Optimization of Transcostal Phased-array Refocusing Using Iterative Sparse Semidefinite Relaxation Method

D. McMahon, M. Almekkawy
School of Electrical Engineering and Computer Science, Penn State University
{dfm5128, mka9}@psu.edu

High Intensity Focused Ultrasound (HIFU) is a means of noninvasively treating cancer by focusing an array of ultrasound transducers onto a focal tumor. The use of large transducer arrays allows for a high focal power deposition, without necessitating undue heating in the intermediary tissue between the transducer array and the target. However, organs that are directly obscured by the ribs present a challenge for HIFU treatment. The ribs distort the ultrasound waves, while also absorbing their energy, causing them to heat and potentially burn the surrounding tissue. As a result of the ribs' presence, the effectiveness of HIFU treatments is limited.

As a result of the challenges presented by the ribs, it is necessary to develop new focusing algorithms. In this project, we endeavor to modify an existing focusing method, the Limited Power Deposition (LPD) method [1], by introducing a sparsity inducing term. The presence of the sparsity inducing term in the objective function of the optimization problem will eliminate some elements from consideration during focusing.

There are numerous advantages to this modification of the problem statement. For example, by removing elements that would otherwise be used for HIFU treatment, we allow them to be used for other purposes, such as motion tracking, in parallel with the treatment. Accordingly, organs that are constantly in motion, like the liver, can receive an improved treatment that is able to remain focused on the moving target with the aid of the motion tracking. Additionally, by removing low efficacy elements from the treatment, the total energy consumed during treatment will be lowered, providing benefits in scenarios like field treatment administration where energy consumption is a constraint.

In this paper, the LPD method, which utilizes semidefinite relaxation to relax non-convex focal constraints into convex form, is modified by appending a sparsity inducing term to the objective function [2], [3]. The sparse algorithm utilizes the one-norm squared as a convex surrogate for the zero-norm. Additionally, the sparsity inducing term will feature an iteratively reweighted matrix that allows for further sparsity induction [4]. The elements of the matrix will be recalculated after each round of sparsity induction so that they are inversely proportional to the magnitude of the excitation of their corresponding transducer element during the previous iteration. Accordingly, in subsequent iterations the cost of using elements that had low excitation will be prohibitively high, driving the excitation of these elements to zero over time.

As a result of the influence of the sparsity inducing term on the objective function, the solution provided during the final iteration of sparsity induction is not optimal. Accordingly, after sparsity induction is complete, the original LPD method, which utilizes its objective function to minimize the magnitude of the array excitation, will be run using only the elements that were left on during sparsity induction. This step removes the influence that the sparsity inducing term had on the elements that remained on, while still retaining the sparsity that it provided.

Simulation results exploring the method’s potential are obtained by using the wave equation and the bio-heat transfer equation. The simulations use a finite-difference time domain method to model the acoustic and thermal properties of the region of interest. The simulation results are then compared to the ray tracing method, a currently known sparsity inducing focusing algorithm [5].

Using iterative sparsity induction, 23 of the elements were switched off. The resulting excitation vector led to a higher focal gain than the ray tracing (shadowing) method. Additionally, the power deposition in the rib plane using the new method was generally lower than the ray tracing method, lowering the risk of burns in this region. The sparse LPD method led to significantly lower heat deposition on the intercostals, reducing the risk of these unprotected regions developing burns.

REFERENCES

[1] M. K. I. Almekkawy. Optimization of focused ultrasound and image based modeling in image guided interventions.

[2] N. D. Sidiropoulos, T. N. Davidson, and Z.-Q. Luo. Transmit beamforming for physical-layer multicasting. IEEE Transactions on Signal Processing, 54(6):2239–2251, 2006.

[3] E. Karipidis, N. D. Sidiropoulos, and Z.-Q. Luo. Quality of service and max-min fair transmit beamforming to multiple cochannel multicast groups. IEEE Transactions on Signal Processing, 56(3):1268–1279, 2008.

[4] O. Mehanna, N. D. Sidiropoulos, and G. B. Giannakis. Multicast beamforming with antenna selection. In Signal Processing Advances in Wireless Communications (SPAWC), 2012 IEEE 13th International Workshop on, pages 70–74. IEEE, 2012.

[5] M. K. Almekkaway, I. A. Shehata, and E. S. Ebbini. Anatomical-based model for simulation of HIFU-induced lesions in atherosclerotic plaques. International Journal of Hyperthermia, 31(4):433–442, 2015.

PennState
College of Engineering

Optimization of Transcostal Phased-array Refocusing Using Iterative Sparse Semidefinite Relaxation Method

Daniel McMahon, Mohamed Almekkawy,

Department of Computer Science and Engineering, Penn State University, University Park, PA

Motivation and Objective

A 64-element ultrasound array can be used for noninvasive therapeutic purposes when focused onto a cancerous target. The objective during these treatments is to expose the target to ample heating, while avoiding exposing the intermediary tissue between the target and the ultrasound array to unnecessary heat deposition. While, inclusion of all 64 elements during treatment leads to a higher focal power deposition, it also occupies the entire transducer array, preventing it from being used for other processes, like motion tracking, in parallel with the High Intensity Focused Ultrasound (HIFU) treatment. Furthermore, inclusion of all available transducer elements leads to a higher power consumption, which can present difficulties in situations, like field treatment administration, where energy is a constraint. Previously researched methods, like the shadowing (ray tracing) method, have sought to focus a reduced sized array by eliminating a predetermined group of transducer elements from consideration. Our goal is to implement an iterative optimization based approach for element selection to determine the most effective reduced size transducer array for targeting a cancer cell that is partially obscured by the ribs. Utilizing optimization will ensure that the most vital elements remain on during treatment. By doing this, the benefits that accompany sparse methods, like the shadowing method, can be obtained without severely reducing the treatment efficacy.

Materials and Method

The focusing of the array is carried out in two steps. First, the subset of elements that are most vital to the treatment are selected using an iterative optimization based approach, where the one-norm squared is used as a convex surrogate for the zero-norm. The form of this problem was achieved using semidefinite relaxation to relax an otherwise nonconvex constraint on the requisite power at the target into convex form. Here, Z is an iteratively reweighted matrix. The elements of Z during each iteration are inversely proportional to the magnitude of their corresponding element in B during the previous iteration, forcing low magnitude excitation transducers off over time. After a solution is found to the previously described optimization problem, the elements corresponding to a zero on the diagonal of B are removed from consideration. In order to eliminate the effect of the one-norm squared on the solution, the original limited power deposition method (shown below) is run using only the subset of elements left on during sparsity induction. The result is the minimum power excitation vector that is capable of successfully delivering the requisite power to the target, while protecting the sensitive rib points.

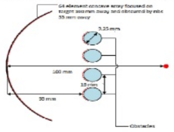
$$
\begin{aligned}
\min_{\mathbf{u}\in\mathbb{C}^N} \quad & \|\mathbf{u}\|_2^2 \\
\text{s.t.}: \quad & |\mathbf{u}^{\mathcal{H}}\mathbf{h}_{ti}|^2 \geq \mathbf{p}_{ti}, \quad i = 1,\ldots,L \\
& |\mathbf{u}^{\mathcal{H}}\mathbf{h}_{rj}|^2 \leq \mathbf{p}_{rj}, \quad j = 1,\ldots,M
\end{aligned}
$$

Shown below is the optimization problem used to achieve sparsity. However, the solution to this problem cannot be used for focusing because it does not minimize the power consumption of the array. Therefore, it is necessary to replace the objective function of this problem with that of the original LPD, once sparsity has been achieved. Doing so will retain the benefits of the sparsity inducing terms, without keeping their influence on the focusing pattern used for treatment.

$$
\begin{aligned}
\min_{\mathbf{A}\in\mathbb{C}^{N\times N}} \quad & tr(Z^{(r)}B^{(r)}) \\
\text{s.t.}: \quad & tr(\mathbf{A}^{(r)}\mathbf{Q}_i) \geq \mathbf{p}_{ti}, \quad i = 1,\ldots,L \\
& tr(\mathbf{A}^{(r)}\mathbf{F}_j) \leq \mathbf{p}_{rj}, \quad j = 1,\ldots,M \\
& \mathbf{A}^{(r)} \succcurlyeq 0 \\
& \begin{bmatrix} B(i,j) - \Re(A(i,j)) & \Im(A(i,j)) \\ \Im(A(i,j)) & B(i,j) - \Re(A(i,j)) \end{bmatrix} \succcurlyeq 0
\end{aligned}
$$

Simulation Setup

Simulations were performed with a 64-element phased-array with a frequency of 1 MHz that was focused upon a target obscured by 4 ribs.



The acoustic simulations had a grid spacing of 0.15 mm, which is one-tenth of the wavelength in water. After the sparsity induction was concluded 23 elements were removed. The heat simulations had a transducer excitation duration of 5 seconds. The model is based upon the bioheat transfer and wave equations shown below. Additionally, the tissue medium and ribs are assumed to have densities of 1090 and 1800 (kg/m^3), specific heats of 3540 and 1300 (J/kg ⁰ C), thermal conductivities of 0.52 and 0.32 (Wm^-1/⁰C), speed of sounds of 1500 and 2800 (m/s), and attenuations of .94 and 4 (dB/cm/MHz), respectively.

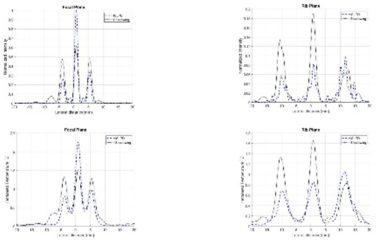
$$
\frac{\partial \mathbf{v}}{\partial t} = \frac{-1}{\rho}\nabla p
$$

$$
\frac{\partial p}{\partial t} = -\rho.c^2\nabla.\mathbf{v} + \alpha p
$$

$$
\rho C_t\frac{\partial T}{\partial t} = k\nabla^2 T - W_b C_b (T - T_b) + Q
$$

Results and Discussion

The field intensity in the rib plane as a whole is less for the iterative sparse LPD than for its shadowing counterpart. Furthermore, the heat rise at the intercostals of the rib plane is significantly reduced in the case of the sparse LPD method. Despite this, the sparse LPD method achieves a higher focal field intensity and temperature rise, while generally reducing the field intensity and resulting temperature rise in other areas of the focal plane. The combination of the higher focal gain and lower rib power deposition indicate that the algorithm proposed in this paper outperforms the ray tracing technique, which removes elements shadowed by the ribs. The relative advantages achieved by the sparse LPD method over the shadowing method indicate that it more appropriately utilizes the degrees of freedom (HIFU transducer selection) while satisfying the optimization constraints.



Conclusion

A new technique for transducer element removal that built upon the LPD method and iteratively used the one-norm squared as a surrogate element counter was introduced in this paper. Using the one-norm squared, it was possible to pose the problem of element selection in a form that could be solved using convex optimization, to preserve the most important transducer elements for safe and effective HIFU cancer therapy. The simulation results achieved during this trial further validate the use of the one-norm squared as a favorable substitute for the ray tracing method for element removal. By comparing the sparse LPD to the shadowing technique, it was found that the utilization of convex optimization produced a more favorable subset of elements to use during treatment. By accounting for the effects of diffraction and interference, the algorithm was able to remove the elements that deposited energy with the least therapeutic benefit. Accordingly, the method was able to achieve a higher focal intensity and therapeutic gain than its ray tracing counterpart.

References

1. P. Gelat, G. Ter Haar, N. Saffari. "The optimization of acoustic fields for ablative therapies of tumors in the upper abdomen," Physics in medicine and biology, vol. 57, no. 24, pp. 8471. 2012.
2. *E. Ebbini and C. Cain, "Multiple-focus ultrasound phased-array pattern synthesis: optimal driving-signal distributions for hyperthermia," Ultrasonics, Ferroelectrics and Frequency Control, IEEE Transactions on, vol. 36, no. 5, pp. 540–548, Sept 1989.*
3. *S. Bobkova, L. Gavrilov, V. Khokhlova, A. Shaw, and J. Hand, "Focusing of high intensity ultrasound through the rib cage using a therapeutic random phased array." Ultrasound in medicine & biology, vol. 36, no. 6, pp. 888–906, 2010.*
4. M. Almekkawy, "Optimization of Focused Ultrasound and Image Based Modeling in Image Guided Interventions," University of Minnesota, 2014.
5. O. Mehanna, N. Sidiropoulos, G. Giannakis, "Multicast beamforming with antenna selection," Signal Processing Advances in Wireless Communications (SPAWC), 2012 IEEE 13th International Workshop on. pp. 70-74, 2012.
6. S. P. Boyd and L. Vandenberghe, Convex optimization. Cambridge university press, 2004.
7. E. Candes, M. Wakin, S. Boyd. "Enhancing sparsity by reweighted l minimization." Journal of Fourier analysis and applications, vol. 14, no. 5-6, pp. 877-905, 2008.
8. N. D. Sidiropoulos, T. N. Davidson, and Z.-Q. Luo, "Transmit beamforming for physical-layer multicasting," Signal Processing, IEEE Transactions on, vol. 54, no. 6, pp. 2239–2251, 2006.
9. P. Tseng, "Further results on approximating nonconvex quadratic optimization by semidefinite programming relaxation," SIAM Journal on Optimization, vol. 14, no. 1, pp. 268–283, 2003.
10. M. K. Almekkaway, I. A. Shehata, E. S. Ebbini, Anatomical based model for simulation of HIFU induced lesions in atherosclerotic plaques, International Journal of Hyperthermia, Taylor & Francis, vol. 31, no. 4, pp. 433-442, 2015.