Name: SOLUTIONS by Janna Shaffer

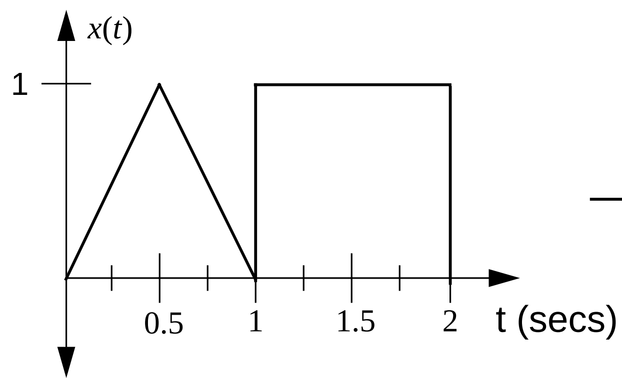
Problem	Points	Score
1a	10	
1b	10	
1c	10	
2a	10	
2b	10	
2c	10	
3a	10	
3b	10	
3c	10	
3d	10	
Total	100	

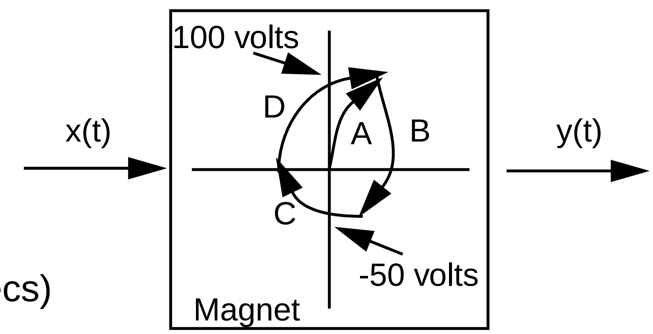
Notes:

1. The exam is closed books/closed notes - except for one page of notes.
2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.
3. Please indicate clearly your answer to the problem. If I can't read it (and I am the judge of legibility), it is wrong. If I can't follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)

Problem No. 1: Classification of Signals and Systems

Consider the signal and system shown below. The system is characterized by an input/output relation. Think of it as follows: when you apply a voltage for the first time, the output starts at zero volts. As you increase the voltage, the output increases as shown on part A of the curve until you reach a maximum. As you subsequently decrease the voltage, the output drops along curve B. As you continue to decrease the voltage, the output varies along curve C, etc. As you continue to cycle the voltage, the output never returns to curve A, but remains in the cycle B,C,D. You might consider this a simple model of a magnet.





(a) Classify the system using as many concepts discussed in Chapters 1-3 as possible Be sure to justify your answer- answers with no justification get no credit.

Answer:

Nonlinear: Superposition does not hold. It can also be seen that the output is not simply a shifted or scaled version of the input.

Dynamic: The output does not depend on past values of input. It is memoryless.

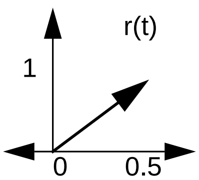
Continuous-time: The input and output are functions of the continuous variable t (which denotes time).

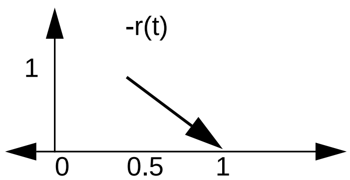
Causal: The output does not depend on future values of the input.

(b) Represent the input signal, $x(t)$, as a combination of singularity functions. Consider this signal the superposition of three pulses:

Answer:

one solution: $x(t) = 2r(t) - 4r(t - 0.5) + 2r(t - 1) + u(t - 1) - u(t - 2)$





First, the signal increases as a function of r(t) from t=0 at a slope of 2 until t=0.5

2r(t)

The increase of r(t) must be “flattened” by adding a r(t) function with a negative slope of 2 at t=0.5.

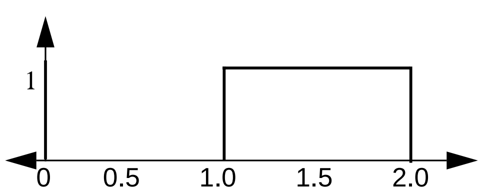
-2r(t-0.5)

To then make the signal decrease again, a r(t) function with a negative slope of 2 must be added at t=0.5.

-2r(t-0.5)

In order to bring the signal back to zero, a r(t) function with slope of 2 must be added at t=1.

2r(t-1)



The remainder of the signal is a unit step function that starts at time t=1.

u(t-1)

In order to bring the signal back to zero at t=2, a unit step must be subtracted.

-u(t-2)

(c) Compute the power and the energy of the input signal.

Answer:

This is an energy signal:

$$E = \lim_{T \to \infty} \int_{-T}^{T} x^2(t)dt = \int_{0}^{0.5} |2t|^2 dt + \int_{0.5}^{1} |-2t|^2 dt + \int_{1}^{2} |1|^2 dt$$

$$= \left.\frac{4t^3}{3}\right|_0^{0.5} + \left.\frac{4t^3}{3}\right|_{0.5}^{1} + t\Big|_1^2 = 2.33J$$

$$P = 0$$

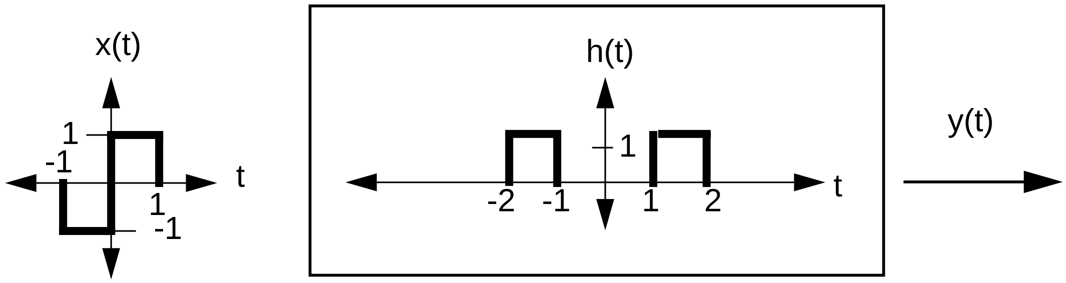
(d) Compute the DC value of the input signal.

Answer:

The DC value of the input signal is zero because this is an aperiodic signal.

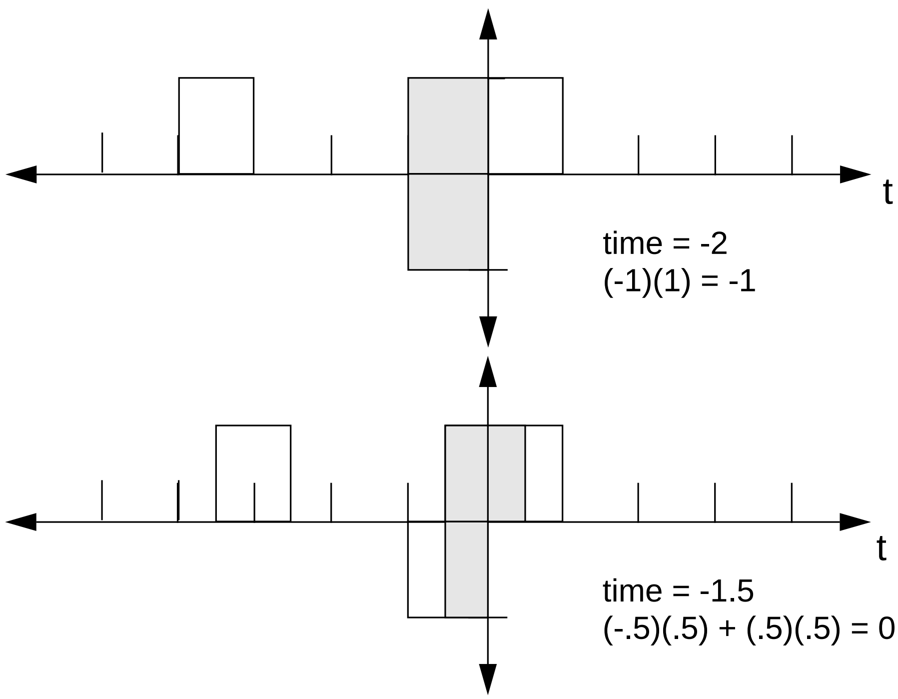
Problem No. 2: Time-Domain Solutions

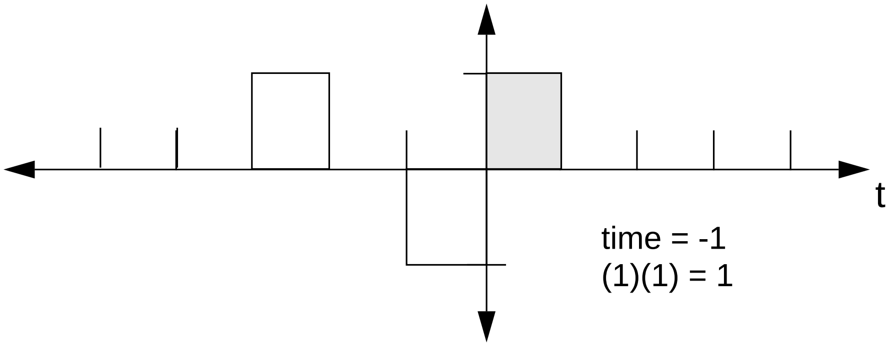
Consider the signal and system (described by its impulse response) shown below:

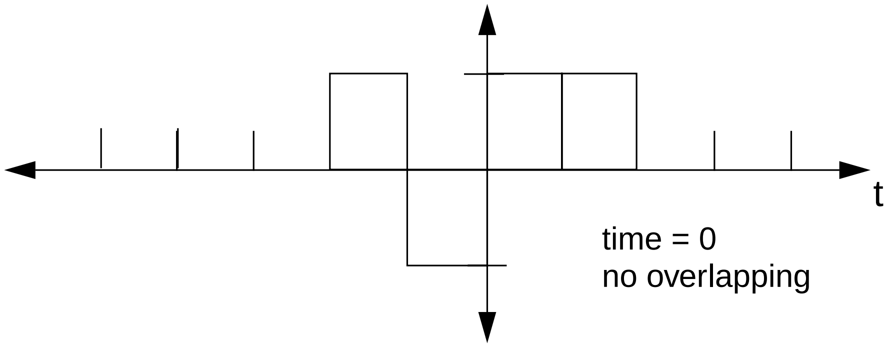


(a) Compute the output, $y(t)$, for the system shown above:

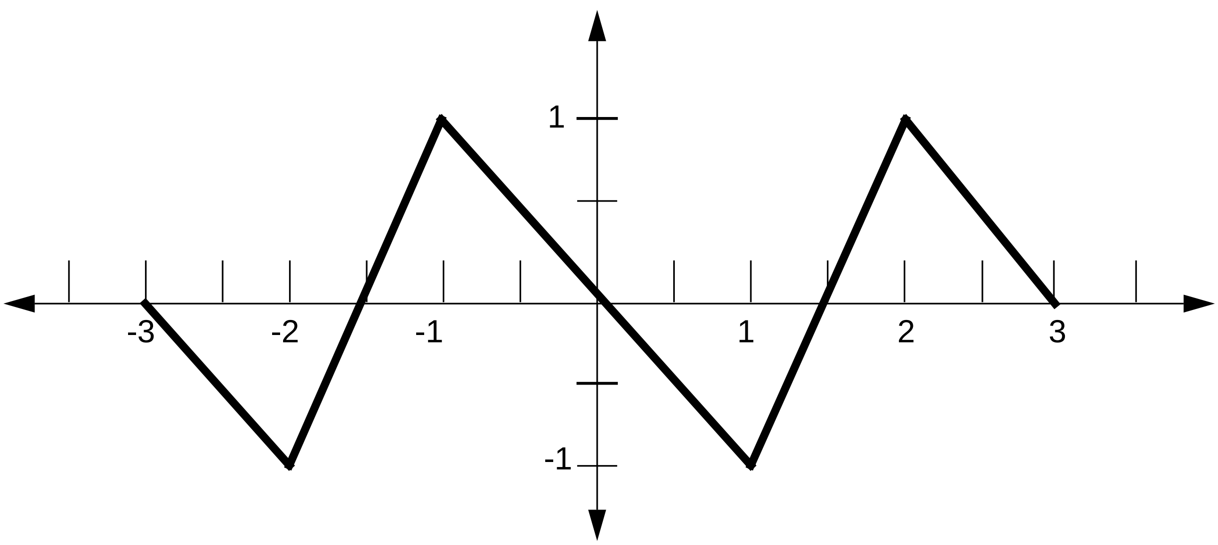
Scratch Space:







Starts all over again when second step function moves across x(t).

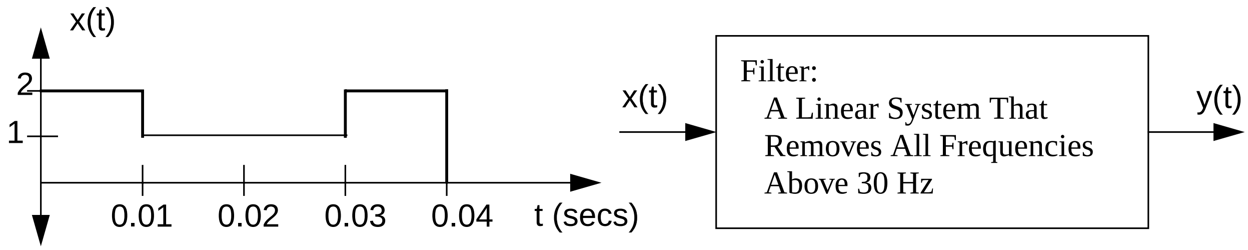


(b) Is the system causal? Explain?

Answer:

The system is definitely not causal. The output starts before input is given to the system.

Problem No. 3: Fourier Series



(a) Using symmetry arguments, explain which Fourier coefficients of the trigonometric Fourier Series will be zero ({an} and {bn}).

Answer:

The signal is half-wave even.

$x(t) = x(-t)$

$b_n = 0$

$a_n = 0$ where n is even

For this filter design, all harmonics are filtered out except for the a_1 term.

(b) Compute the first term in the trigonometric Fourier series, a_0.

Answer:

The a_o term is the dc value of a signal. It is the average value of the signal over one period.

$$a_0 = \frac{1}{T}\int_{t_0}^{t+T_0} x(t)dt = \frac{1}{0.04}\left[\int_0^{0.01} 2u(t)dt + \int_{0.01}^{0.03} u(t)dt + \int_{0.03}^{0.04} 2u(t)dt\right]$$

$$= \frac{1}{0.04}[2t|_0^{0.01} + t|_{0.01}^{0.03} + 2t|_{0.,03}^{0.04}] = \frac{1}{0.04}[0.02 + 0.02 + 0.02]$$

$$= 1.5$$

(c) Compute the remaining coefficients of the trigonometric Fourier series.

Answer:

$$a_n = \frac{2}{T}\int_{t_0}^{t+T_0} x(t)\cos(nw_0t)dt$$

$$a_n = 2\left[\frac{2}{0.04}\left[\int_0^{0.01} 2\cos(50\pi nt)dt + \int_{0.01}^{0.02}\cos(50\pi nt)dt\right]\right]$$

$$= \frac{4}{0.04}\left[\frac{2}{50\pi n}\sin(50\pi nt)\Big|_0^{0.01} + \frac{1}{50\pi n}\sin(50\pi nt)\Big|_{0.01}^{0.02}\right]$$

$$= 100\left(\frac{2}{50\pi n}\sin\frac{\pi n}{2} + \frac{1}{50\pi n}\sin\pi n - \frac{1}{50\pi n}\sin\frac{\pi n}{2}\right)$$

$$= \frac{2}{\pi n}\langle \sin\frac{\pi n}{2} + \sin\pi n\rangle$$

Therefore the solution to a_1 is $\frac{2}{\pi}$.

From the general solution you can see that all values of a_n for even values of n will be zero.

(d) Compute the power of the output signal, $y(t)$.

Answer:

Using Parseval's Theorem:

$$P_{av} = \frac{1}{T_0}\int_{T_0} |x(t))|^2 dt = X_0^2 + 2\sum_{n=1}^{\infty} |X_n|^2$$

$$P_{av} = 1.5^2 + 2\left(\frac{2}{\pi}\right)^2 = 3.06W$$

note: All harmonics are filtered except for when n=1.