(t)$$

(c) Determine whether the system is stable or unstable. Show ALL work — the correct answer with no supporting work gets no points. Be as detailed as possible.

$$H(s) = \frac{5}{((s+3)(s-2))}$$

By inspection , one can look at the terms in the denominator and see that the system is unstable because one of the poles is in the right hand plane(s=2).

(d) Sketch the frequency response of the system using Bode plots.

Analyzing the transfer function, H(s), we can calculate the poles and zeros associated with it which are necessary to sketch the frequency response of the system using a Bode plot:

Zeros:

There is a constant, K, value: K = 5/6

Therefore, to find the dc offset in dB: 20 log(5/6) = -1.584 dB

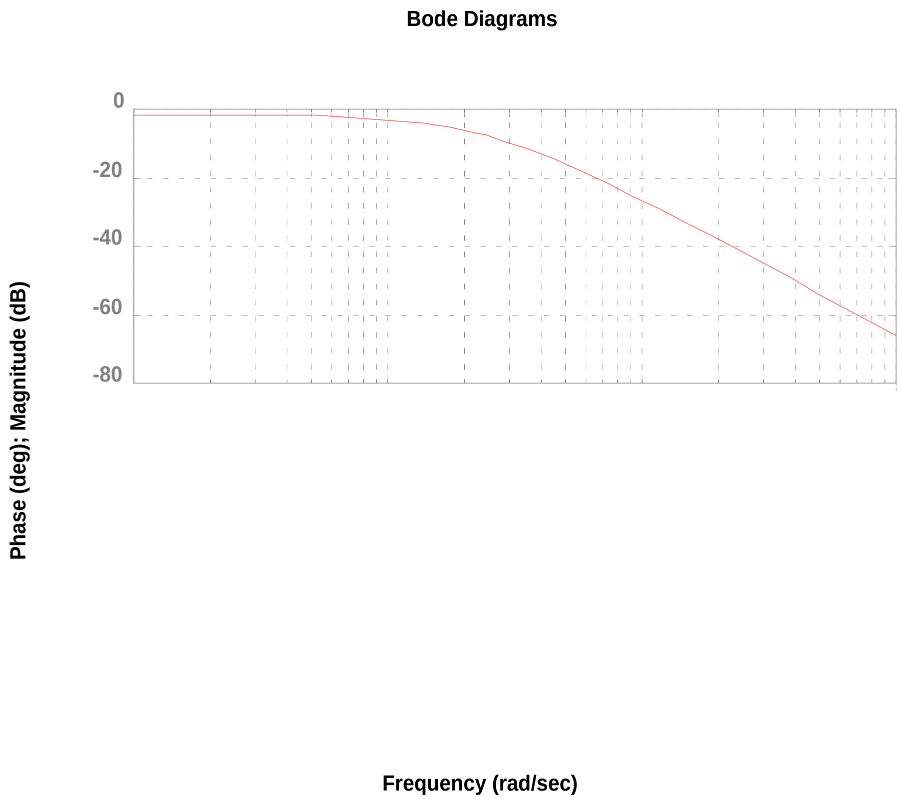
Poles:

There are two simple poles: $\omega\tau = 2, -3$ where $\omega\tau$ is the break frequecy

The negative break frequency is simply mirrored about the vertical axis.

The frequency response for both poles is -20 dB/decade.

As shown on the graph below, there is a pole at 2 and at 3. Each pole gives a -20 dB/decade slope at each particular pole to give a total of -40dB/decase slope.


Bode Diagrams
0
-20
-40
-60
-80
Phase (deg); Magnitude (dB)
Frequency (rad/sec)

(b) Find the impulse response.

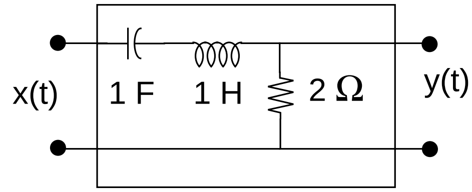
$H(s) = \frac{5}{((s+3)(s-2))}$ Using Partial Fraction Expansion we get

$$\frac{5}{((s+3)(s-2))} = \frac{A}{s+3} + \frac{B}{s-2}$$

$$\left.\frac{5}{(s-2)} = A + B\frac{(s+3)}{(s-2)}\right|_{s=-3}$$ so $A = -1$

$$\left.\frac{5}{s+3} = A\frac{(s-2)}{s+3} + B\right|_{s=2}$$ so $B = 1$

Problem No. 2: Circuit analysis using Fourier transforms.



For the circuit shown above:

(a) State all the Fourier transform theorems that are invoked when you compute the transfer function of this circuit. You must give specific evidence to support each theorem described (and I must be able to understand your logic!).

Linearity: this is shown by $ax(t) + bh(t) = aX(f) + bH(f)$

Differentation: $\frac{(d^n x(t))}{dt^n} = s^n X(s) - s^{n-1}x(0)\text{-...-}x^{(n-1)}(0)$

Which is shown by $Z = \frac{V}{I} = \frac{\left(\frac{d}{dt}i(t)\right)}{i(t)} = j\omega L$

Integration:

$$\int_{-\infty}^{t} x(\lambda)d\lambda = \frac{(X(s))}{s} + \frac{(x^{-1}(0))}{s}$$

Which is shown by $Z = \frac{V}{I} = \frac{\left(\frac{1}{C}\int i(t)dt\right)}{\left(C\frac{dv}{dt}\right)} = \frac{1}{(j\omega C)}$

(b) State and prove the Frequency Translation Theorem.

$$F\{x(t)e^{j\omega_0 t}\} = X(\omega - \omega_0)$$

$$\int_{-\infty}^{\infty} [x(t)e^{j\omega_0 t}]e^{-j\omega t}dt = X(\omega - \omega_0)$$

$$\int_{-\infty}^{\infty} xt)e^{-j(\omega-\omega_0)}dt = X(\omega-\omega_0)$$

Therefore, the righthandsideof the equation is simply the Fourier Transform of x(t) with ω replaced by $(\omega-\omega_0)$.

(c) Find the impulse response of the circuit using Fourier Transforms.

$$H(j\omega) = Y\frac{(j\omega)}{X(j\omega)} = \frac{R}{\left(R+\left(j\omega L+\frac{1}{j\omega C}\right)\right)}$$

$$\frac{R}{\left(\frac{(j\omega RC+(j\omega)^2LC+1)}{j\omega C}\right)} = Hj\omega)$$

$$H(j\omega) = \frac{(j\omega RC)}{(j\omega)^2LC+j\omega RC+1}$$

Inputing L and C values we get

$$H(j\omega) = \frac{(j\omega 2)}{(j\omega)^2+2j\omega+1} = \frac{(j\omega 2)}{(j\omega+1)^2}$$

Now using Partial Fraction Expansion,

$$H(j\omega) = \left(\frac{A}{j\omega+1}+\frac{B}{(j\omega+1)^2}\right)$$

Finding the coefficients:

$$2j\omega = A(j\omega + 1) + B$$

$A = 2$ and $B = -A = -2$

Therefore,

$$H(j\omega) = \left(\frac{2}{j\omega + 1} - \frac{2}{(j\omega + 1)^2}\right)$$

$$h(t)) = F^{-1}\{H(j\omega)\} = F^{-1}\left\{\frac{2}{j\omega + 1}\right\} - F^{-1}\left\{\frac{2}{(j\omega + 1)^2}\right\}$$

$$h(t) = [2e^{-t} - 2te^{-t}]u(t)$$

Problem No. 3: The Dreaded Thought Problem

Signal to Noise (SNR) ratio is defined as the ratio of the power of a signal and the power of the noise in a system, computed on a log scale and measured in dB:

$$SNR|_{dB} = 10\log_{10}\left(\frac{P_{signal}}{P_{noise}}\right)$$

Assume the signal is given by $x(t) = \sin\omega_o t$, and the noise is given by $w(t) = e^{-\alpha|t|}$.

(a) Compute SNR in the time domain.

For A=1,

$$P[x(t)] = \frac{A^2}{2} = \frac{1}{2}$$

$$P[w(t)] = \int_0^1 e^{-2\alpha|t-n|}dt$$

let n=0

$$P[w(t)] = -\frac{1}{(2\alpha)}e^{-2\alpha t}\Big|_0^1 = -\frac{1}{(2\alpha)} + \frac{1}{(2\alpha)}e^{-2\alpha}$$

Therefore;

$$SNR|_{dB} = 10\log\left(\frac{(P[x(t)])}{(P[w(t)])}\right) = \frac{\left(\frac{1}{2}\right)}{\left(\left(-\frac{1}{(2\alpha)}\right) + \left(\frac{1}{2\alpha}\right)e^{-2\alpha}\right)}$$

$$SNR|_{dB} = 10\log\left(\frac{1}{\left(-\frac{1}{\alpha} + \frac{1}{\alpha}e^{-2\alpha}\right)}\right)$$

$$SNR|_{dB} = 10\log\left(\frac{1}{\left(\left(-\frac{1}{\alpha}\right)(1 - e^{-2\alpha})\right)}\right)$$

$$SNR|_{dB} = 10\log\left(\frac{\alpha}{(1-e^{-2\alpha})}\right)$$

(b) Compute SNR in the frequency domain and prove it is equivalent to the time domain calculation.

$$x(t) = \sin\omega_o t$$

$$(s) = \frac{1}{2j}\delta(f-fo) - \frac{1}{2j}\delta(f+fo)$$

$$\int_{-\infty}^{\infty}\left|\frac{1}{2j}\delta(f-fo) - \frac{1}{2j}\delta(f+fo)\right|^2 d$$

When taking the absolute value and squaring, we find that the middle term$\frac{1}{2j}\delta(f-fo)\Big)\Big(\frac{1}{2j}(\delta(f+fo))$ goes to zero.

Since the area of a delta function is 1 the integral reduces down to:

$$P(X(s)) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

To find the power of the $e^{-\alpha(t-N)}$ in the freqency domain, a Complex Fourier series was taken as follows:

$$\left(\frac{1}{To}\right)\int_0^{To} e^{-\alpha t - jm\omega t}$$

We let $\omega = 2\pi fo$ where $fo = 1$
and $To = 1$ to get the equation in the form of

$$\int_0^1 e^{-\alpha t - jm2\pi t}$$

$$=\int_0^1 e^{t(-\alpha - jm2\pi)}$$

$$=\frac{1}{-\alpha - jm2\pi}e^{t(-\alpha - jm2\pi)}\Big|_0^1$$

$$X(n)=\frac{1}{-\alpha - jm2\pi}(1-e^{-\alpha - jm2\pi})$$

According to Parseval's theorem, power in the frequency domain will be equal to power in the time domain. This is done by Integrating the following formula: $\int_{-\infty}^{\infty} |X(n)|^2 df$ The details of the Integral were Looked at by inspection and by using Matlab but the integral could still not be solved by this student.

Another way to attack this problem would be to take one period of the signal and convolve it with a pulse train of delta funtions. This convolution in the time domain would result in multiplication in the frequency domain. This would now give a correct function in the frequency domain to be input into to Parseval's equation to find power. This Integral was also unsolvable by this student by inspection or by Matlab.

Although lacking in the total mathematical proof, this student believes that Parsevals does hold true, therefore: $SNR|_{dB} = 10\log\left(\frac{\alpha}{(1-e^{-2\alpha})}\right)$

(c) Explain how the SNR varies with ω_0 and α.

As alpha increases, the SNR also increases as shown in the graph below. As alpha goes to infinity, so does the SNR.

Because the power of a sine wave is independent of ω_0, the SNR will not vary with change in ω_0. Therefore the signal to noise ratio is proportional to alpha but is independent of ω_0.