Name: Solution Key

Problem	Points	Score
1a	10	
1b	10	
1c	10	
2a	10	
2b	10	
2c	10	
3a	10	
3b	10	
3c	10	
3d	10	
Total	100	

Notes:

1. The exam is closed books/closed notes - except for one page of notes.

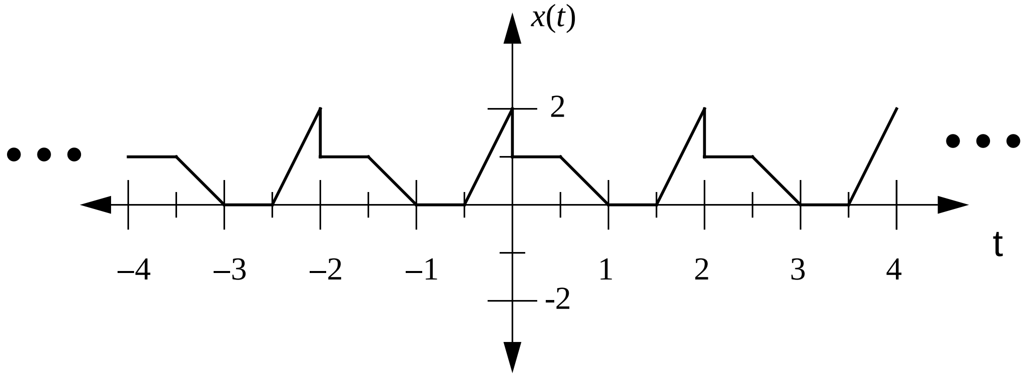
2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

3. Please indicate clearly your answer to the problem.

I hereby promise not to discuss this exam with anyone in the MWF section of EE 3133.

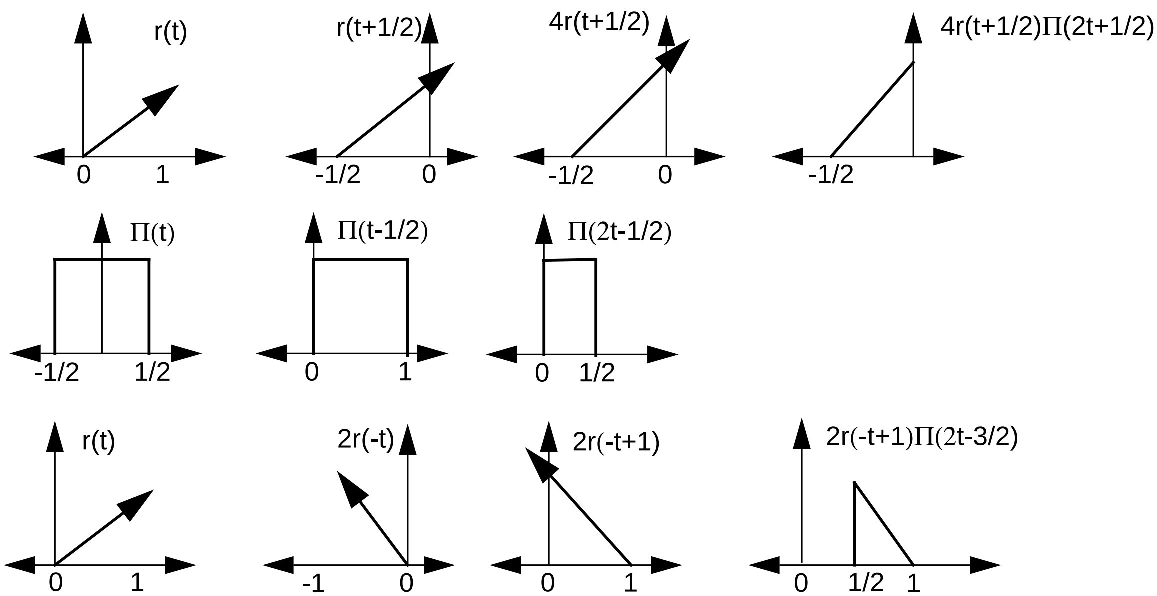
Signature: ____

Problem No. 1: Signal Models



(a) Express the waveform shown above in terms of $u(t), r(t), \Pi(t)$:

One possible solution is:



$$\therefore\ g(t) = 4r(t+\tfrac{1}{2})\Pi(2t+\tfrac{1}{2}) + \Pi(2t-\tfrac{1}{2}) + 2r(-t+1)\Pi(2t-\tfrac{3}{2})$$

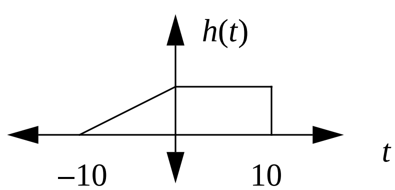
To make the function periodic,

$$x(t) = \sum_{n=-\infty}^{\infty} g(t-2)$$

(b) Is $x(t)$ an energy signal or a power signal? Explain.

It is a power signal because it is a periodic signal, and the signal is bounded in amplitude.

(c) $y(t)$ is the output of the convolution of $x(t)$ in (a) and $h(t)$:



Is $y(t)$ (circle all that apply):

(2 pts) Continuous-time Discrete-time

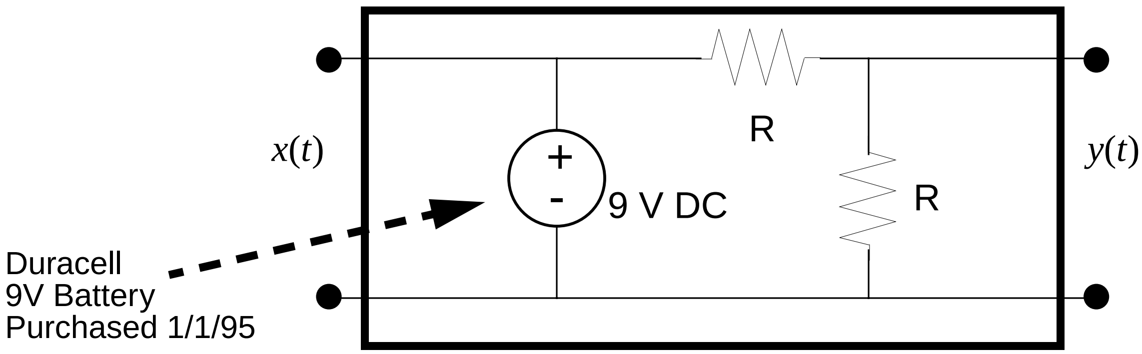
(2 pts) Continuous amplitude Quantized in amplitude

(6 pts) Periodic Aperiodic

Note: A periodic signal input to a linear system results in a periodic output.

Problem No. 2: Linear Systems

(a) Is the system shown below:



Linear? Explain.

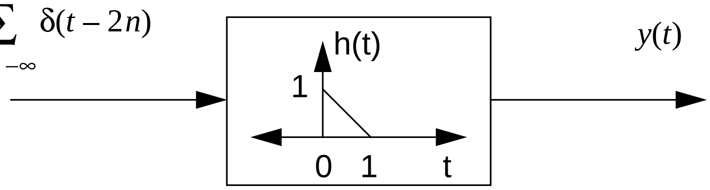
Let's assume an ideal battery. Since there is a DC voltage source inside the system, the output will not be a linear function of the input. If x(t)= 1V DC, the output is 5V DC. If x(t) = 2V DC, the output is 5.5V DC, which is not twice the output for x(t) = 1V. Therefore, the system is NOT linear.

Time-varying? Explain.

Yes. The output voltage varies with the output voltage of the internal battery. Eventually, the battery will deplete itself, and the output voltage due to the battery will decay to zero. The act of the battery draining itself constitutes a time-varying system.

(b) Find $y(t)$:

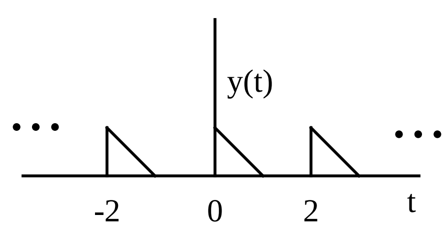
$$x(t) = \sum_{n=-\infty}^{\infty} \delta(t-2n)$$



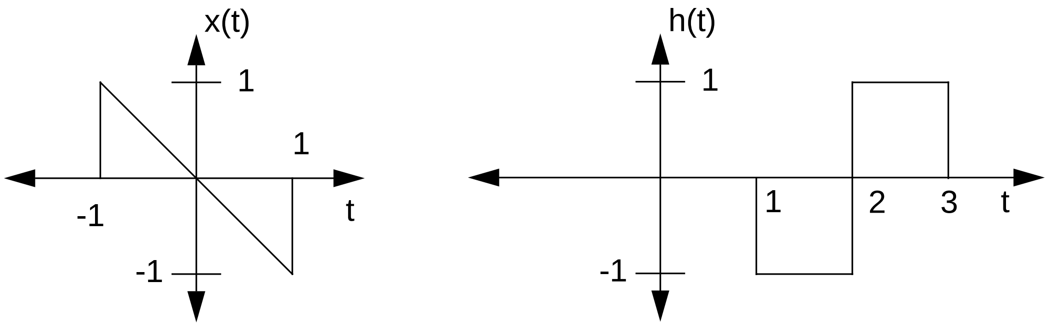
The input signal is a periodic pulse train with a period of two seconds. The output to each pulse will be the system impulse response. Hence, the output is:

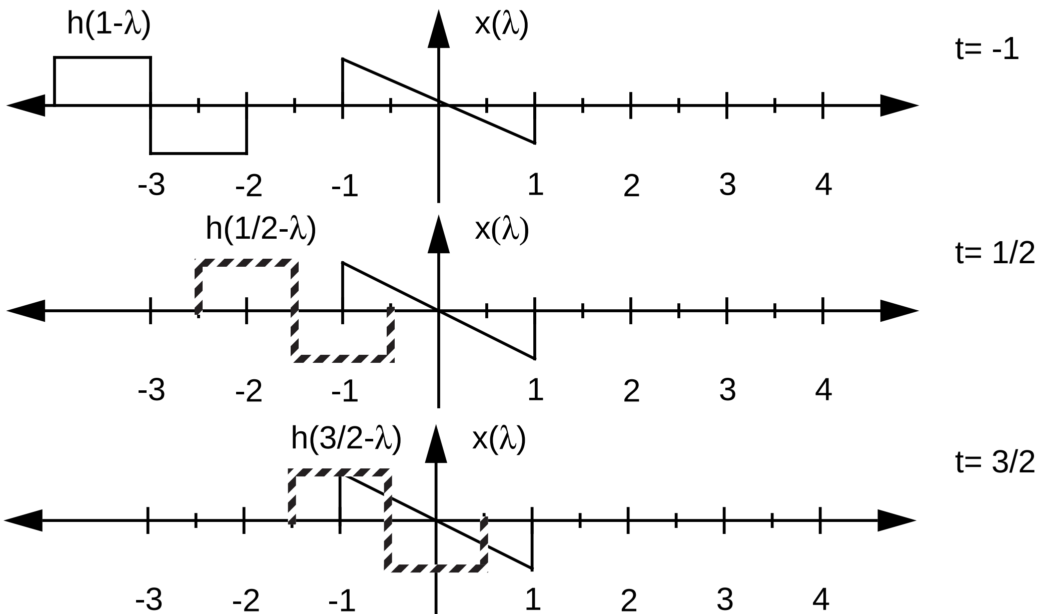
$$y(t) = \int_{-\infty}^{\infty} x(t)h(t-\lambda)d\lambda = \int_{-\infty}^{\infty}\left(\sum_{n=-\infty}^{\infty} \delta(t-2n)\right)h(t-\lambda)d\lambda$$

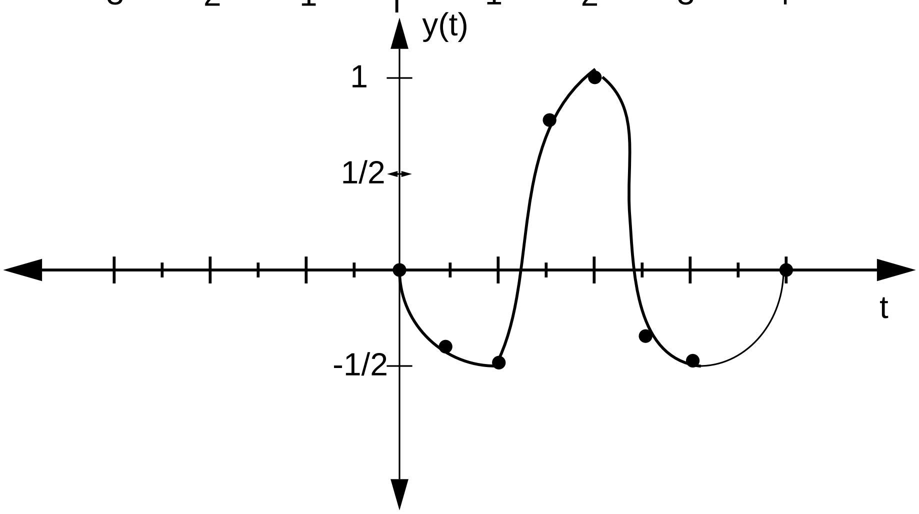
$$= \sum_{n=-\infty}^{\infty} h(t-2n)$$



(c) Sketch the output of the system show below:

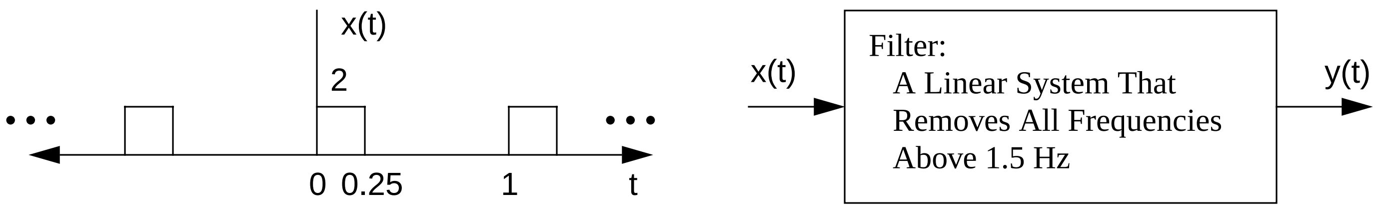






Problem No. 3: Fourier Series

For the signal and system shown below:



(a) Compute the DC value of the output:

$$DC\ Value = a_0 = \frac{1}{T}\int_0^{1/4}(1)dt = \left(\frac{1}{4}\right)(2) = \frac{1}{2}$$

(b) Compute the output y(t):

Use a Fourier Series. Note that the fundamental frequency is 1 Hz
⇨ only need to compute the first harmonic:

$$a_1 = \frac{1}{1}\int_0^{0.25}(2)\cos 2\pi t = (2)\left(\frac{1}{2\pi}\right)\sin 2\pi t\Big|_0^{0.25} = \frac{1}{\pi}\sin\frac{\pi}{2} = \frac{1}{\pi}$$

$$b_1 = \frac{1}{1}\int_0^{0.25}(2)\sin 2\pi t = (2)\left(\frac{-1}{2\pi}\right)\cos 2\pi t\Big|_0^{0.25} = \frac{1}{\pi}$$

∴ $$y(t) = \frac{1}{2} + \frac{1}{\pi}\cos 2\pi t + \frac{1}{\pi}\sin 2\pi t$$

(c) Compute the energy and power of y(t):

The output is a periodic signal, which is a power signal. Hence,

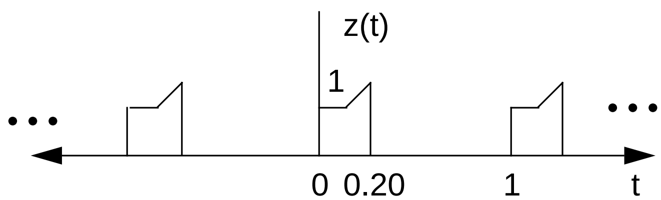
$$E = \infty$$

The power can be found from Parseval's Theorem:

$$X_1 = \frac{1}{2}(a_1 - jb_1) = \frac{1}{2}\left(\frac{1}{\pi} - j\frac{1}{\pi}\right)$$

$$|X_1| = \frac{1}{2}\sqrt{\frac{2}{\pi^2}} = \frac{1}{(\sqrt{2})\pi}$$

Therefore, $P = X_0^2 + 2|X_1|^2 = \left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{2\pi^2}\right) = \frac{1}{4} + \frac{1}{\pi^2}$



(d) Discuss the differences in the spectra of the signal shown below and x(t).

- The "duty cycle" of z(t), given by $\left(\frac{\tau}{T} = \frac{0.2}{1} = 0.2\right)$, is smaller than the duty cycle for x(t). Therefore, the rate at which the spectrum attenuates as a function of frequency, or harmonic number, will be smaller than x(t). Hence, the effective bandwidth will be larger.

- The sharp discontinuity will cause z(t) to have high frequency harmonics with larger amplitudes than x(t).

Hence, the net effect will be a line spectrum with a large bandwidth than x(t).