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Problem	Points	Score
1a	10	
1b	10	
2a	10	
2b	10	
2c	10	
3a	10	
3b	10	
3c	10	
3d	10	
3e	10	
Total	100	

Notes:

- 1.The exam is closed books/closed notes - except for one page of notes.
- 2.Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.
- 3.Please indicate clearly your answer to the problem. If I can't read it (and I am the judge of legibility), it is wrong. If I can't follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)

Problem No. 1: Show the following properties of the Fourier transform for **real** signals:

$$(a) |X(f)| = |X(-f)|$$

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt \quad \text{Using Euler's Theorem}$$

$$X(f) = \int_{-\infty}^{\infty} x(t) \cos(2\pi ft) dt - j \int_{-\infty}^{\infty} x(t) \sin(2\pi ft) dt$$

Taking the magnitude

$$|X(f)| = \int_{-\infty}^{\infty} x(t) \cos(2\pi ft) dt$$

This is a real and even function of f. The magnitude's dependency on frequency is through the cosine function. Substituting -t for t yields:

$$|X(f)| = \int_{-\infty}^{\infty} x(-t) \cos(-2\pi ft) dt$$

Since $\cos(-t) = \cos(t)$, from the definition of an even function, the conclusion

$$|X(f)| = |X(-f)| \quad \text{can be drawn.}$$

$$(b) \theta(f) = -\theta(-f)$$

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt \quad \text{Using Euler's Theorem}$$

$$X(f) = \int_{-\infty}^{\infty} x(t) \cos(2\pi ft) dt - j \int_{-\infty}^{\infty} x(t) \sin(2\pi ft) dt$$

Taking the phase

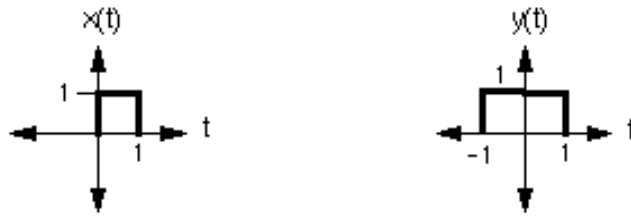
$$\theta(f) = -j \int_{-\infty}^{\infty} x(t) \sin(2\pi ft) dt$$

This is an odd function of f due to the dependency through the sine. Substituting -t for t yields

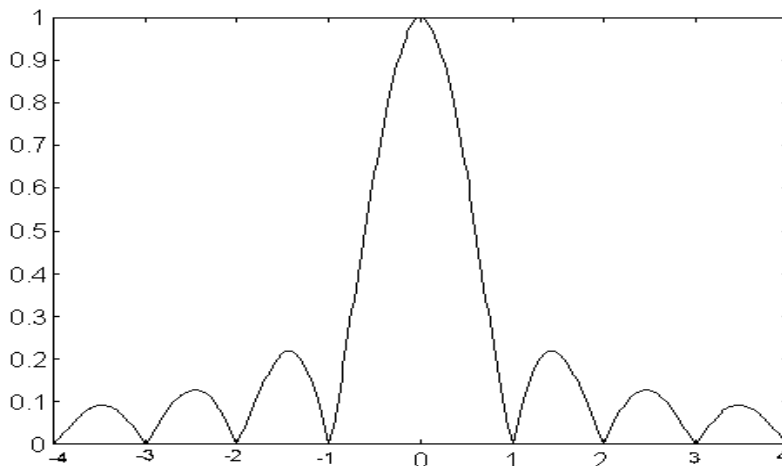
$$\theta(-f) = -j \int_{-\infty}^{\infty} x(-t) \sin(-2\pi ft) dt \quad \text{and can conclude that}$$

$$\theta(f) = -\theta(-f)$$

Problem No. 2: For the following two signals:



(a) Plot the magnitude spectrum of $x(t)$:

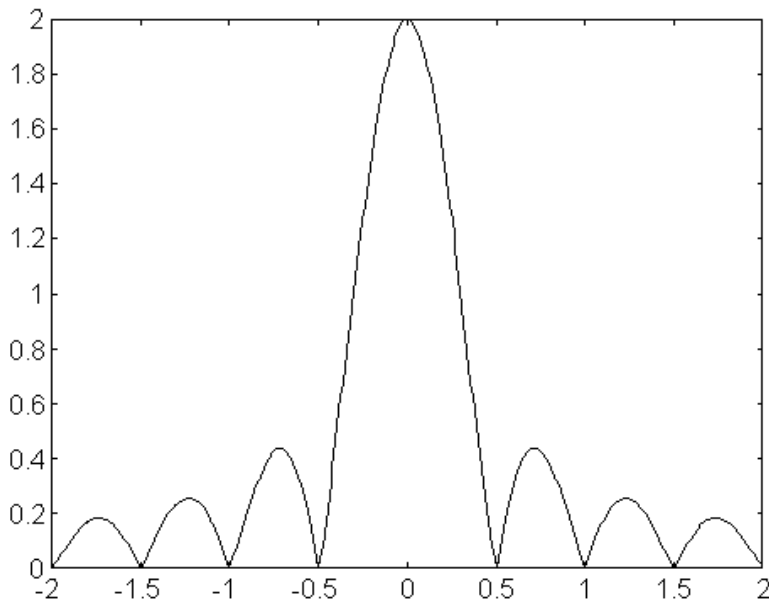


$$x(t) = \Pi\left(t - \frac{1}{2}\right)$$

$$X(f) = \frac{\sin \pi f}{\pi f}$$

$$X(f) = \text{sinc}(f)$$

(b) Plot the magnitude spectrum of $y(t)$:



$$y(t) = \Pi\left(\frac{t}{2}\right)$$

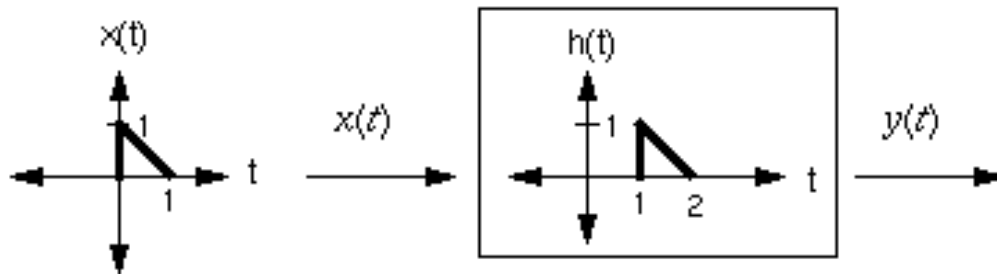
$$Y(f) = \frac{2 \sin 2\pi f}{2\pi f}$$

$$Y(f) = 2\text{sinc}2f$$

(c) Compare and contrast these plots (note that you can answer this question correctly without getting the correct answers for (a) and (b)).

$y(t)$ has twice the area under the curve as $x(t)$ so the magnitude of $Y(f)$ is $\frac{1}{2}$ as wide as the magnitude of $X(f)$. The time shift from $x(t)$ to $y(t)$ has no effect on the magnitude spectrum, instead it causes a shift in the phase response).

Problem No. 3: For the following system,



(a) Find $X(s)$:

$$y(x) = ax + b$$

$$x(t) = (-t + 1)\{0 < x < 1\}$$

$$X(S) = \int_0^{\infty} x(t)e^{-st} dt$$

$$X(S) = \int_0^1 (1-t)e^{-st} dt$$

$$X(S) = \frac{1}{s} - \frac{1}{s^2}$$

Using the time-shift theorem

$$h(t) = (-t + 2) = x(t - 1)$$

$$H(S) = X(S)e^{-s}$$

$$H(s) = \left(\frac{1}{s} - \frac{1}{s^2}\right)e^{-s}$$

(b) Find $Y(s)$:

Using the Laplace transform of the convolution of two signals:

$$y(t) = x(t) * h(t)$$

$$Y(s) = X(s)H(s)$$

substituting the values for X(S) and H(S) gives:

$$Y(S) = \left(\frac{1}{s} - \frac{1}{s^2}\right)\left(\frac{1}{s} - \frac{1}{s^2}\right)e^{-s}$$

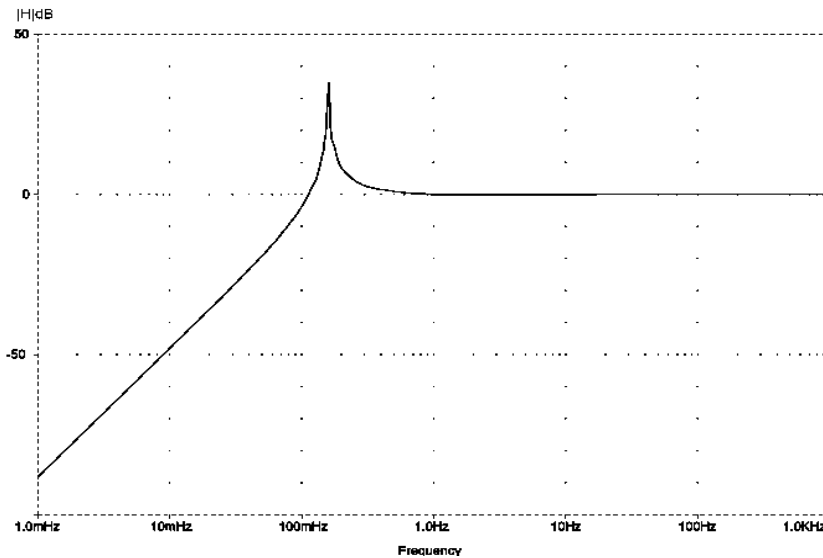
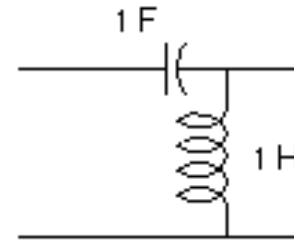
$$Y(S) = \left(\frac{1}{s} - \frac{1}{s^2}\right)^2 e^{-s}$$

(c) For the circuit shown, construct the Bode magnitude plot.

Start with slope of +40dB/dec.

Pole at $\omega=1$ rad/sec ($f=1/2\pi=.159$ Hz)

with $\zeta=0.5$



(d) How many poles and zeros does the system have?

Poles: There are 2 poles. They are a complex conjugate pair.

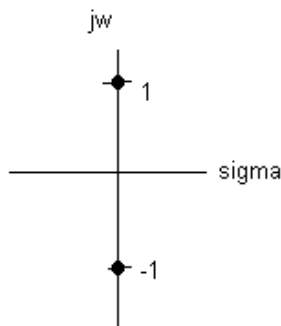
$$s^2+1=0, \text{ or } s^2=-1, \text{ or } s=j \text{ and } s=-j$$

Zeros: There are 2 zeros. Both are at $s^2=0$

(e) Using concepts introduced in Chaps. 4 - 6, determine the stability of the system.

Zeros have no effect on the stability of the system.

To determine the stability, plot the poles, $(s-1)$ and $(s+1)$, shown below.



they are plotted on the imaginary axis, which makes the system marginally stable.