Name:

| Problem | Points | Score |
|---|---|---|
| 1a | 10 | |
| 1b | 10 | |
| 1c | 10 | |
| 2a | 10 | |
| 2b | 10 | |
| 2c | 10 | |
| 2d | 10 | |
| 3a | 10 | |
| 3b | 10 | |
| 3c | 10 | |
| Total | 100 | |

Notes:

1. The exam is closed books/closed notes - except for one page of notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

3. Please indicate clearly your answer to the problem.

I hereby promise not to discuss this exam with anyone in the MWF section of EE 3133.

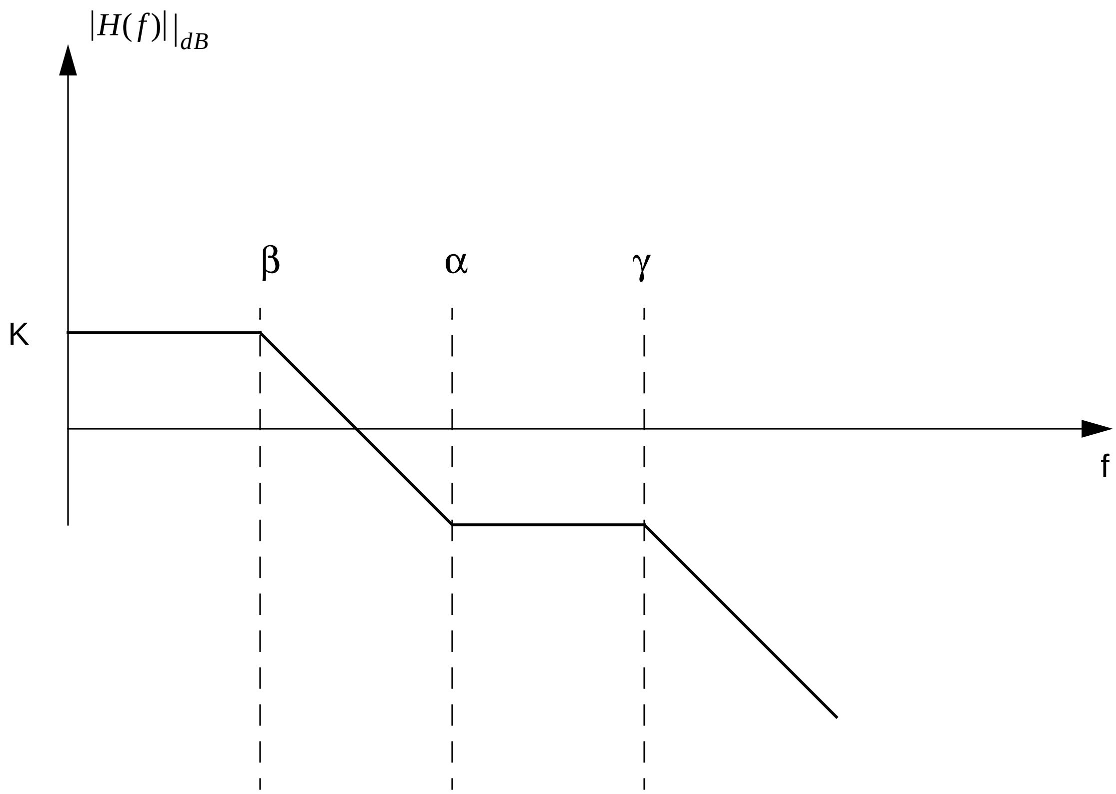
Signature: ____________________________________________________

**Problem No. 1**:

(a) For the transfer function shown below, sketch the Bode plots:

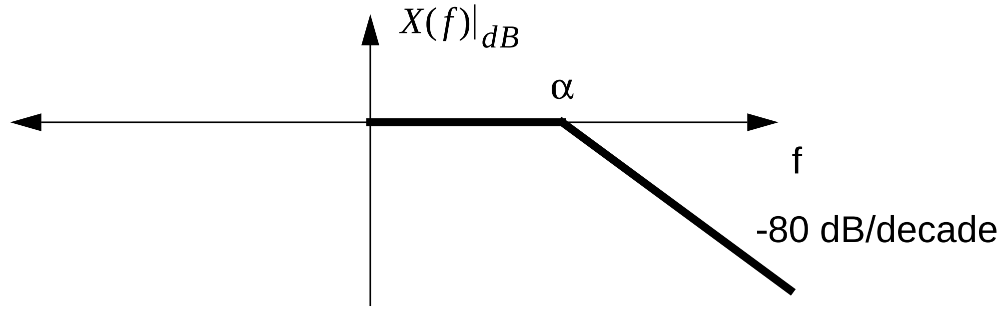
$$H(s) = \frac{C(s+\alpha)}{(s+\beta)(s+\gamma)}$$

Assume $\beta \ll \alpha \ll \gamma$, and that $\alpha,\beta,\gamma$ are real.




Note that $K = 20\log\left(\frac{C\alpha}{\beta\gamma}\right)$, and that the slopes of the lines are 20 dB/decade.

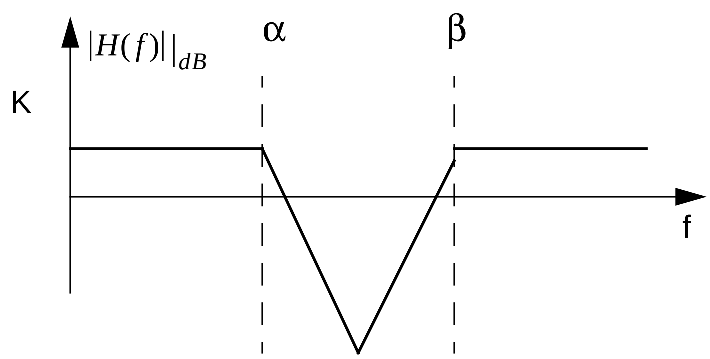
(b) Describe the transfer function for the system that has the following Bode plot (ignore $\angle X(f)$):




$$H(s) \approx \frac{K}{(s+\alpha)^4}$$

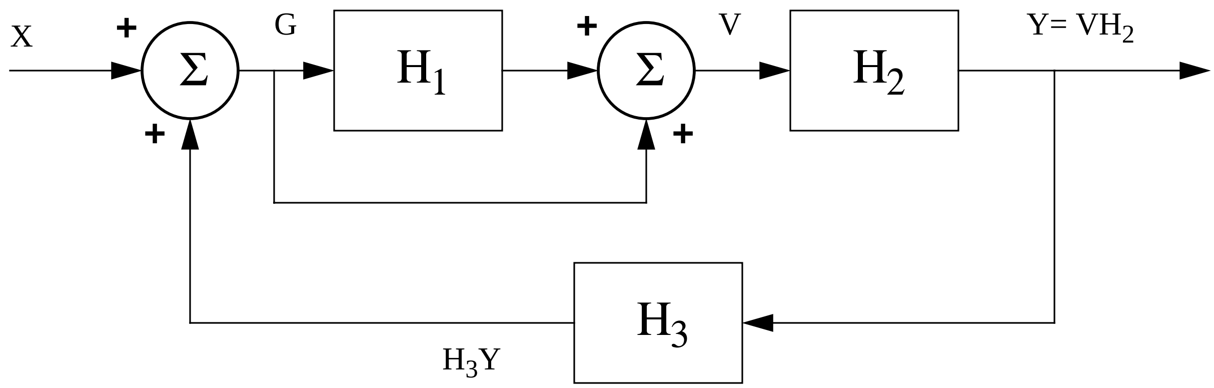
(c) What would you do to convert the system to a filter that rejects frequencies in the range $\alpha < f < \beta$, and passes all other frequencies?

Add 4 zeros at $f = \beta$ to create a notch filter:



**Problem No. 2**:

(a) Compute the equivalent transfer function for the block diagram below:




From the first summation unit $X + H_3Y = G$.

Note that $V = (1 + H_1)G = (1 + H_1)(X + H_3Y)$ ,

and $Y = VH_2 = H_2((1 + H_1)(X + H_3Y))$ .

After simplification:

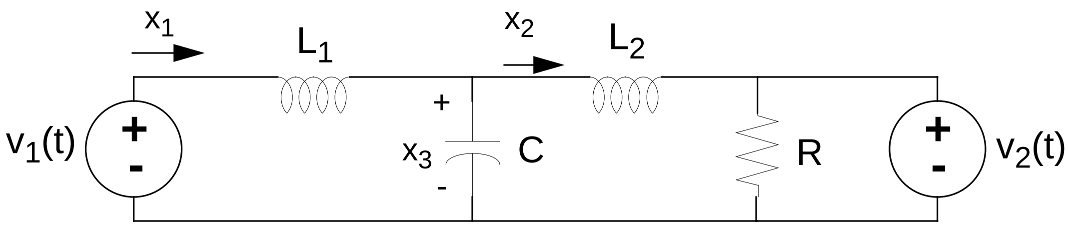
$$Y(1 - H_2H_3 - H_1H_2H_3) = (H_2 + H_1H_2)X .$$

Therefore,

$$H = \frac{Y}{X} = \frac{H_1H_2 + H_2}{1 - H_2H_3 - H_1H_2H_3}$$

(b) Explain how you would use the Nyquist criterion to determine the stability of this system.

See the lecture notes. The denominator of the Laplace transform of the transfer function must be evaluated along a contour encircling the right-half plane. The complex values of this contour must be plotted in the $H(\hat{s})$ plane (real/imaginary axes). The number of counter-clockwise encirclements of the origin must be counted. If the number of net counterclockwise encirclements (R) is equal to the number of poles with positive real parts (P, which is usually zero), the system is stable.




(c) How many state variables would you need to represent the circuit shown above? Explain.

This circuit can be represented by a third-order differential equation because it contains one capacitor and two inductors.

Therefore, the number of state variables is 3.

(d) Derive a state variable representation for this circuit. Note that there are two inputs, $v_1(t)$ and $v_2(t)$. Assume there are two desired outputs, the voltage across the resistor, R, and the current through the resistor.

(1) $-v_1 + L_1\dot{x}_1 + x_3 = 0$

(2) $-x_3 + L_2\dot{x}_2 + v_2 = 0$

(3) $x_2 - x_1 = C\dot{x}_3$

After rearrangement:

$$\dot{x}_1 = 0 + 0 + \left(-\frac{1}{L_1}\right)x_3 + \left(\frac{1}{L_1}\right)v_1$$

$$\dot{x}_2 = 0 + 0 + \left(\frac{1}{L_2}\right)x_3 + \left(-\frac{1}{L_2}\right)v_2$$

$$\dot{x}_3 = \left(-\frac{1}{C}\right)x_1 + \left(\frac{1}{C}\right)x_2 + 0$$

Let $\bar{u} = \begin{bmatrix} v_1 & v_2 \end{bmatrix}^T$.

In matrix form:

$$\dot{\bar{x}} = \begin{bmatrix} 0 & 0 & -\frac{1}{L_1} \\ 0 & 0 & \frac{1}{L_2} \\ -\frac{1}{C} & \frac{1}{C} & 0 \end{bmatrix} \bar{x} + \begin{bmatrix} \frac{1}{L_1} & 0 \\ 0 & \frac{-1}{L_2} \\ 0 & 0 \end{bmatrix} \bar{u}$$

The output equations are easy:

(1) $y_0 = v_r = v_2$

(2) $y_1 = \left(\frac{1}{R}\right) v_2$

In matrix form:

$$\bar{y} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \bar{x} + \begin{bmatrix} 0 & 1 \\ 0 & \frac{1}{R} \end{bmatrix} \bar{u}$$

**Problem No. 3**:

(a) For the signal $x(t) = 3e^{-\alpha t}\sin 2\pi 1000t$ where $\alpha < 1$, compute the minimum sample frequency required for perfect reconstruction of the signal when it is converted from an analog signal to a discrete-time signal, and reconverted back to an analog signal.

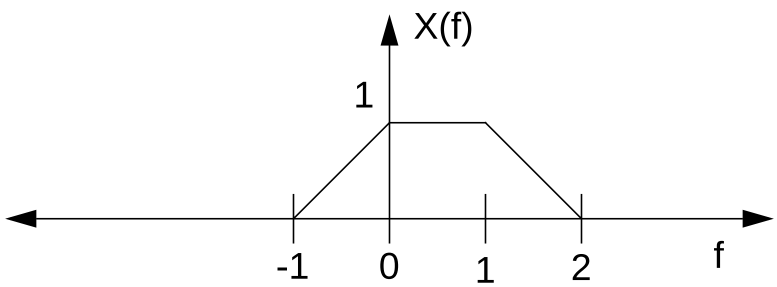
The Fourier transform of $x(t)$ is:

$$X(f) = \frac{1}{j2\pi f + \alpha} \otimes \left[\frac{1}{2}(\delta(f - 1000) + \delta(f + 1000))\right]$$

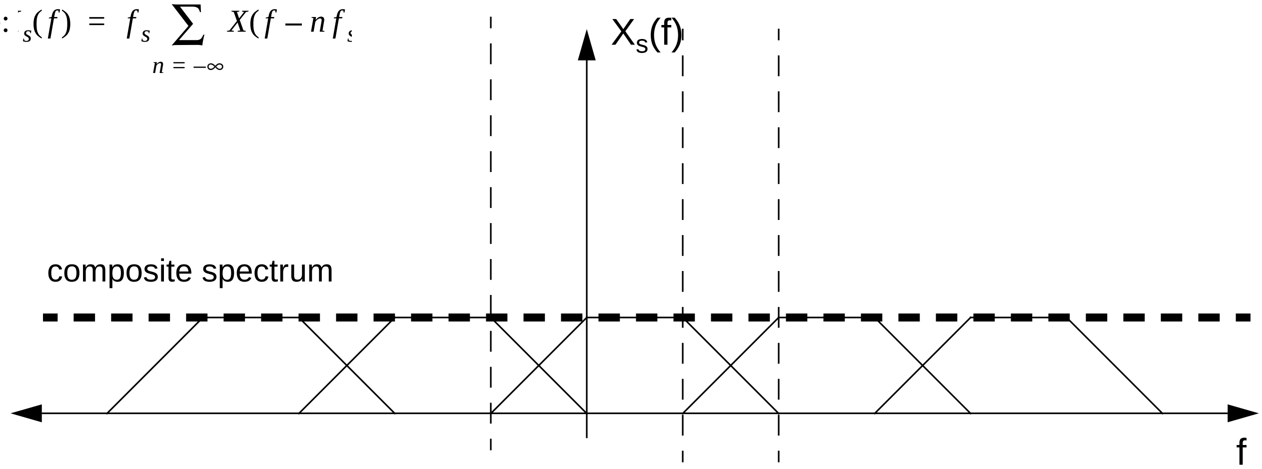
This is not a bandlimited signal. It cannot be sampled without aliasing.

The minimum sampling frequency is ___$\infty$______________________________ Hz.

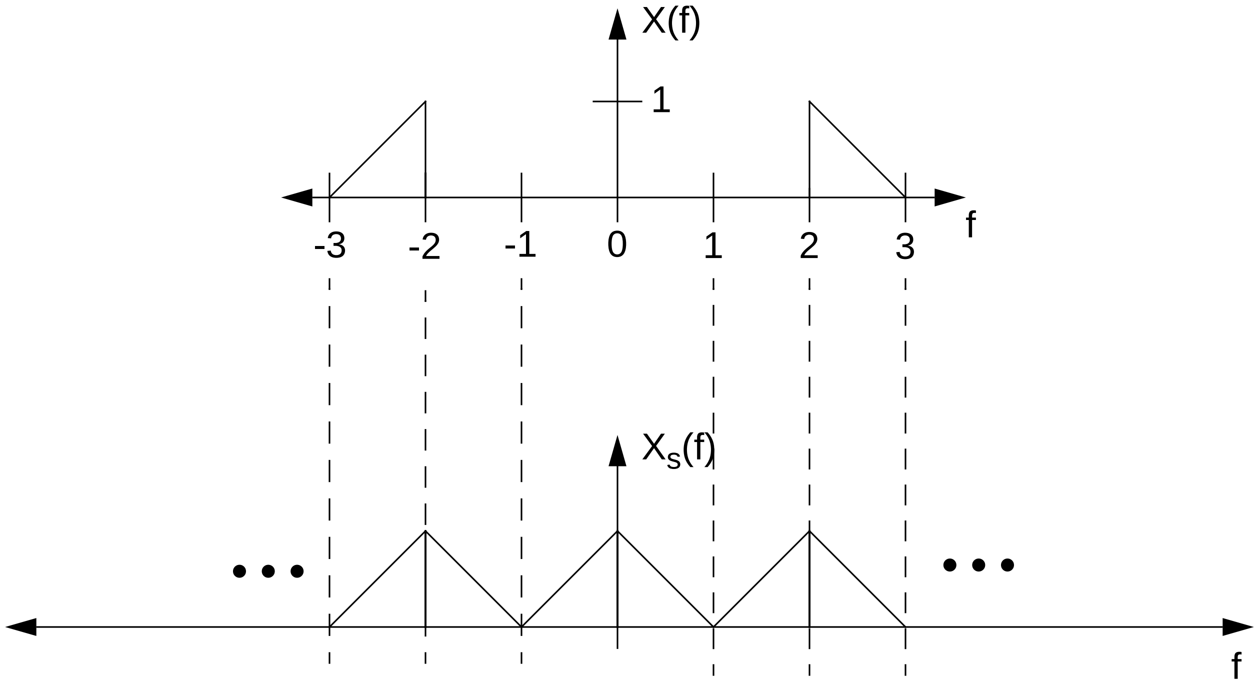
(b) Plot the magnitude spectrum for the signal shown below if it is sampled at 2 Hz.



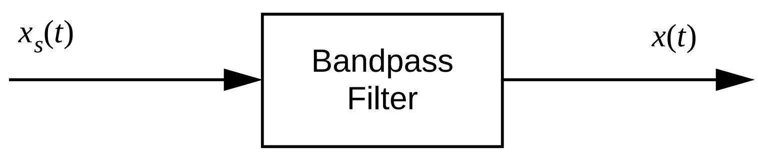

Note: $X_s(f) = f_s \sum_{n=-\infty}^{\infty} X(f - n f_s)$

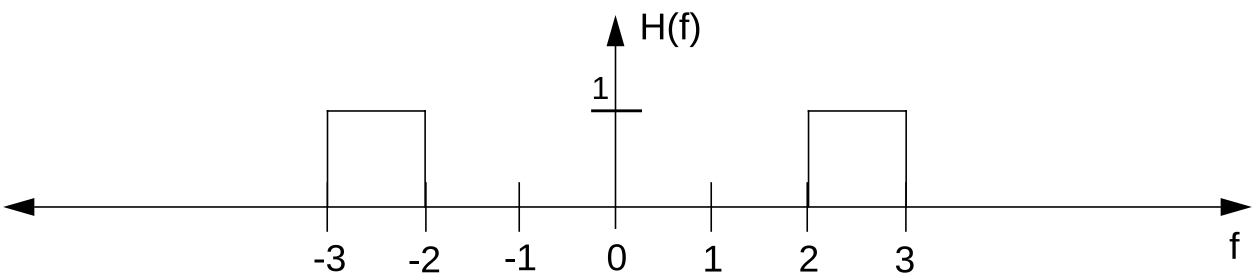


(c) The signal shown below is sampled using a sample frequency of 2 Hz. Plot the spectrum of the sampled signal.




Can the original signal reconstructed from the sampled signal with no error? If so, explain how. If not, explain why.




The ORIGINAL signal can be reconstructed by passing the sampled signal through a bandpass filter that removes the replicas of the spectrum created by the sampling process. This is a demonstration of the bandpass sampling theorem — which states that the minimum sampling frequency required to sample a signal with no aliasing is a function of the lowest and highest frequencies for which the signal has spectral energy. In this case, the signal has a bandwidth of 1 Hz, and the lower and upper frequencies are multiples of a common factor, so the minimum sample frequency is 2 Hz.