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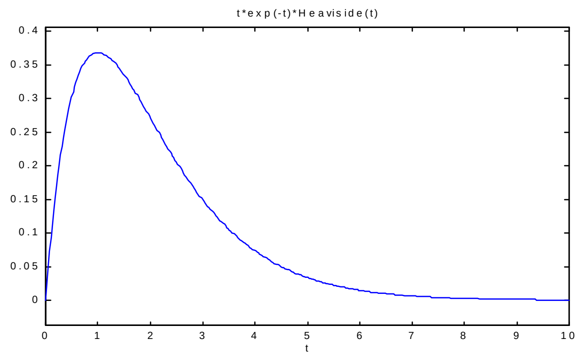
NetIII Test 1 rework

Problem 1.

(a) Prove whether the signal $x(t) = te^{-\alpha t}u(t)$ is an energy or power signal.

x(t) is an energy signal iff $0 < E < \infty$ so that Power = 0.

Lets take a look at the plot of this function.....



As the plot shows, as t increases, x(t) converges to zero.

$$\text{Energy} = \lim_{T \to \infty} \int_{-T}^{T} | Ax(t)|^2 \, dt \text{ * joules}$$

$$= \lim_{T \to \infty} \int_{0}^{T} A^2 \; t^2 * e^{-2t} \, dt \text{ * joules}$$

$$= \lim_{T \to \infty} A^2 \int_{0}^{T} t^2 * e^{-2t} \, dt \text{ * joules}$$

$$= \lim_{T \to \infty} A^2 (-½ T^2 - ½ T - 1/4) * e^{-2T} + 1/4$$

$$= A^2/4$$

This shows that x(t) has finite energy. Also, by looking at the plot, the signal has no average power over time. Therefore the signal is an Energy signal.

(b) Is the signal $x(t) = \sin^2 w_o t$ periodic? If so, what is its period? If not explain.

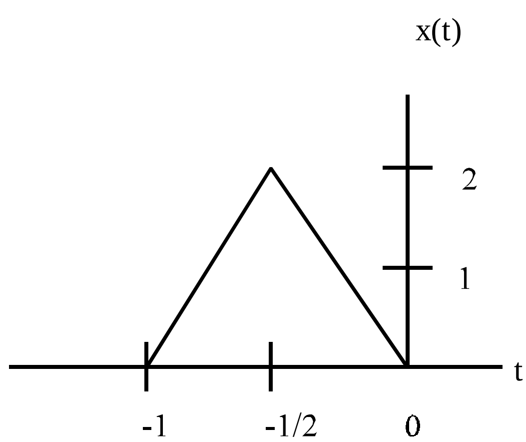
Using a trig identity, $\sin^2 w_o t = ½ - (½)\cos 2w_o t$.

For finding the period, the ½ offset and ½ magnitude can be ignored.

This leaves the function $\cos 2w_o t$. Let $w_o = 2\pi f_o$. $2 w_o = 4\pi f_o$. To find the period,

$w = 4\pi f_o$, $2\pi f = 4\pi f_o$, $f = 4\pi f_o/2\pi$, $f = 2f_o$, Therefore $T = 1/(2f_o)$

(c) Write the following signal in terms of a weighted sum of a combination of one or all of the following functions: *$\delta(t)$, $u(t)$, $r(t)$*.



The first portion of the signal is a ramp function. The unit ramp function rises 1 unit per 1 unit run (i.e. slope = rise/run = 1). Since the first portion rises 2 units per ½ unit run...

slope = rise/run = 2/.5 = 4.

The signal begins at time equal -1 therefore the portion of the signal from -1 to -1/2 is

4r(t+1)

The portion of the signal from -1/2 to 0 has a negative slope of 4. The second portion must also cancel out the first. Including the ½ time shift, the second portion of the signal is

-8r(t+1/2)

Finally, the signal even levels off at time = 0. This is achieved by canceling the negative slope ramp function. Since the portion of the signal from -1/2 to 0 has the form -4r(t), it is canceled by adding the negative of itself. At time = 0, the following function is added...

$$\mathbf{4r(t)}$$

The complete signal is the sum of these singularity functions.

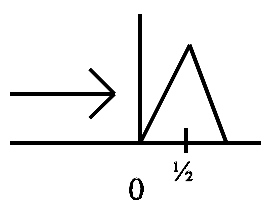
$$\mathbf{x(t) = 4r(t+1) - 8r(t+1/2) + 4r(t)}$$

(d) Compute the energy value of the signal in (c).

Energy is given by the following equation...

$$\text{Energy} = \lim_{T \to \infty} \int_{-T}^{T} | Ax(t)|^2 \, dt \; * \text{ joules}$$

To simplify the analysis, the signal is shifted to start at time t = 0;



To simplify even further, the signal is symmetric about t = ½, therefore for integration purposes, the function can be evaluated from 0 to ½ and multiplied by 2.

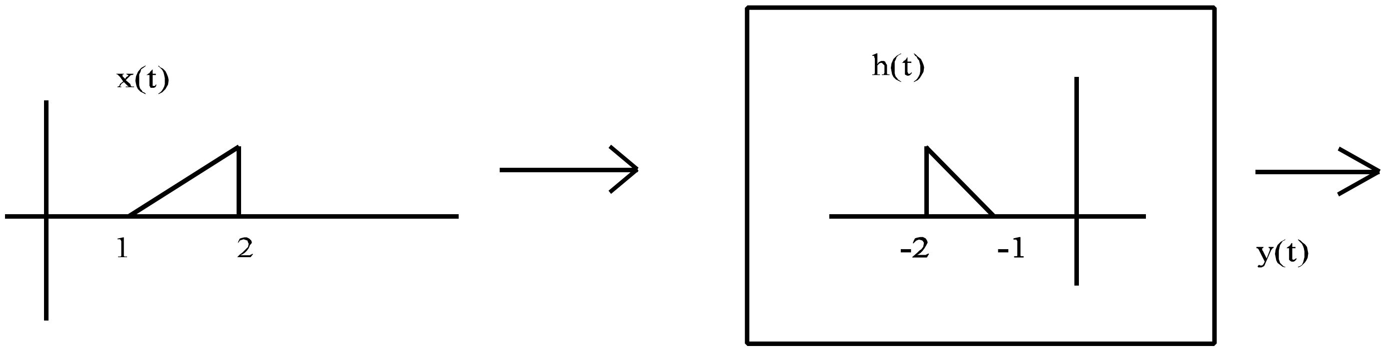
$$\mathbf{Energy = \lim_{T \to \infty} 2 \int_{-T}^{T} | 4t|^2 \, dt \; * \text{ joules}}$$

$$\mathbf{= 2 * 16t^3 / 3}$$

$$\mathbf{= 4/3 \; * joules}$$

Problem No. 2: Time-Domain Solutions

Consider the signal and system (completely described by its impulse response):



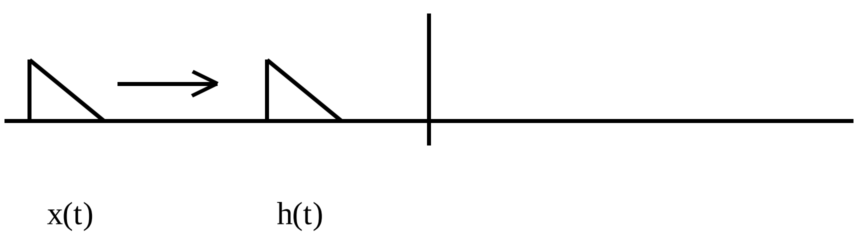
(a) Compute and plot the output, y(t) for the system shown above.

The formal definition of convolution is shown by the convolution integral.

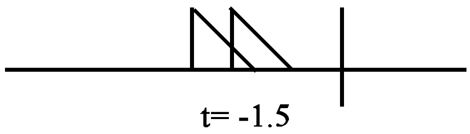
$$y(t) = \int x(\lambda)h(t-\lambda)d\lambda$$

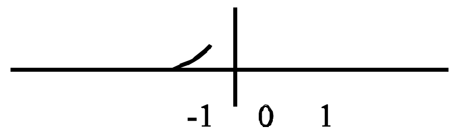
The principle of convolution and the derivation of the output y(t) is best shown by graphical convolution.

1) The input is flipped and set at time negative to the response.



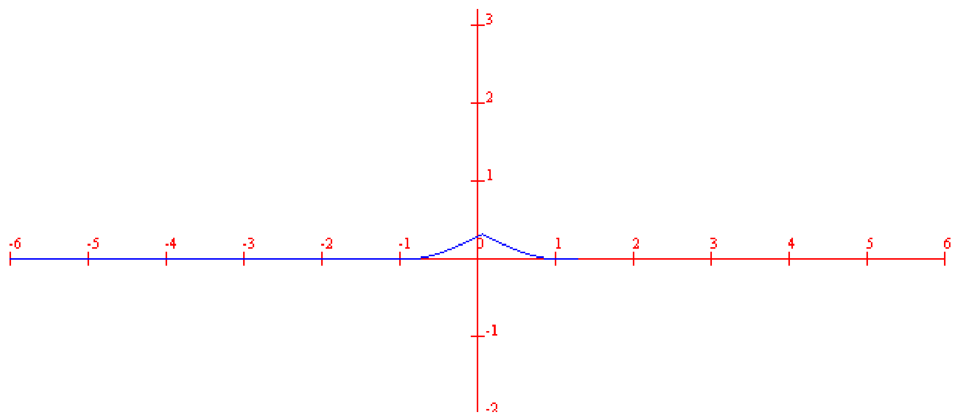
The x(t) signal approaches the h(t) signal meeting at time t = -1. This can be found by simply subtracting the starting time of the h(t) signal from the input signal x(t).





At time t = 0 the two signals will overlap one another with allowing the maximum overlapping area possible. The overlapping areas decrease past time t = 0, decreasing till they no longer over lap at time t = 1.

The output y(t) is the plot of the areas as the two signals are convolved.



(b)   Without using the answer to part (a), explain whether the system is causal.

A system is non-causal when the output anticipates the input. By looking at h(t), the system clearly shows a response before t = 0.   Therefore the system is non-causal.

(c) Use you answer to part (a) to support your reasoning given in (b).

The graph of the output shows y(t) responds to values of the input before time t = 0. There is an output before time t = 0 when the only input is at time t > 0.

(d) Describe the system using as many system modeling concepts as possible (for examle, linearity). Justify your answers.

As above, the system is non-causal.

The system is time invariant. $y(t) = y(t-\tau)$. In other words, if the input is shifted, the output will be of the same form, just shifted in time also.

The system is linear. The system will follow the properties of linearity for example
y(t) = x1(t) and y2(t) = x2(t)

The system is instantaneous or non-dynamic because x(t) does not contain differential equations i.e. x(t) and h(t) are linear over time. An example of a dynamic system would contain inductors or capacitors which have a certain value based on time. These "pre-condition" values must be taken into account during analysis.

Problem No. 3 Fourier Series

Given the signal $x(t) = 3\sin(1.5w_1t) + (5\cos(1.75w_1\, t)$,

(a) Using symmetry arguments, explain which Fourier coefficients of the trigonometric Fourier Series will be zero ({an} and {bn}). Be careful and be precise.

In order to derive information of F.S. coefficients by inspection, the signal must have some symmetric property. Examples would be an even function such as a cosine wave x(t) = x(-t) and an odd function such as a sine wave x(-t) = -x(t). For an even function, bn = 0 and for an odd function an = 0. Signals may also exhibit halfwave properties which further affect which particular an's and bn's are affected. For example, for a wave which is even and halfwave even, bn = 0 for even coefficients.

The signal given above is neither even or odd. The signal is simply a sum of two sinusoids, one of which has a different frequency. This shift in frequency prohibits the signal from being symmetrical about x = 0. To determine the coefficients, closer analysis will need to be applied.

(b) Compute the Fourier series coefficients.

Some analysis of the signal is needed first. The fundamental frequency of the signal can be found by taking the difference of the frequencies in the sum of sinusoids.

w = 1.75 - 1.5 = .25 **(Let w_1 = 1)**

fo = .25/(2*pi) = .03978

To = 1/fo = 25.1327

Using this information (with the help of Matlab) the coefficients are found to be

ao = 0

a7 = 5

b6 = 3

The results can be found without doing the F.S. computations but rather by inspection. The signal will be periodic for some value n1 and n2 for which f1 = n1fo and f2 = n2f0. For the signal, integers 6 and 7 satisfy this.

6 * 1.75 = 7 * 1.5

As far as relevance in the frequency spectrum, the signal will only exhibit magnitude at these multiples.

This can be verified by simply looking at the form of Fourier coefficients... an + jbn. The sin function is odd and corresponds to the imaginary bn coefficient. Likewise, the real cosine function corresponds to the real an coefficient.

Simply stated, a periodic signal can be represented by a sum of sinusoids. To reproduce the signal above, you only need the sum two signals - the separate signals explicitly shown in x(t). The periods of the sinusoids are directly related to the coefficients of the signal Fourier series.