Name:

Problem	Points	Score
1a	10	
1b	10	
1c	10	
1d	10	
2a	10	
2b	10	
2c	10	
3a	10	
3b	10	
3c	10	
Total	100	

Notes:

1. The exam is closed books/closed notes - except for one page of notes.
2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.
3. Please indicate clearly your answer to the problem. If I can’t read it (and I am the judge of legibility), it is wrong. If I can’t follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade :)

Problem No. 1: Fourier Transform

(a) Prove the following property of the Fourier transform: $F\left\{\frac{d}{dt}x(t)\right\} = (j\omega)F(\omega)$

$$F\left\{\frac{d}{dt}x(t)\right\} = \int_{-\infty}^{\infty}\left[\frac{\partial}{\partial t}x(t)\right]e^{-j2\pi ft}dt$$

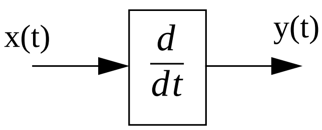
Using integration by parts, with $u = e^{-j2\pi ft}$ and $dv = \left(\frac{dx}{dt}\right)dt$,

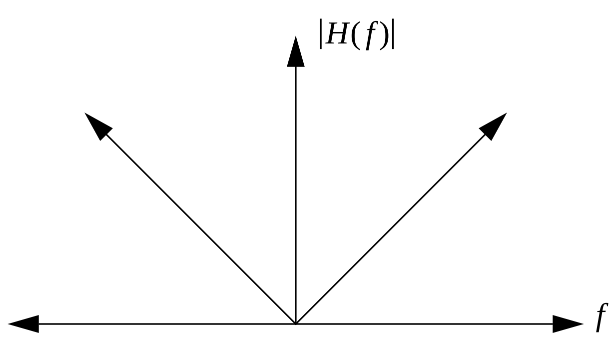
$$F\left\{\frac{d}{dt}x(t)\right\} = x(t)e^{-j2\pi ft}\Big|_{-\infty}^{\infty} + j2\pi f\int_{-\infty}^{\infty}x(t)e^{-j2\pi ft}dt$$

If $x(t)$ is absolutely integrable, $\lim_{t\to\pm\infty}|x(t)| = 0$. Therefore,

$$F\left\{\frac{d}{dt}x(t)\right\} = j2\pi f\int_{-\infty}^{\infty}x(t)e^{-j2\pi ft}dt$$

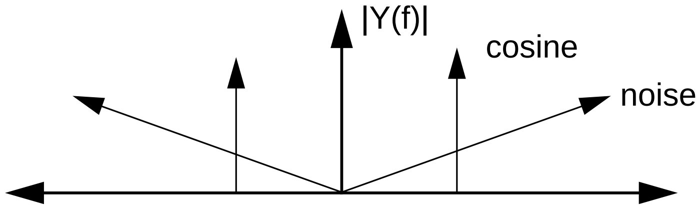
(b) For the system shown below, sketch $|H(f)|$:





(c) Suppose the input signal in part (b), $x(t)$, is a sinewave (amplitude of 1) plus white noise (regard white noise as a signal whose magnitude spectrum has an amplitude of 1). Describe the spectrum of the output signal, $y(t)$ in this case.

The derivative of a sinewave is a cosinewave. The spectrum of the derivative of the noise signal is a noise signal whose amplitude spectrum increases linearly with frequency:



Hence, the total noise power is very large, and swamps the signal. This is why differentiation is considered an inherently noisey process — it amplifies high frequency information (which is often noisy).

(d) Consider a signal, $f_e(t)$, that is an even function: $f(t) = f(-t)$. Derive a simplified expression for the Fourier transform of $f_e(t)$.

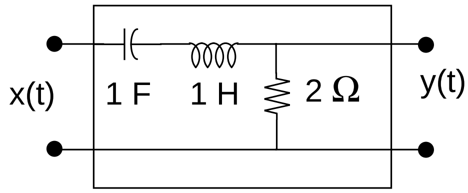
$$F\{f_e(t)\} = \int_{-\infty}^{\infty} f_e(t)e^{-j2\pi ft}dt$$

$$= \int_{-\infty}^{\infty} f_e(t)\cos(2\pi ft)dt - j\int_{-\infty}^{\infty} f_e(t)\sin(2\pi ft)dt$$

$$= 2\int_{0}^{\infty} f_e(t)\cos(2\pi ft)dt$$

Problem No. 2: Laplace Transform

For the circuit shown below, assume the initial conditions are zero.



(a) FInd $y(t)$ if $x(t) = \delta(t)$:
Use Laplace transforms and a voltage divider:

$$H(s) = \frac{R}{\frac{1}{sC} + sL + R} = \frac{sRC}{s^2LC + sRC + 1} = \frac{2s}{s^2 + 2s + 1}$$

Use partial fractions:

$$\frac{2s}{s^2 + 2s + 1} = \frac{A}{(s+1)} + \frac{B}{(s+1)^2}$$

Multiply by the denominator and equate coefficients:

$$2s = A(s+1) + B$$

which implies $A = 2$ and $B = -A = 2$.

Hence,

$$H(s) = \frac{2}{s+1} - \frac{2}{(s+1)^2}$$

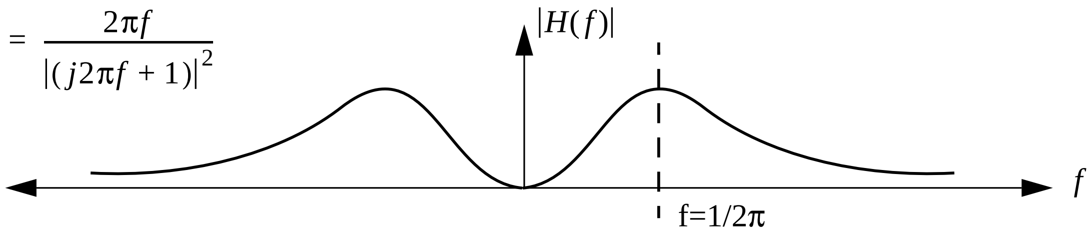
Since the input is an impulse function, the output is the impulse response, which can be found by applying the inverse Laplace transform:

$$y(t) = L^{-1}\left\{\frac{2}{s+1}\right\} - L^{-1}\left\{\frac{2}{(s+1)^2}\right\}$$

$$= [2e^{-t} - 2te^{-t}]u(t)$$

(b) Sketch $|H(f)|$:

$$|H(f)| = \frac{2\pi f}{|(j2\pi f + 1)|^2}$$



(c) If $x(t) = u(t)$, find $y(t)$.

Note that $L\{u(t)\} = \frac{1}{s}$.

Therefore,

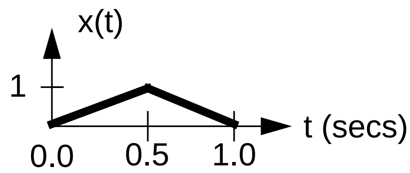
$$Y(s) = sH(s) = \frac{2}{s^2 + 2s + 1} = \frac{2}{(s+1)^2}$$

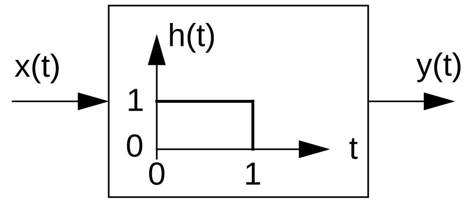
Applying the inverse Laplace transform:

$$y(t) = L^{-1}\left\{\frac{2}{(s+1)^2}\right\}$$

$$= 2te^{-t}u(t)$$

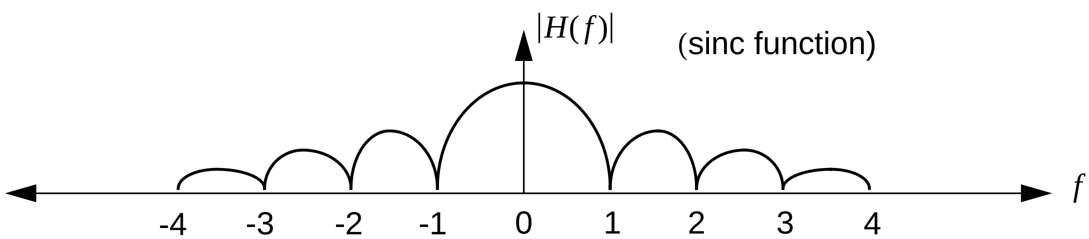
Problem No. 3: Integration of Knowledge





x(t) is a **periodic signal** whose shape for one period is shown above.

(a) Sketch $|H(f)|$.



(b) Sketch $|Y(f)|$. (Be careful — if your answer to part (a) is wrong you won't get partial credit for an incorrect answer. If you are unsure of your answer to part (a), think of an independent method for computing the answer to (b).)

$y(t)$ is nothing more than a DC value — the harmonics of $x(t)$ are cancelled by the zeros of $H(f)$.

(c) Suppose $h(t)$ is equal to $x(t)$. Explain how your answer to part (b) would change.

The output will always be a DC value (or zero) as long as the harmonics of $x(t)$ match the zeros of $H(f)$.