K-complex Detection using Sparse Optimization

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This work describes a method using sparse optimization for the detection of K-complex in sleep EEG. K-complex is an important feature in sleep stage identification, which is helpful to sleep disorder diagnostics process. In this work, a discrete-time sleep EEG signal $\mathbf{y} \in \mathbb{R}^N$ is modeled as:

$$\mathbf{y} = \mathbf{x}_1 + \mathbf{x}_2 + \mathbf{w}, \tag{1}$$

where $\mathbf{x}_1$ is the baseline trend, $\mathbf{x}_2$ is composed of K-complexes, and $\mathbf{w}$ presents noise. More specifically, $\mathbf{x}_1$ is piecewise smooth comprising a lowpass signal component $\mathbf{f}$, and a sparse order-K_1 derivative component $\mathbf{g}_1$, i.e., $\mathbf{x}_1 = \mathbf{f} + \mathbf{g}_1$. Further, $\mathbf{x}_2$ is assumed as a transient waveform with a negative wave followed by a positive wave, and modeled as a 'wavelet' (e.g. Fig. 1(b)). Moreover, $\mathbf{x}_2$ is modeled as the output of a high-pass filter, i.e., $\mathbf{x}_2 = \mathbf{H}_2\mathbf{g}_2$. In addition, we assume the order-K_1 derivative of $\mathbf{g}_1$ is sparse, and we likewise assume the order-K_2 derivative of $\mathbf{g}_2$ is sparse. In another word, $\mathbf{g}_1$ and $\mathbf{g}_2$ are sparse-derivative signals, where $\mathbf{u}_1 = \mathbf{D}_1\mathbf{g}_1$, and $\mathbf{u}_2 = \mathbf{D}_2\mathbf{g}_2$ are both sparse. Adopting the zero-phase filter design techniques discussed in Ref. [3], and the idea of morphological component analysis (MCA) [4], we formulate the optimization problem:

$$\{\mathbf{u}_1^*, \mathbf{u}_2^*\} = \arg\min_{\mathbf{u}_1, \mathbf{u}_2} \frac{1}{2}\left\|\mathbf{H}_1\mathbf{y} - \mathbf{A}_1^{-1}\mathbf{B}_1\mathbf{u}_1 - \mathbf{A}_2^{-1}\mathbf{B}_2\mathbf{u}_2\right\|_2^2 + \lambda_1 \sum_n \rho_1([\mathbf{u}_1]_n) + \lambda_2 \sum_n \rho_2([\mathbf{u}_2]_n). \tag{2}$$

where ρ_1 and ρ_2 denote penalty functions. The high-pass filters are expressed as $\mathbf{H}_1 = \mathbf{A}_1^{-1}\mathbf{B}$, $\mathbf{H}_2 = \mathbf{A}_2^{-1}\mathbf{B}$, with $\mathbf{B} = \mathbf{B}_1\mathbf{D}_1 = \mathbf{B}_2\mathbf{D}_2$. Using the solution from (2), we recover $\mathbf{x}_1$ and $\mathbf{x}_2$ by:

$$\hat{\mathbf{x}}_1 = \mathbf{y} - \mathbf{H}_1\mathbf{y} + \mathbf{A}_1^{-1}\mathbf{B}_1\mathbf{u}_1^*, \quad \hat{\mathbf{x}}_2 = \mathbf{A}_2^{-1}\mathbf{B}_2\mathbf{u}_2^*. \tag{3}$$

Problem (2) both decomposes the data $\mathbf{y}$ into $\mathbf{x}_1$ and $\mathbf{x}_2$, and performs denoising. It can be solved iteratively by majorization-minimization (MM) [2]. Further, the proposed algorithm is computationally efficient as it makes use of banded matrices. We use an asymmetric penalty function to capture the morphology of K-complex, and implement a simple detector by thresholding the local energy of $\mathbf{x}_2$. We test the proposed method by the public dataset collected in [1]. It achieves a better accuracy (F-measurement) than the result reported in [1]. An example is illustrated in Fig. 2.

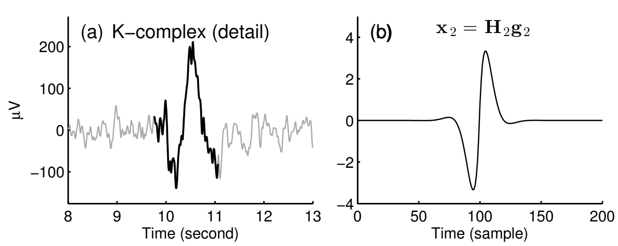

Fig. 1. (a) K-complex detail, (b) transient component signal model.

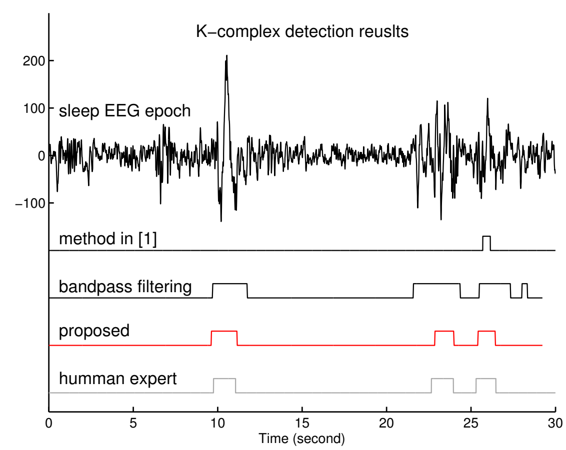

Fig. 2. Comparison of detection results: method proposed in [1], bandpass filtering, our proposed method, expert.

References

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