Name: Eldric Jefferson

Problem	Points	Score
1a	10	
1b	10	
1c	10	
2a	10	
2b	10	
2c	10	
2d	10	
3a	10	
3b	10	
3c	10	
Total	100	

Notes:

1. The exam is closed books/closed notes - except for one page of notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

3. Please indicate clearly your answer to the problem.

Problem No. 1: For the linear time-invariant system: $H(s) = \dfrac{s+2}{(s^2 - 2s - 3)}$

(a) Find the state variable description of the system.

$$H(s) = \frac{(s+2)}{s^2 - 2s - 3} = \frac{(s+2)}{(s+1)(s-3)} = \frac{A}{s+1} + \frac{B}{s-3}$$

Using partial fractions,

$$A = \left.\frac{(s+2)}{s-3}\right|_{s = -s-1} = \frac{(-s-1+s+2)}{-s-1+s-3} = -\frac{1}{4}$$

$$B = \left.\frac{(s+2)}{s-3}\right|_{s = -s+3} = \frac{(-s+3+s+2)}{-s+3+8+1} = \frac{5}{4}$$

$$H(s) = \frac{Y(s)}{X(s)} = -\frac{25}{s+1} + \frac{1.25}{s-3} = -25X1(s) + 1.25X2(s)$$

$$X1(s) = \frac{1}{s+1}, -X1 + u\ X2(s) = \frac{1}{s-3}, 3X2 + u$$

$$\dot{x}1 = -X1 + u, \dot{x}2 = 3X2 + u, y = 0.25X1 + 3.75X2 + 2u$$

$$\begin{bmatrix} \dot{x}1 \\ \dot{x}2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} X1 \\ X2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} (u)\ y = \begin{bmatrix} 0.25 & 3.75 \end{bmatrix} \begin{bmatrix} X1 \\ X2 \end{bmatrix} + 2u$$

$$A = \begin{bmatrix} 0 & -1 \\ 0 & 3 \end{bmatrix}$$

b) Compute the state transition matrix, $\Phi(t)$.
First,

$$\Phi(s) = (sI - A)^{-1} = \begin{bmatrix} s+1 & 0 \\ 0 & s-3 \end{bmatrix}^{-1} = \frac{1}{s^2 - 2s - 3}\begin{bmatrix} s+1 & 0 \\ 0 & s-3 \end{bmatrix} = \begin{bmatrix} \frac{1}{s-3} & 0 \\ 0 & \frac{1}{s+1} \end{bmatrix}$$

Afterwards, the state transition matrix can be found: $\Phi(t) = \begin{bmatrix} e^{-3t} & 0 \\ 0 & e^{-t} \end{bmatrix}$

(c) Using the state variable representation, implement this as an RLC circuit.

As one can see from this problem, there are instabilities in the system. Therefore, implementing the state variable representation as an RLC circiut can not be done.

Problem No. 2: This problem deals with various aspects of Z-Transforms.

(a) **Derive** the expression for the Z-Transform of $x(n) = na^{-n}u(n)$.

$x(n) = na^{-n}u(n)$

First, transform into Z: $Z[na^{-n}u(n)] = \frac{1}{e^{-at}z^{-1}} = \frac{1}{1 - Kz^{-1}}$

We know that this equals:

$(x(nT) = e^{-at}), a > 0, n \geq 0$

$$X(z) = \sum_{n=0}^{\infty} e^{-ant}z^{-n} = \sum_{n=0}^{\infty} (e^{-at}z^{-1})^n$$

$$(x = e^{-at}z^{-1}), X(z) = \frac{1}{1 - e^{-at}z^{-1}}, |z| > e^{-at} \quad K = e^{-at}$$

$$X(z) = \Im[K^n] = \frac{1}{1 - Kz^{-1}}, |z| > K$$

$$\frac{1}{1 - Kz^{-1}} = \frac{z}{z - K}$$

Things we also know:
X(z) has a zero at z=0.
X(z) has a pole at z=K.

(b) For the transfer function, $H(z) = \frac{1 - (1/2)z^{-1}}{1 - (3/2)z^{-1} - z^{-2}}$, find a closed-form expression for $h(n)$ (don't use long division).

$$\frac{H(z)}{z} = \left(\frac{z^2}{z^2}\right)\frac{1 - .5z^{-1}}{1 - 1.5z^{-1} - z^{-2}}$$

$$\frac{H(z)}{z} = \frac{z^2 - 0.5z}{-1 - 1.5z + z^2}$$

$$\frac{H(z)}{z} = \frac{z^2 - 0.5z}{(z-2)(z+0.5)} = \frac{A}{(z+0.5)} + \frac{B}{(z-2)}$$

$$A = \frac{z^2 - 0.5z}{(z-2)} \text{ with } z = -0.5$$

A= -0.2

$$B = \frac{z^2 - 0.5z}{(z+0.5)} \text{ with } z = 2$$

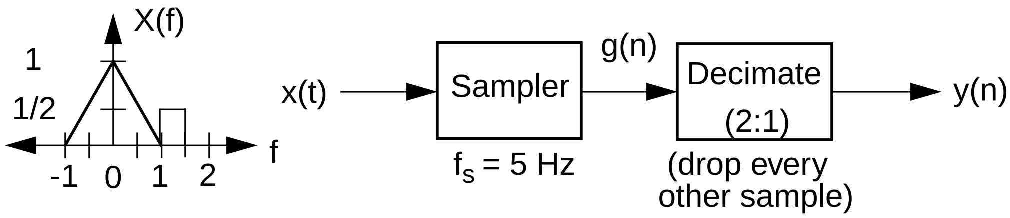
B= 1.2

$$\frac{H(z)}{z} = \frac{-0.2}{(z+0.5)} + \frac{1.2}{(z-2)}$$

$$H(z) = \frac{-0.2z}{(z+0.5)} + \frac{1.2z}{(z-2)}$$

$$H(z) = \frac{-0.2}{1+0.5z^{-1}} + \frac{1.2}{1-2z^{-1}}$$

$$X(n)T = 1.2(2)^n - 0.2(-0.5)^n,\ n \ge 0$$



(c) Is the system stable?

No, this system definitely has some instabilities.

(d) Is the system causal? Explain.

Yes, the system is causal. This system begins at t=0.

Problem No. 3: For the system shown:

(a) Plot the spectrum of g(n):

(b) Plot the spectrum of y(n):

(c) How do you explain the fact that the sample frequency of y(n) is less than the Nyquist rate, yet there is no distortion?

The Nyquist rate will not cause such distortion if one dropped every other sample frequency. In this case, we would be sampling at a frequency fs=2.5, instead of the frequency fs=5. By dropping those other samples, we can see that the Nyquist rate to be 2fs.