The Effect of Graph Frequencies on Dynamic Structures in Graph Signal Processing

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IEEE SPMB, December 3, 2022

Principal idea

- Use graph (spatial) structure to understand multivariate signals
- *Multivariate* signals:
  - e.g., neurophysiological signals such as:
    - electroencephalography (EEG), magnetoencephalography (MEG), ...
  - multiple time signals from various sensors
  - shape: # sensors × # time samples
  - dynamic structure crucial for EEG (1):
    - alpha-waves, beta-waves etc.
  - spatial structure (2):
    - connections between sensors → correlations in data
    - commonly disregarded for data classification
- investigate novel method: *Graph Signal Processing* (GSP) (3; 4; 5)

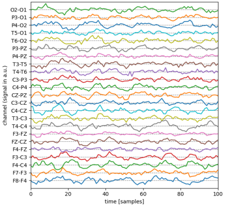


Figure: Multivariate EEG signal

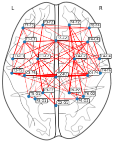
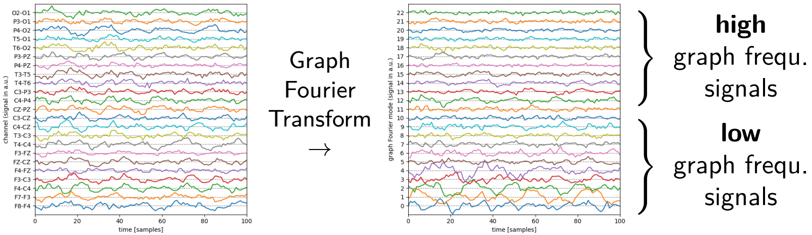


Figure: EEG spatial structure: correlations between channels

Contents

Graph Signal Processing (GSP)

- Retrieve adjacency matrix $\mathbf{A}$ of graph structure:
  - e.g., $\mathbf{A}_{ij}$ as pairwise correlation between channels i and j
- *Graph Fourier Transform* (GFT) (6; 7; 8):
  - project multivariate signal $\mathbf{X}$ onto *graph Fourier modes* $\mathbf{v}_i$ (= linear spatial transformation, $\tilde{\mathbf{X}} = [\mathbf{v}_1, ..., \mathbf{v}_N]^\top \mathbf{X}$)
  - modes $\mathbf{v}_i$ are eigenvectors of *Laplacian matrix* $\mathbf{L} = \deg(\mathbf{A}) - \mathbf{A}$
  - modes can be ordered by their eigenvalue: "low graph Fourier mode", ...
  - transforms N time signals to N ordered *graph frequency signals*
- GFT + signal processing = GSP



Simulated data

- Necessary due to sparsity of data
- Simulate multivariate time signals with:
  - spatial structure: control correlation between pairwise channels
  - spectral structure: control power spectral density
- Control of spatial structure:
  - use random, fixed adjacency matrix A as graph shift operator
    - shifts sensor configuration from one time step to the next
    - enforces spatial structure in simulated data (see Figure)

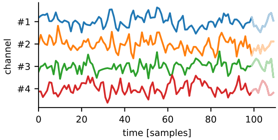


Figure: Simulated time series

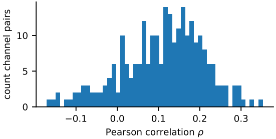


Figure: Spatial structure: correlations between channel pairs

Simulated data

- Necessary due to sparsity of data
- Simulate multivariate time signals with
  - spatial structure: control correlation between pairwise channels
  - spectral structure: control power spectral density
- Control of spectral structure:
  - add coloured noise (before performing shift)
  - coloured noise profiles allow to simulate conditions
  - noise profiles of two conditions closer to each other:
    - data harder to classify

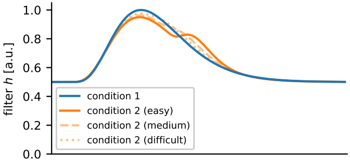


Figure: Filters used to colour the noise

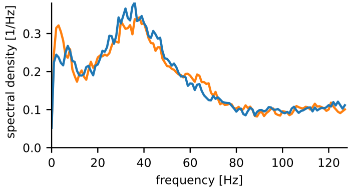


Figure: Noise profile of simulated data for two conditions

Experiment

- Goal: test whether dynamic structures are preserved / amplified after GFT *depending on the graph frequency*
  $\rightarrow$ Compare graph frequency signals between each other
- Measure dynamic structures for each graph frequency signal indirectly as:
  - ability to train a classifier on dynamic structures of signal
  - dynamic structures (spectral features): derived from *power spectral density (Welch's method)*
  - classifier: *support vector machine*
- Accuracy of classifier as a proxy for dynamic structures in transformed data!
- Compare low with high graph frequencies
- Compare low / high frequencies to no GSP (baselines)

Baseline framework

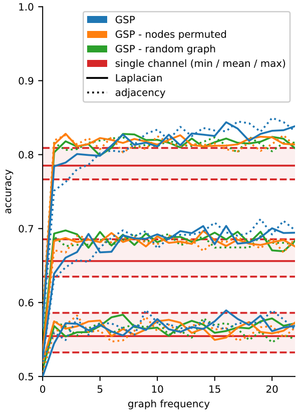
- Goal: isolate effect of using graph on classification accuracy
- Baseline 1: permuted graph
  - GSP
  - same weights
  - invalid graph structure
- Baseline 2: random graph
  - GSP
  - not same weights
  - invalid graph structure
- Baseline 3: identity transformation (= no transformation)
  - no GSP

↓ less use of graph structure

Contents

Results

- Reminder: classification accuracy as proxy for spectral features in signals
- Results for 3 different classification difficulties
- (Dotted lines: variation of GSP)
- low graph frequency < high graph frequency
  → no amplification of spectral features when mixing nearby channels
- model ≈ baseline permuted & baseline random
  → no apparent advantage of GSP
- single channel (no GSP) < all others
  → mixing of channels accumulates information



Conclusion

- Application of GSP for neurophysiological data may be limited:
  - GFT mixes channels
  - interference presumably impairs dynamic structures
- Higher graph frequencies better preserved dynamic structures
- Baseline framework:
  - crucial for validating application of GSP
  - results showed necessity of using several baselines:
    - single channel baseline “failed”

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