Name: Jason R. Lee

Problem	Points	Score
1a	10	
1b	10	
1c	10	
1d	10	
2a	10	
2b	10	
2c	10	
2d	10	
3a	10	
3b	10	
Total	100	

Notes:

1. The exam is closed books/closed notes - except for one page of notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

3. Please indicate clearly your answer to the problem. If I can't read it (and I am the judge of legibility), it is wrong. If I can't follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)

Problem No. 1: Modeling Concepts

(a) Prove whether the signal $x(t) = te^{-\alpha t}u(t)$ is an energy signal or power signal.

By definition, the Energy in $x(t) = te^{-\alpha t}u(t)$ is $\int_0^\infty |te^{-\alpha t}|^2 dt = \int_0^\infty t^2 e^{-2\alpha t} dt$

Using the identity: $$\int_0^\infty t^m e^{-\alpha t} = \frac{m!}{\alpha^{n+1}}$$

we can see that:

$$E = \frac{1}{4\alpha^3}$$

Therefore, since the energy of the signal is finite, and the power of the signal is zero, this is an ENERGY SIGNAL.

(b) Is the signal $x(t) = \sin^2\omega_0 t$ periodic? If so, what is its period? If not, explain.

This signal is periodic because it satsifies the following conditions:

$$x(t) = x(t + T_0)$$

$$x(t + T_0) = (\sin\omega_0(t + T_0))^2$$

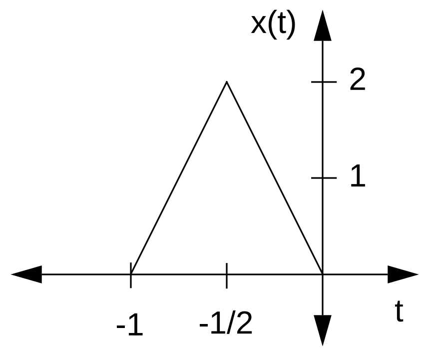
$$= \left(\sin\omega_0\left(t + 2\frac{\pi}{\omega_0}\right)\right)^2$$

$$= (\sin\omega_0 t)^2$$

Therefore, the condition holds, proving that this signal is periodic. The period can be found by saying:

$$(\sin\omega_0 t)^2 = \frac{1}{2} - \frac{1}{2}\cos 2\omega_0 t \text{ so } T_0 = \frac{\pi}{\omega_0} = \frac{1}{2f_0}$$

(c) Write the following signal in terms of a weighted sum of a combination of one or all of the following functions: $\delta(t), u(t), r(t)$.



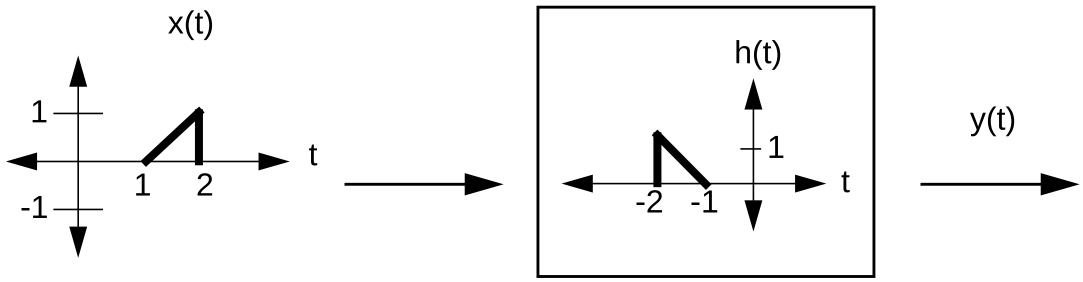
$$x(t) = 4r(t+1) - 8r\left(t + \frac{1}{2}\right) + 4r(t)$$

(d) Compute the energy value of the signal in (c).

$$E = 2\int_0^{0.5} (4t)^2 dt \quad = \quad 2\left(\left(\frac{16}{3}\right)t^3\Big|_0^{0.5}\right) \quad = 1.33 \text{ Joules}$$

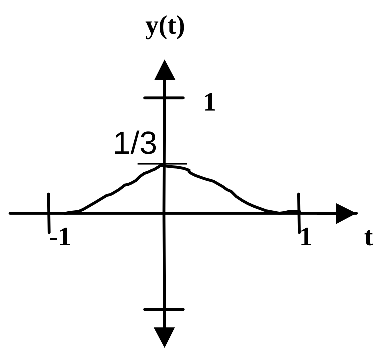
Problem No. 2: Time-Domain Solutions

Consider the signal and system (completely described by its impulse response):



(a) Compute and plot output, $y(t)$, for the system shown above.

The lower time limit of y(t), which is the time that the output signal starts, is



(Graph should be symmetric about the y axis)

equal to the lower time limit of x(t) + the lower limit of h(t). 1+(-2)=-1, so -1 is where the output starts and ends at 1. The maximum amplitude is equal to the maximum area shared between x(t) and a convolved h(t). The output is computed by the following:

$$y(t) = 0, (t < -1), (t > 1)$$
$$x(t) = (t-1), 1 \le t \le 2$$
$$h(t) = (t+2), -2 \le t \le -1$$
$$\int_{-2}^{(t-1)} (t - \lambda - 1) d\lambda = -\frac{t^2}{6} + \frac{t}{2} + \frac{1}{3}, -1 \le t \le 0$$
$$-\int_{(t-2)}^{-1} (t - \lambda - 1)(\lambda + 1) d\lambda = \frac{t^2}{6} - \frac{t}{2} + \frac{1}{3}, 0 \le t \le 1$$

(b) Without using the answer to part (a), explain whether the system is causal.

The system is obviously non-causal because h(t) is not 0 for negative values of time. In other words, the output y(t) 'anticipates' the input because it occurs before the input. The system response h(t) begins at t=-2, while the input x(t) begins at t=1.

(c) Use your answer to part (a) to support your reasoning given in (b).

Looking at the graphs, output y(t) anticipates input x(t). We know that y(t) is anticipatory because y(t) begins at -1, while x(t) does not begin until 1. This is because h(t) occurs before t=0. Looking at y(t), we see that it will undoubtedly occur before t=0, because x(t) convolved with h(t) will produce a function that begins before t=0. Therefore, the system is noncausal.

(d) Describe the system using as many system modeling concepts as possible (for example, linearity). Justify your answers.

Time invariant - input--output relationship does not change with time. In other words, convolving $h(t-\tau)$ with $x(t-\tau)$ is equal to $y(t-\tau)$.

Aperiodic - this is because there is no fundamental period, and $y(t) \neq y(t+T_0)$

Instantaneous - there are no memory elements present, and the output is a function of the input at the present time only.

Linear - since the difference between x(t) and y(t) is the same for two separate times, t and $(t-\tau)$.

Non-causal - explained above

Problem No. 3: Fourier Series

Given the signal $x(t) = 3\sin(1.5\omega_1 t) + 5\cos(1.75\omega_1 t)$,

(a) Using symmetry arguments, explain which Fourier coefficients of the trigonometric Fourier Series will be zero ({an} and {bn}). Be careful and be precise :)

By inspection, the signal has no constant, or DC value. The signal propagates about zero with no offset, so this signal has an average value of zero. Therefore, $a_0 = 0$, and all other coefficients are nonzero. This argument is furthered by the mathematical evidence below:

If x(-t) = x(t), then the function is even, and if x(-t) = -x(t), then the function is odd.

$$x(t) = (3\sin(1.5\omega_1 t)) + (5\cos(1.75\omega_1 t))$$
$$x(-t) = (3\sin(-1.5\omega_1 t)) + (5\cos(-1.75\omega_1 t))$$
$$-x(t) = -(3\sin(1.5\omega_1 t)) - (5\cos(1.75\omega_1 t))$$

since x(-t) does not equal x(t), and x(-t) does not equal -x(t), neither a_n nor b_n will be zero.

(b) Compute the Fourier series coefficients.

The signal x(t) is already in standard form for a Trigonometric Fourier Series:

$$x(t) = a_0 + \sum_{n=1}^{\infty} ((a_n \cos n\omega_0 t) + (b_n \sin n\omega_0 t))$$

$$x(t) = (3\sin 1.5\omega_1 t) + (5\cos 1.75\omega_1 t)$$

It can easily be seen that the above series is periodic with a fundamental frequency, $f_0 = \frac{f_1}{4}$, which can be transformed into $\omega_1 = 4\omega_0$. Using this relationship, we can now determine the Fourier coefficients of the series. Knowing the properties of integrals involving the products of sines and cosines:

$$I_1 = \int_{T_0} (\sin m\omega_0 t)\sin n\omega_0 t dt = 0, m \neq n$$

and

$$I_1 = \frac{T_0}{2}, m = n \neq 0$$

$$I_2 = \int_{T_0} (\cos m\omega_0 t)(\cos n\omega_0 t) dt = 0, m \neq n$$

and

$$I_2 = \frac{T_0}{2}, m = n \neq 0$$

we know that $1.75\omega_1 t = n\omega_0 t$, so the following is true:

$$1.75\omega_1 = n\omega_0$$

$$n = 1.75\frac{\omega_1}{\omega_0}$$

$$n = 1.75\frac{(4\omega_0)}{\omega_0}$$

$$n = 7$$

Therefore, a_7 will be the only a-coefficient.
Using the same argument for b_n, we see that:

$$1.5\omega_1 = n\omega_0$$

$$n = 1.5\frac{\omega_1}{\omega_0}$$

$$n = 1.5\frac{(4\omega_0)}{\omega_0}$$

$$n = 6$$

Therefore, b_6 will be the only b-coefficient. So there are only two coefficients for this series, a_7 and b_6. All other coefficients are zero. To determine what the values for these coefficients are, we can simply compare the original equation with the characteristic equation of a Trigonometric Fourier Series:

Characteristic Equation: $x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t$

Original Equation Given: $x(t) = a_7 \cos 1.75\omega_1 t + b_6 \sin 1.5\omega_1 t$

From the above equations we see that the values for a_7 and b_6 can quickly be determined taking into account the relationship between ω_0 and ω_1 mentioned above and by inspection as follows:

$$a_7 = 5$$

$$b_6 = 3$$