Frequency Domain Eigenspace-based Projection Minimum Variance for Ultrasound Imaging

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In recent years, because of safety and timeliness of the ultrasound imaging, this technology has been widely used in the field of medical diagnosis [1]. In the process of ultrasound imaging, the beamforming process

is the most important part, which directly determines the imaging quality [2]. At present, the most widely used algorithm is the traditional delay-and-sum (DAS), but DAS has some inherent disadvantages in low resolution and obviously artifacts [3]. For the purpose of solving these deficiencies, many advanced imaging methods have been proposed. Among them, the minimum variance (MV) designed by Capon is a kind of very potential algorithm due to its high resolution [4]. However, the effect of MV algorithm is mainly depended on the accuracy of the preset desired directional vector and the calculation of covariance matrix. Therefore, the MV has the problem of insufficient robustness [5]. In subsequent studies, many innovative methods had been used to overcome the shortcomings of MV algorithm [6], such as eigenspace-based MV (ESBMV).



Figure 1. The images of point targets simulation generated by (a) DAS, (b) MV, (c) ESBMV, (d) the proposed FDEBMV algorithm.

In this paper, for the propose of further enhancing the resolution and contrast of ultrasound imaging, we proposed the frequency domain eigenspace-based projection minimum variance (FDEBMV). Initial, we use the short time fourier transform method to converting time domain echoes into frequency domain. Secondly, the signal subspace in frequency domain is obtained to improve the weights of MV in frequency domain. Finally, the frequency domain output is converted to the time domain for imaging, and the imaging



Figure 2. The images of point targets simulation generated by (a) DAS, (b) MV, (c) ESBMV, (d) the proposed FDEBMV algorithm.

process is based on the synthetic aperture.

Fig. 1 is the images of point targets simulation, which were obtained by the DAS, MV, ESBMV and FDEBMV. The element number is 64, the center frequency is 7 MHz. The focus depth is set in 60mm. The element kerf is 0.1mm. The height of element is 5mm. The speed of sound is set to 1540m/s. Meanwhile, the sampling frequency is 100MHz.

From Fig. 1, the image generated by DAS has obvious sidelobe artifacts, and its resolution is the lowest. MV has some improvement in resolution, but has a few increase in sidelobe artifacts suppression. FDEBMV has the best resolution. Fig. 2 exhibits the lateral resolution curve of Fig. 1 in the depth of 50mm, the full width at half maxima in -6dB (FWHM) of each algorithm at 40,50,60 mm is recorded in Table 1.

Combining the Table 1 with the Fig. 2, we can see that DAS has the widest mainlobe and highest sidelobe, which means it has the lowest resolution and contrast.

MV has narrower mainlobe width than DAS, but less improvement in sidelobe level than DAS. ESBMV further improve the contrast compared with MV. In particular, FDEBMV can acquire the minimum



Figure 3. The images of point targets simulation generated by (a) DAS, (b) MV, (c) ESBMV, (d) the proposed FDEBMV algorithm.

able 1. The FWHM of different algorithms in	
40mm,50mm,60mm depth	

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Method	40mm	50mm	60mm
DAS	0.790	0.860	0.885
MV	0.235	0.195	0.230
ESBMV	0.255	0.215	0.245
FDEBMV	0.100	0.115	0.165

 Table 2. The anechoic cyst indexes for difference algorithms

Method	CR	CNR
DAS	30.50	2.03
MV	37.99	1.96
ESBMV	43.41	1.91
FDEBMV	47.30	2.04

mainlobe width and the best sidelobe suppression. In Table 1, the FWHM of FDEBMV in 40mm depth is improved by 87.34% and 57.45% than that of DAS and MV algorithm. It indicates FDEBMV has the best resolution.

Fig. 3 is the anechoic cyst images, which were generated by the above algorithms. The parameters set is almost the same as the ones used in point targets simulation. As is shown in Fig. 3, the images of DAS and MV have a lot of sidelobe artifacts, the sidelobe suppression ability of ESBMV and FDEBMV are better than DAS and MV. Among them, FDEBMV has the best suppression in noises and interferences.

Table 2 shows the main index of cyst simulation. In Table 2, the CR of DAS is lowest. FDEBMV can get the best CR. MV and DAS have higher CNR than ESBMV, the CNR of ESBMV is the lowest. The CR of the proposed FDEBMV is enhanced by 55.08% (16.8dB), 24.51% (9.31dB)

and 8.96% (3.89dB) than that of DAS, MV and ESBMV. In addition, FDEBMV has obvious improvement in CNR than other algorithms.

This work proposed an improved algorithm by combining the projection method with the frequency domain minimum variance algorithm. It makes the ESBMV algorithm meet the narrow-band application condition of minimum variance, so as to further enhance the resolution and contrast of the algorithm. The proposed FDEBMV has higher CR and better FWHM than other traditional algorithms. The developed method proved to be promising for the formation of ultrasound imaging, but still need further research than those used in this work.

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Frequency Domain Eigenspace-based Projection Minimum Variance for Ultrasound Imaging

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Abstract

- A frequency domain eigenspace-based projection minimum variance beamformer (FDEBMV) for ultrasound imaging is proposed in our study.
- Firstly, the ultrasound echoes are transformed into the frequency domain by short time fourier transform method.
- Secondly, we obtain frequency domain signal subspace to further optimize the final weight vector.
- The simulated results show that the proposed method can greatly improve the resolution and contrast of algorithm.
- The maximal improvements of full width at half maxima (FWHM in -6dB) is 87.34% in the point targets simulation.
- The contrast ratio (CR) of FDEBMV is improved by 24.51% and 55.08% than traditional minimum variance (MV) and delay-and-sum (DAS) beamformer respectively.

Introduction and Background

- In recent years, because of low cost safety and timeliness of the ultrasound imaging, this technology has been widely used in the field of medical diagnosis.
- In the process of ultrasound imaging, the beamforming process is the most important part, which directly determines the imaging quality.
- Minimum variance (MV) is the potential and mainstream adaptive beamformer proposed by Capon, which has attracted extensive attention because of its high-resolution performance.
- However, the effect of MV algorithm is mainly determined by the accuracy of the preset desired direction vector and the calculated sample covariance matrix. Therefore, the robustness of MV beamformer is often a problem.
- In this paper, in order to further improve the resolution and contrast of ultrasound imaging, we proposed the frequency domain eigenspace-based projection minimum variance beamformer (FDEBMV).

Method and proposed algorithm

- The basic principle of the minimum variance (MV) algorithm is to minimize the output power of the beamformer while keeping the desired directional gain unchanged.
- In theory, MV is a distortionless beamformer. However, in practice, there are often errors in the estimation of sample covariance matrix, which often leads to serious sidelobe artifacts and low contrast.
- In order to further improve the contrast and resolution, we introduce the eigenspace-based method into frequency domain.

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• The echo data of a single aperture shall be
converted as follows:
S(h, \omega) = \sum_{k=-\infty}^{\infty} x(k) z(k-h) e^{-i\omega k}
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- So the *m* narrowband signal for the *n* element can be expressed as:
 SP_{n(m)} = [S_n(m,1),...,S_n(m,ω),...,S_n(m,W)]
- Therefore, the array signal in the time-frequency point can be expressed as:

 $XS(m,\omega) = \left[S_1(m,\omega), S_2(m,\omega), \dots, S_N(m,\omega) \right]$

 The ultrasound array is divided into several subarrays, and the frequency-domain sample covariance matrix in each time-frequency point is

$$\boldsymbol{RS}(\boldsymbol{m},\boldsymbol{\omega}) = \frac{1}{N-L+1} \sum_{i=1}^{N-L+1} \boldsymbol{RS}_{i}(\boldsymbol{m},\boldsymbol{\omega})$$
$$= \frac{1}{N-L+1} \sum_{i=1}^{N-L+1} \boldsymbol{XS}_{i}(\boldsymbol{m},\boldsymbol{\omega}) \boldsymbol{XS}_{i}(\boldsymbol{m},\boldsymbol{\omega})^{\mathrm{H}}, L \leq \frac{N}{2}$$

The diagonal loading method is used to increase the robustness

 $RS_{DL}(m,\omega) = RS(m,\omega) + \eta I$

The RS_{DL} is decomposed into signal subspace and noise subspace, which is as follows

 $RS_{DL}(m, w) = FE_{s}FA_{s}FE_{s}^{H} + FE_{n}FA_{n}FE_{n}^{H}$

$$=\sum_{i=1}^{L}\lambda_{i}\boldsymbol{f}\boldsymbol{e}_{i}\boldsymbol{f}\boldsymbol{e}_{i}^{H}+\sum_{i=\gamma+1}^{L}\lambda_{i}\boldsymbol{f}\boldsymbol{e}_{i}\boldsymbol{f}\boldsymbol{e}_{i}^{H}$$

• The output of the frequency domain minimum variance beamformer can be expressed as:

$$\boldsymbol{w}_{FDS-MV} = \frac{\boldsymbol{RS}_{DL}(\boldsymbol{m}, \boldsymbol{\omega})^{-1} \boldsymbol{fa}}{\boldsymbol{fa}^{\mathrm{H}} \boldsymbol{RS}_{DL}(\boldsymbol{m}, \boldsymbol{\omega})^{-1} \boldsymbol{fa}}$$

 We project the w into the modified frequency domain signal subspace

 $\boldsymbol{w}_{FDEBMV} = \boldsymbol{MFE}_{s}(\boldsymbol{m}, \boldsymbol{w}) \boldsymbol{MFE}_{s}(\boldsymbol{m}, \boldsymbol{w})^{H} \boldsymbol{w}_{FDS-MV}$

 The final output of beamformer in frequency domain is:

$$\boldsymbol{y}_{FDEBMV}(\boldsymbol{m},\boldsymbol{\omega}) = \frac{1}{N-L+1} \sum_{l=1}^{N-L+1} \boldsymbol{w}_{FDEBMV}^{H} \boldsymbol{X} \boldsymbol{S}_{l}(\boldsymbol{m},\boldsymbol{\omega})$$

 Finally, we need transform the frequency domain output to time domain:

 $\mathbf{y}_{FDEBMV}(k) = ISTFT(\mathbf{y}_{FDEBMV}(m,\omega))$

$$=\frac{1}{2\pi}\sum_{m=-\infty}^{\infty}\sum_{\omega=-\infty}^{\infty}\mathbf{y}_{FDEBMV}(m,\omega)\mathbf{e}$$

Result and discussion

- The element number is 64, the center frequency is 7 MHz and the sampling frequency is 100MHz. The focus depth was set in 60mm.
- The element kerf is 0.1mm. The height of elements is 5mm. The speed of sound was set to 1540m/s.
- The images of point targets simulation were shown as follow, which were obtained by the DAS, MV, ESBMV and FDEBMV.
- The image generated by DAS has obvious sidelobe artifacts, and its resolution is the lowest.



The lateral resolution curves of point target simulation in the depth of 50mm are shown as follows:



- DAS has the widest mainlobe and highest sidelobe, which means it has the lowest resolution and contrast.
- MV has narrower mainlobe width than DAS, but less improvement in sidelobe level than DAS.
- ESBMV further improve the contrast compared with MV.
- In particular, FDEBMV has the narrowest mainlobe width and lowest sidelobe level, which indicate FDEBMV has the best resolution than other traditional agorithms.
- The full width at half maxima in -6dB(FWHM) of each algorithm at 40,50,60mm as shown below:

Method	40mm	50mm	60mm
DAS	0.790	0.860	0.885
MV	0.235	0.195	0.230
ESBMV	0.255	0.215	0.245
FDEBMV	0.100	0.115	0.165

- FDEBMV has the lowest FWHM in all depth.
- The anechoic cyst images obtained by the DAS, MV, ESBMV and FDEBMV were shown as follows:



 The images of DAS and MV have a lot of sidelobe artifacts, the sidelobe suppression of ESBMV and FDEBMV are better than that of DAS and MV. Among them, FDEBMV has the best suppression in noises and interferences.

Method	CR	CNR
DAS	30.50	2.03
MV	37.99	1.96
ESBMV	43.41	1.91
FDEBMV	47.30	2.04

 FDEBMV has the best CR. Meanwhile, FDEBMV has obvious improvement in CNR compared with DAS, MV and ESBMV.

Summary

- This work proposed an improved algorithm by combining the eigenspace-based method with the frequency domain minimum variance beamformer.
- It makes the ESBMV algorithm meet the narrow-band application condition of minimum variance, so as to further improve the resolution and contrast of the algorithm.
- Compared with traditional DAS, MV, and ESBMV, the proposed FDEBMV has higher CR and better FWHM.
- The developed method proved to be promising for the formation of ultrasound imaging, but still need further research than those used in this work.

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